# Linear Block Codes

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1. Generator and Parity check Matrices
2. Encoding circuits

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3. Syndrome and Error Detection
4. Minimum Distance Considerations

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11. Product codes and Interleaved codes

# Introduction to Linear Block Codes

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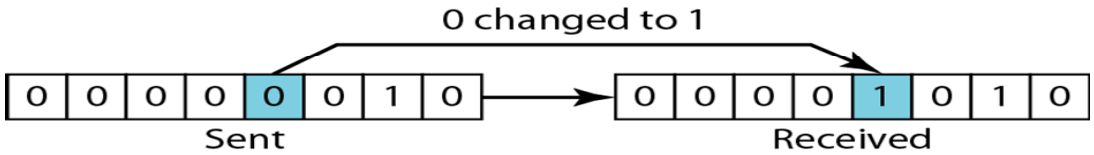

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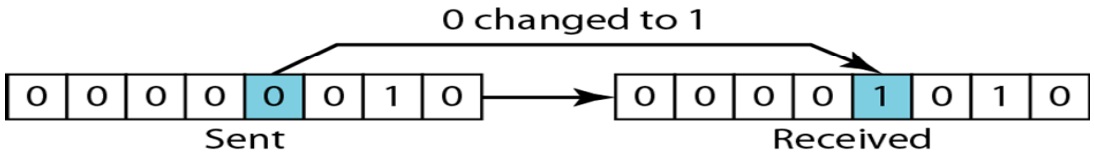


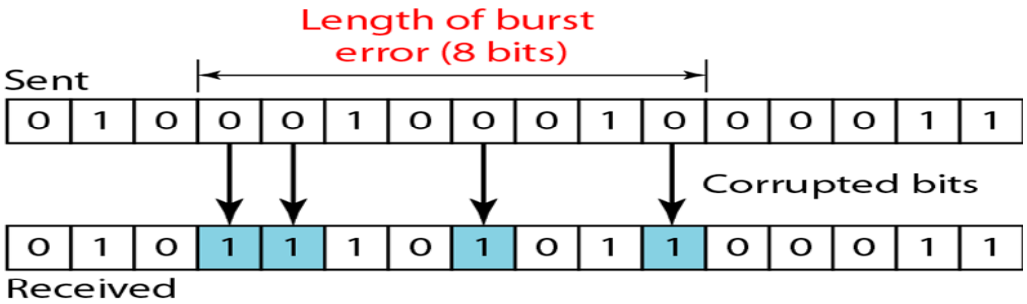

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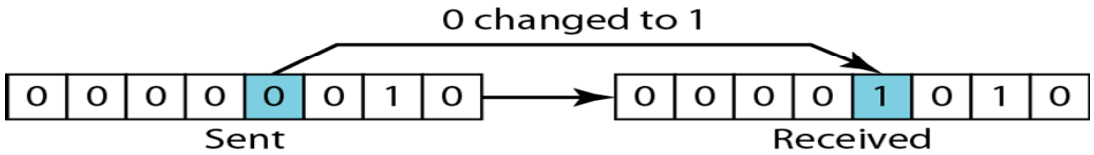


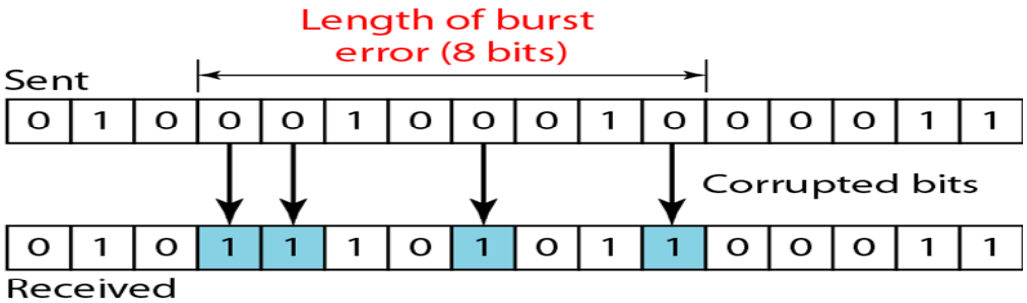


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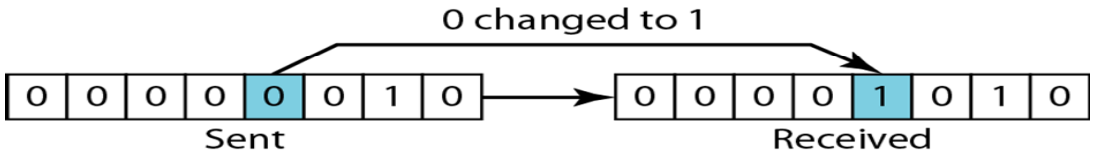


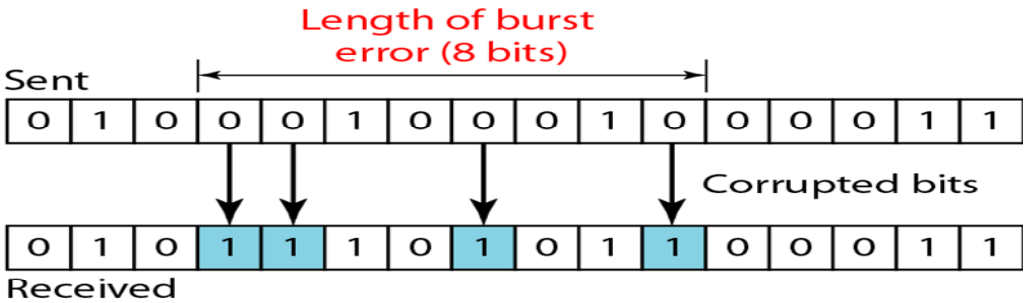


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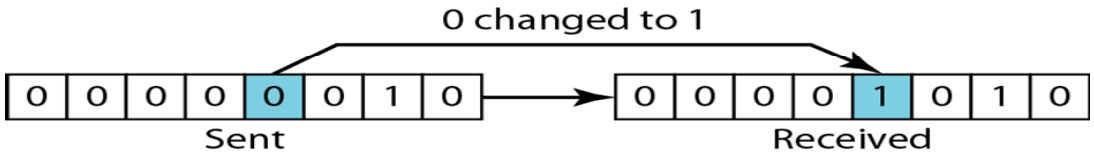


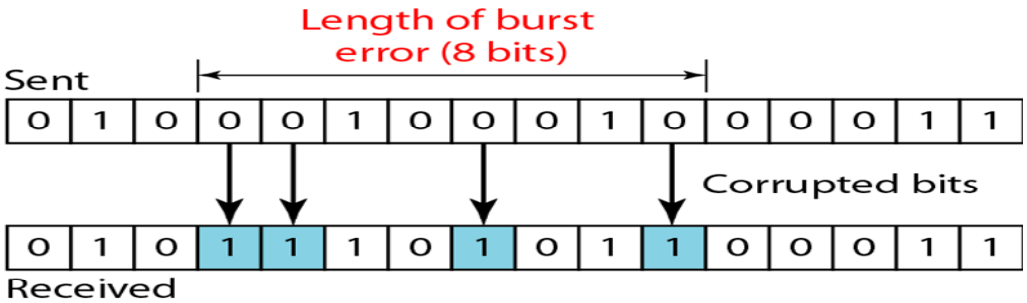


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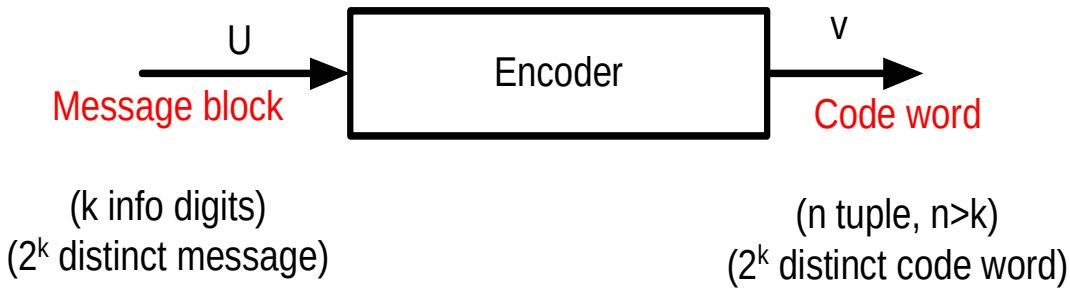


Figure: The encoder

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| Message | Codewords |
|---|---|
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| (1 0 0 0) | (1 1 0 1 0 0 0) |
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| (1 1 0 0) | (1 0 1 1 1 0 0) |
| (0 0 1 0) | (1 1 1 0 0 1 0) |
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| (1 1 1 0) | (0 1 0 1 1 1 0) |
| (0 0 0 1) | (1 0 1 0 0 0 1) |
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where $g_i = (g_{i0}, g_{i1}, \ldots g_{in-1})0$ or 1 for $0 \leq i < k$

If $u = (u_0, u_1, \ldots, u_{k-1})$ is the message to be encoded, the corresponding code word

$$v = U.G = \begin{pmatrix} u_0, & u_1, \ldots, & u_{k-1} \end{pmatrix} . \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{k-1} \end{bmatrix} \tag{3}$$

$v = u_0 g_0 + u_1 g_1 + \ldots u_{k-1} g_{k-1}$

- Because the rows of G generate the (n, k) linear code C, the matrix G is called a generator matrix for C
- Note that any k linearly independent code words of an (n, k) linear code can be used to form a generator matrix for the code
- It follows from (3.3) that an (n, k) linear code is completely specified by the k rows of a generator matrix G

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- =1.(1 1 0 1 0 0 0)+ 1.(0 1 1 0 1 0 0)+ 0.(1 1 1 0 0 1 0)+1. (1 0 1 0 0 0 1)
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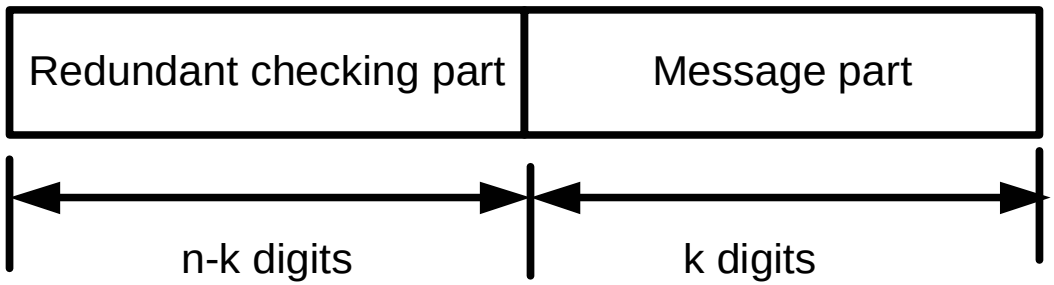


Figure: Systematic format of a codeword

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$$= \underbrace{\begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0,n-k-1} \\ p_{10} & p_{11} & \cdots & p_{1,n-k-1} \\ p_{20} & p_{21} & \cdots & p_{2,n-k-1} \\ \vdots & & & \\ p_{k-1,0} & p_{k-1,1} & \cdots & p_{k-1,n-k-1} \end{bmatrix}}_{P \quad Matrix} \underbrace{\begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ & & & & \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix}}_{kxk \quad Identity \quad Matrix} \tag{4}$$

where $p_{ij} = 0$ or 1

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- An (n, k) linear code is completely specified by its parity check

$$v.H^T = (v_0, v_1, \ldots v_{n-k-1}, u_0, u_1, \ldots u_{k-1}).H^T = 0$$

$$H^T = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \ldots & 1 \\ p_{00} & p_{01} & p_{02} & \cdots & p_{0,n-k-1} \\ p_{10} & p_{11} & p_{12} & \cdots & p_{1,n-k-1} \\ \vdots & & & & \\ p_{k-1,0} & p_{k-1,1} & p_{k-1,2} & \cdots & p_{k-1,n-k-1} \end{bmatrix}$$

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$$v_0(1) + v_1(1) + v_{n-1}(1) + u_0(p_{00}\ p_{01}\ \ldots\ p_{0,n-k-1}) +$$
$$+u_1(p_{10}\ p_{11}\ p_{12} \ldots p_{1,n-k-1}) + u_2(p_{20}\ p_{21}\ p_{22} \ldots p_{2,n-k-1})$$
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$$v_j + u_0 p_{0j} + u_1 p_{1j} \ldots + u_{k-1} p_{k-1j} = 0 \qquad \text{for } 0 \leq j < n-k$$

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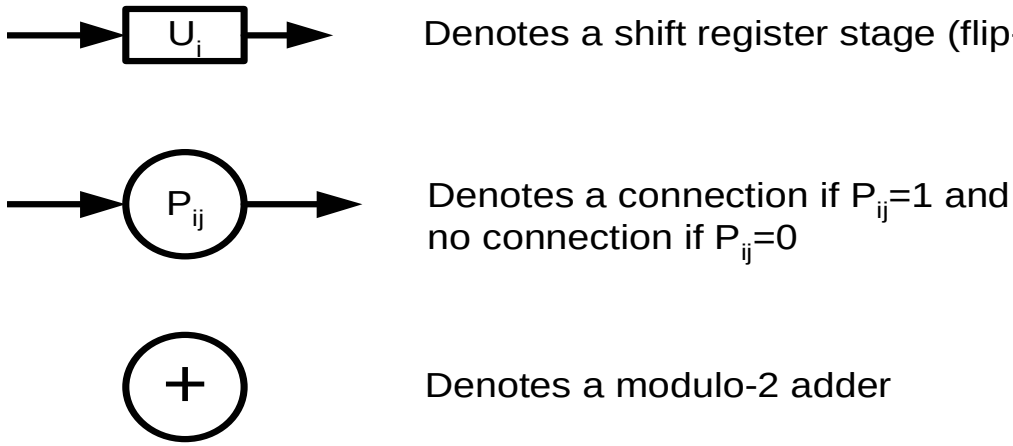

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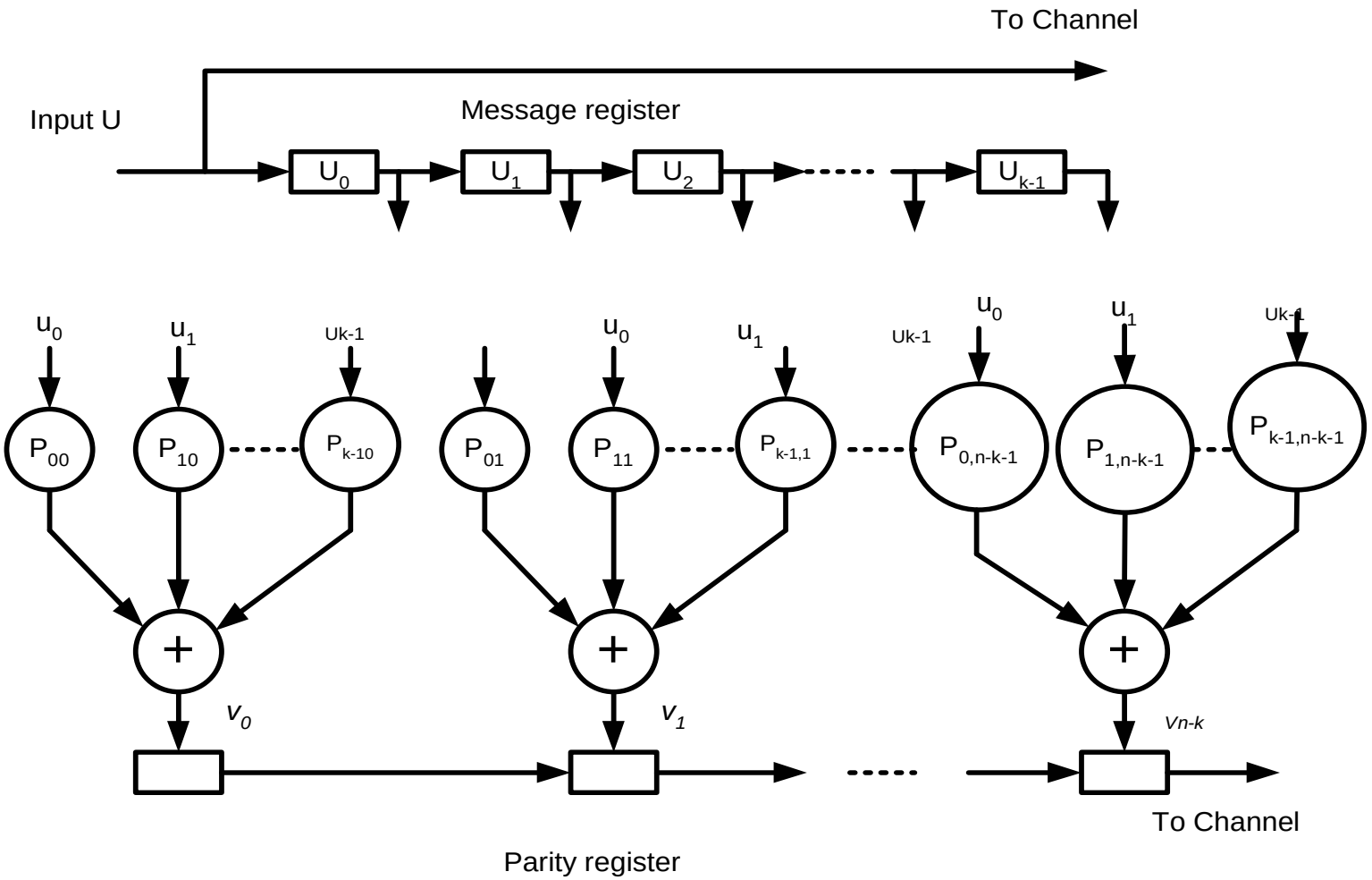


Figure: The encoding circuit for a liner system (n,k) code

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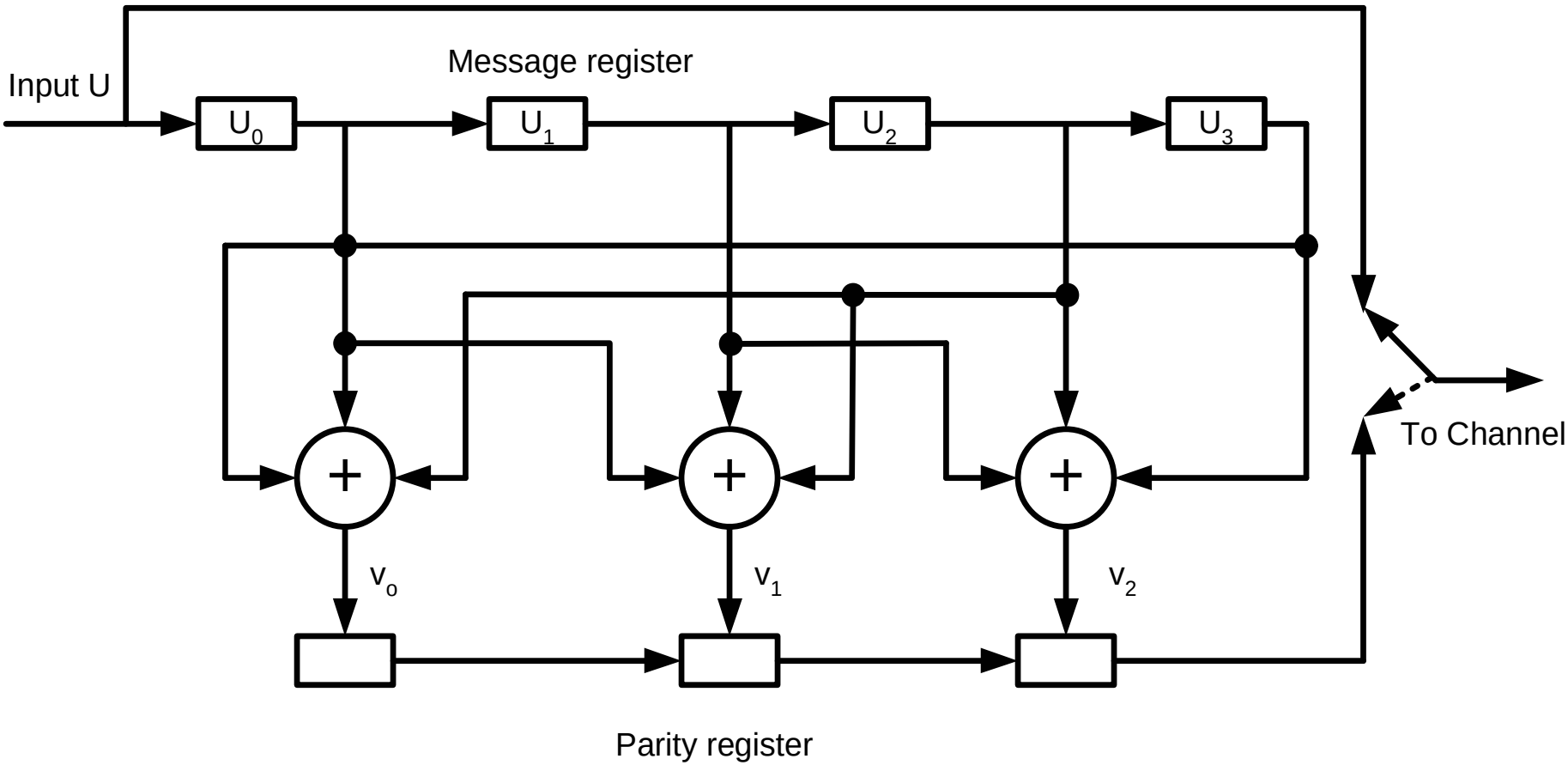


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# Syndrome and Error Detection

- $v = (v_0, v_1, \ldots, v_{n-1})$ be a codeword transmitted over a noisy channel.

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$$e = r + v = (e_0, e_1, \ldots, e_{n-1}) \tag{9}$$

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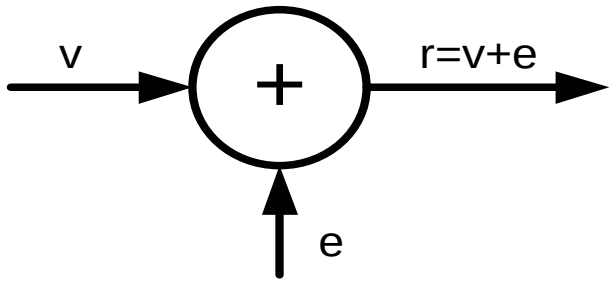


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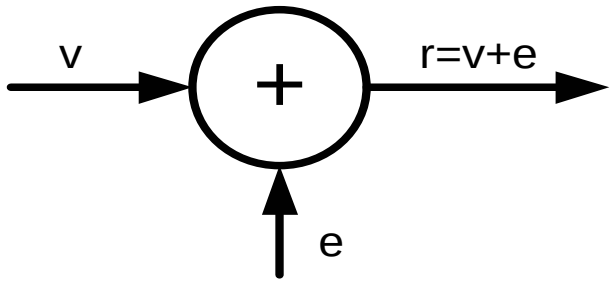


- The decoder first determine whether r contains errors.
- If errors are detected, correct errors (FEC) or Request for a retransmission of v (ARQ).

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- When r is received, the decoder computes the following (n-k)-tuple:

$$s = r \cdot H^T = (s_0, s_1, \ldots, s_{n-k-1}) \tag{10}$$

- which is called the syndrome of r
- $s = 0$ if and only if r is a code word and receiver accepts r as the transmitted code word
- $s \neq 0$ if and only if r is not a code word and the presence of errors has been detected
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$$S = (r_0, r_1, \ldots, r_{n-1}).H^T = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \ldots & 1 \\ p_{00} & p_{01} & p_{02} & \cdots & p_{0,n-k-1} \\ p_{10} & p_{11} & p_{12} & \cdots & p_{1,n-k-1} \\ \vdots & & & & \\ p_{k-1,0} & p_{k-1,1} & p_{k-1,2} & \cdots & p_{k-1,n-k-1} \end{bmatrix}$$

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$$s_0 = r_0 + r_{n-k}p_{00} + r_{n-k+1}p_{10} + \ldots + r_{n-1}p_{k-1,0}$$
$$s_1 = r_1 + r_{n-k}p_{01} + r_{n-k+1}p_{11} + \ldots + r_{n-1}p_{k-1,1}$$

$$s_{n-k-1} = r_{n-k-1} + r_{n-k}p_{0,n-k-1} \ldots + r_{n-1}p_{k-1,n-k-1} \quad (11)$$

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- The syndrome s is the vector sum of the received parity digits $(r_0, r_1, \ldots, r_{n-k-1})$ and the parity-check digits recomputed from the received information digits $(r_{n-k}, r_{n-k+1} \ldots, r_{n1})$
- A general syndrome circuit is shown in Fig. 5

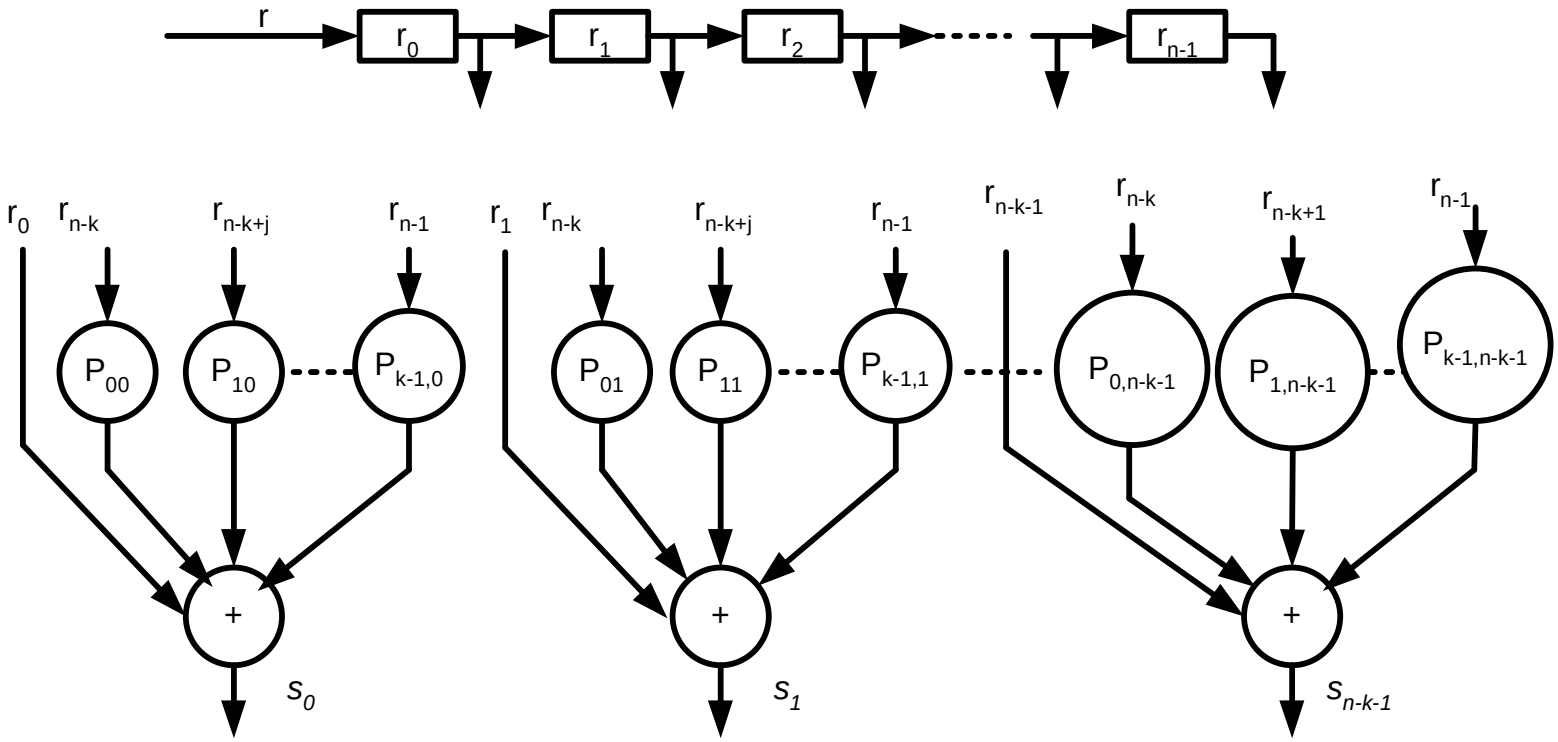


Figure: Syndrome circuit for a liner system (n,k) code

## Example 3.4

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$$S = (s_0, s_1, s_2) = (r_0, r_1, r_2, r_3, r_4, r_5, r_6) \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

- $s_0 = r_0 + r_3 + r_5 + r_6$
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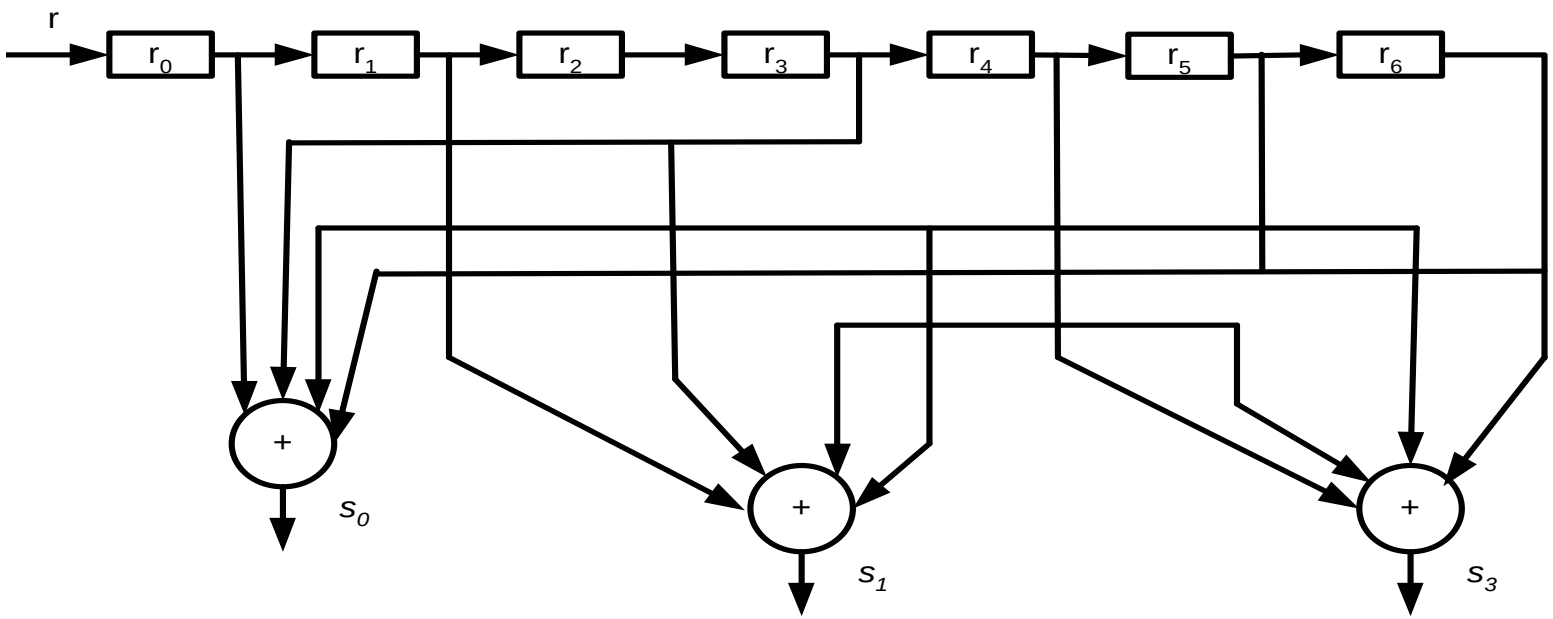
- The syndrome circuit for this code is shown below

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- To minimize the probability of a decoding error, the most probable error pattern that satisfies the equations of (3.13) is chosen as the true error vector

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- Now, since the channel is Binary Symmetric Channel (BSC), Then the most probable error pattern that satisfies the system above is e=(1 0 0 0 0) which has the smallest number of nonzero digits.
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- There are $2^4 = 16$ error patterns that satisfy the equations above.

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- where v* is the actual transmitted code word

# The Minimum Distance of a Block Code

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$$d(v, w) + d(w, x) \geq d(v, x) \qquad (3.14) \qquad (14)$$

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$$d_{min} = min\{d(v, w) : v, w \in C, v \neq w\} \qquad (3.16) \qquad (16)$$

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Theorem 3.1

- *The minimum distance of a linear block code is equal to the minimum weight of its nonzero code words.*
- The (7,4) code has minimum weight of 3.

## Theorem 3.2

- Let C be an (n, k) linear code with parity-check matrix H. For each code vector of Hamming weight *l*, there exist *l* columns of H such that the vector sum of these l columns is equal to the zero vector. Conversely, if there exist *l* columns of H whose vector sum is the zeros vector, there exists a code vector of Hamming weight *l* in C.

## Proof

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The Minimum Distance of a Block Code

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$$\begin{aligned} 0 &= v.H^T \\ &= v_0 h_0 + v_1 h_1 + \ldots + v_{n-1} h_{n-1} \\ &= v_{i1} h_{i1} + v_{i2} h_{i2} + \ldots + v_{il} h_{il} \\ &= h_{i1} + h_{i2} + \ldots + h_{il} \end{aligned}$$

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- It following from (3.18) that $x.H^T = 0$, x is code vector of weight l in C

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- However, it cannot detect all the error pattern of $d_{min}$ errors because there exists at least one pair of code vectors that differ in $d_{min}$ places and there is an error pattern of $d_{min}$ errors that will carry one into the other.
- The random-error-detecting capability of a block code with minimum distance $d_{min}$ is $d_{min}$-1.

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- If the minimum distance of C is $d_{min}$, then $A_1$ to $A_{dmin} - 1$ are zero.

Error-Detecting and Error-Correcting Capabilities of a Block Code

- Consider the (7,4) code given in table. The weight distribution is: $A_0 = 1, A_1 = A_2 = 0, A_3 = A_4 = 7, A_5 = A_6 = 0$,and $A_7 = 1$

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- If $p = 10^{-2}$ then $P_u(E) = 7x10^{-6}$ this means, if 1 million codewords are transmitted over a BSC with $p = 10^{-2}$ on average seven erroneous codewords pass through the decoder without being detected.

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- The inequality above says that if an error pattern of $t$ or fewer errors occurs, the received vector $r$ is closer (in Hamming distance) to the transmitted code vector v than to any other code vector w in C.

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$$2t + 1 \leq d_{min} \leq 2t + 2 \tag{19}$$

**Fact 1:**

- The code C is capable of correcting all the error patterns of $t$ or fewer errors.

**Proof:**

- Let **v** and **r** be the transmitted code vector and the received vector, respectively and **w** be any other code vector in C.

$$d(v, r) + d(w, r) \geq d(v, w) \tag{20}$$

- Suppose that an $t^{'}$ errors occurs during the transmission of v. Then $d(v, r) = t^{'}$.
- Since v and w are code vectors in C, we have

$$d(v, w) \geq d_{min} \geq 2t + 1. \tag{21}$$

Combining equation 20 and equation 21

$$d(v, r) + d(w, r) \geq 2t + 1$$

and $d(v, r) = t^{'}$

$$d(w, r) \geq 2t + 1 - t^{'}$$

$$if \quad t^{'} \leq t \Rightarrow d(w, r) > t$$

- The inequality above says that if an error pattern of $t$ or fewer errors occurs, the received vector $r$ is closer (in Hamming distance) to the transmitted code vector v than to any other code vector w in C.
- For a BSC, this means that the conditional probability $P(r|v)$ is greater than the conditional probability $P(r|w)$ for $w \neq v$.

Error-Detecting and Error-Correcting Capabilities of a Block Code

## Fact 2:

Error-Detecting and Error-Correcting Capabilities of a Block Code

**Fact 2:**

- The code is not capable of correcting all the error patterns of $l$ errors with $l > t$, for there is at least one case where an error pattern of $l$ errors results in a received vector which is closer to an incorrect code vector than to the actual transmitted code vector.

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- 1. $e_1 + e_2 = v + w$
  2. $e_1$ and $e_2$ do not have nonzero components in common places.
- We have

$$w(e_1) + w(e_2) = w(v + w) = d(v, w) = d_{min}.(3.23) \qquad (22)$$

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$$w(e_2) = d_{min} - w(e_1) \leq (2t + 2) - (t + 1) = t + 1$$

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- Combining (3.24) and (3.25) and using the fact that $w(e1) \geq t+1$ and $w(e2) \leq t+1$, we have

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- This inequality say that there exists an error pattern of $l (l > t)$ errors which results in a received vector that is closer to an incorrect code vector than to the transmitted code vector.
- Based on the maximum likelihood decoding scheme, an incorrect decoding would be committed.

Error-Detecting and Error-Correcting Capabilities of a Block Code

- A block code with minimum distance $d_{min}$ guarantees correcting all the error patterns of $t = [(d_{min} - 1)/2]$ or fewer errors, where $[(d_{min} - 1)/2]$ denotes the largest integer no greater than $(d_{min} - 1)/2$

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- A block code with random-error-correcting capability $t$ is usually capable of correcting many error patterns of $t + 1$ or more errors.
- For a t-error-correcting (n, k) linear code, it is capable of correcting a total $2^{n-k}$ error patterns.

# Standard Array and Syndrome Decoding

- Let $V_1, V_2, V_3, \ldots, V_{2^k}$ be the code vector of C i.e $C = \{V_1, V_2, , \ldots, V_{2^k}\}$. Each code vector i.e for example $V_1 = (v_0, v_1, \ldots, v_{n-1})$
- Any decoding scheme used at the receiver is a rule to partition the $2^n$ possible received vectors into $2^k$ disjoint subsets $D_1, D_2, \ldots, D_{2^k}$ such that the code vector $v_i$ is contained in the subset $D_i$ for $1 \leq i \leq 2^k$.
- Each subset $D_i$ is one-to-one correspondence to a code vector $v_i$.
- If the received vector r is found in the subset $D_i$, r is decoded into $v_i$.
- Correct decoding is made if and only if the received vector r is in the subset $D_i$ that corresponds to the actual code vector transmitted.

- A method to partition the $2^n$ possible received vectors into $2^k$ disjoint subsets such that each subset contains one and only one code vector is described here.

  **Step 1.**

- First, the $2^k$ code vectors of C are placed in a row with the all-zero code vector $v_1 = (0, 0, \ldots, 0)$ as the first (leftmost) element.

| | | | |
|---|---|---|---|
| $D_1$, | $D_2, \ldots,$ | $D_i$, | $D_2 k$ |
| $v_1 = (00...0)$ | $v_2, \ldots,$ | $v_i$, | $v_2 k$ |

  **Step 2.**

- From the remaining $2^n - 2^k$ n-tuple, an n-tuple $e_2$ of minimum weight is chosen and is placed under the zero vector $v_1$.
- A second row is formed by adding $e_2$ to each code vector $v_i$ in the first row and placing the sum $e_2 + v_i$ under $v_i$

## Step 3.

- An unused n-tuple $e_3$ is chosen from the remaining n-tuples and is placed under $e_2$.
- Then a third row is formed by adding $e_3$ to each code vector $v_i$ in the first row and placing $e_3 + v_i$ under $v_i$.
- Continue this process until all the n-tuples are used.
- Then we have an array of rows and columns as shown in Fig 3.6
- This array is called a standard array of the given linear code C

$$
\begin{array}{ccccccc}
v_1 = 0 & & v_2 & \dots & v_i & \dots & v_{2^k} \\
e_2 & & e_2 + v_2 & \dots & e_2 + v_i & \dots & e_2 + v_{2^k} \\
e_3 & & e_3 + v_2 & \dots & e_3 + v_i & \dots & e_3 + v_{2^k} \\
\vdots & & & & & & \\
e_l & & e_l + v_2 & \dots & e_l + v_i & \dots & e_l + v_{2^k} \\
\vdots & & & & & & \\
e_2^{n-k} & & e_2^{n-k} + v_2 & \dots & e_2^{n-k} + v_i & \dots & e_2^{n-k} + v_{2^k}
\end{array}
$$

Theorem 3.3: *No two n-tuples in the same row of a standard array are identical. Every n-tuple appears in one and only one row.* **Proof:**

- The first part of the theorem follows from the fact that all the code vectors of C are distinct
- Suppose that two n-tuples in the $I^{th}$ rows are identical, say $e_I + v_i = e_I + v_j$ with $i \neq j$
- This means that $v_i = v_j$, which is impossible, therefore no two n-tuples in the same row are identical

Proof

- It follows from the construction rule of the standard array that every n-tuple appears at least once
- Suppose that an n-tuple appears in both lth row and the mth row with $l < m$
- Then this n-tuple must be equal to el + vi for some i and equal to $e_m + v_j$ for some j
- As a result, $e_l + v_i = e_m + v_j$
- From this equality we obtain $em = el + (vi + vj)$
- Since $vi$ and $vj$ are code vectors in C, $vi + v$ is also a code vector in C, say vs
- This implies that the n-tuple em is in the lth row of the array, which contradicts the construction rule of the array that em, the first element of the mth row, should be unused in any previous row
- No n-tuple can appear in more than one row of the array

- From Theorem 3.3 we see that there are $2^n/2^k = 2^{n-k}$ disjoint rows in the standard array, and each row consists of $2^k$ distinct elements
- The $2_{n-k}$ rows are called the cosets of the code C
- The first n-tuple $e_j$ of each coset is called a coset leader
- Any element in a coset can be used as its coset leader

- Consider the (6, 3) linear code generated by the following matrix:

$$G = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Message is of

| $u_0$ | $u_1$ | $u_2$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

The coded message is of $U.G =$
(000000, 011100,101010, 110001, 110110, 101101, 011011, 000111)
The standard array of this code is shown in Table.

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| **Coset leader** | | | | | | | |
|---|---|---|---|---|---|---|---|
| 000000 | 011100 | 101010 | 110001 | 110110 | 101101 | 011011 | 000111 |
| 100000 | 111100 | 001010 | 010001 | 010110 | 001101 | 111011 | 100111 |
| 010000 | 001100 | 111010 | 100001 | 100110 | 111101 | 001011 | 010111 |
| 001000 | 010100 | 100010 | 111001 | 111110 | 100101 | 010011 | 001111 |
| 000100 | 011000 | 101110 | 110101 | 110010 | 101001 | 011111 | 000011 |
| 000010 | 011110 | 101000 | 110011 | 110100 | 101111 | 011001 | 000101 |
| 000001 | 011101 | 101011 | 110000 | 110111 | 101100 | 011010 | 000110 |
| 100100 | 111000 | 001110 | 010101 | 010010 | 001001 | 111111 | 100011 |

# Standard Array Decoding

- Consider (011100) is the transmitted codeword and the received word is (001100) which lies in $2^{nd}$ column whose coset leader e=(010000). So e is correctable error pattern. v=r+e= (001100)+(010000)=(011100).
- Consider (011100) is the transmitted codeword and the received word is (010100) which lies in $2^{nd}$ column whose coset leader e=(001000). So e is correctable error pattern. v=r+e= (010100)+(001000)=(011100).
- Consider (011100) is the transmitted codeword and the received word is (001010) which lies in $2^{nd}$ column whose coset leader e=(100000). v=r+e= (001010)+(100000)=(101010), in which there are 3 errors ocuur in the received vector that is equal to $d_{min}$, hence it is undetectable.
- Again consider (011100) is the transmitted codeword and the received word is (101100) and for this error pattern there is no coset leader in the standard array, so e is uncorrectable error pattern.

- A standard array of an (n, k) linear code C consists of $2^k$ disjoint columns
- Let Dj denote the jth column of the standard array, then

$$D_j = \{v_j, e_2 + v_j, e_3 + v_j, \ldots, e_{2^{n-k}} + v_j\} \qquad (3.27)$$

- $v_j$ is a code vector of C and $e_2, e_3, \ldots e_{2^{n-k}}$ are the coset leaders
- The $2^k$ disjoint columns $D_1, D_2, \ldots, D_{2^k}$ can be used for decoding the code C.
- Suppose that the code vector $v_j$ is transmitted over a noisy channel, from (3.27) we see that the received vector r is in $D_j$ if the error pattern caused by the channel is a coset leader
- If the error pattern caused by the channel is not a coset leader, an erroneous decoding will result

- The decoding is correct if and only if the error pattern caused by the channel is a coset leader
- The $2^{n-k}$ coset leaders (including the zero vector 0) are called the correctable error patterns.
  **Theorem 3.4** *Every (n, k) linear block code is capable of correcting* $2^{n-k}$ *error pattern.*
- To minimize the probability of a decoding error, the error patterns that are most likely to occur for a given channel should be chosen as the coset leaders
- When a standard array is formed, each coset leader should be chosen to be a vector of least weight from the remaining available vectors

# Syndrome Decoding

- The syndrome of an n-tuple is an (n-k)-tuple and there are $2^{n-k}$ distinct (n-k)-tuples.
- From theorem 3.6 that there is a one-to-one correspondence between a coset and an (nk)-tuple syndrome
- Using this one-to-one correspondence relationship, we can form a decoding table, which is much simpler to use than a standard array
- The table consists of $2^{n-k}$ coset leaders (the correctable error pattern) and their corresponding syndromes
- This table is either stored or wired in the receiver

**The decoding of a received vector consists of three steps:**
**Step 1.**

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- Compute the syndrome $S$ of the received word r,

$$S = r.H^T = H^T.r$$

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- Locate the coset leader $e_l$ whose syndrome is equal to $r.H^T$, then $e_l$ is assumed to be the error pattern caused by the channel.

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- Locate the coset leader $e_l$ whose syndrome is equal to $r.H^T$, then $e_l$ is assumed to be the error pattern caused by the channel.

**Step 3.**

- Decode the received vector r into the code vector v. i.e., $v = r + e_l$
- The decoding scheme described above is called the syndrome decoding or table-lookup decoding

Example 3.8

- Consider the (7, 4) linear code given in Table 3.1, the parity-check matrix is given in example 3.3
- The code has $2^3 = 8$ cosets.
- There are eight correctable error patterns (including the all-zero vector)
- Since the minimum distance of the code is 3, it is capable of correcting all the error patterns of weight 1 or 0
- All the 7-tuples of weight 1 or 0 can be used as coset leaders.
- The number of correctable error pattern guaranteed by the minimum distance is equal to the total number of correctable error patterns.

Table: Decoding table for the (7,4) linear code.

| **Syndrome** | **Coset Leader** |
|---|---|
| (100) | (1000000) |
| (010) | (0100000) |
| (001) | (0010000) |
| (110) | (0001000) |
| (011) | (0000100) |
| (111) | (0000010) |
| (101) | (0000001) |

- Suppose that the code vector v = (1 0 0 1 0 1 1) is transmitted and r = (1 0 0 1 1 1 1) is received code vector.

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- For decoding r, we compute the syndrome of r.

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- For decoding r, we compute the syndrome of r.

$$S = (1\ 0\ 0\ 1\ 1\ 1\ 1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = (0\ 1\ 1)$$

- From Table 3.2 we find that (0 1 1) is the syndrome of the coset leader e = (0 0 0 0 1 0 0), then r is decoded into

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$$\begin{aligned} v^* &= r + e \\ &= (1001111) + (0000100) \\ &= (1001011) \end{aligned}$$

- which is the actual code vector transmitted
- The decoding is correct since the error pattern caused by the channel is a coset leader.

- Suppose that v = (0 0 0 0 0 0 0) is transmitted and r = (1 0 0 0 1 0 0) is received code vector.

- Suppose that v = (0 0 0 0 0 0 0) is transmitted and r = (1 0 0 0 1 0 0) is received code vector.
- We see that **two** errors have occurred during the transmission of v.

- Suppose that v = (0 0 0 0 0 0 0) is transmitted and r = (1 0 0 0 1 0 0) is received code vector.
- We see that **two** errors have occurred during the transmission of v.
- The error pattern is not correctable and will cause a decoding error.

- Suppose that v = (0 0 0 0 0 0 0) is transmitted and r = (1 0 0 0 1 0 0) is received code vector.
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- When r is received, the receiver computes the syndrome.

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$$s = r.H^T = (111)$$

- From the decoding table we find that the coset leader e = (0 0 0 0 0 1 0) corresponds to the syndrome s = (1 1 1).

- r is decoded into the code vector.

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$$\begin{aligned} v^* &= r + e \\ &= (1000100) + (0000010) \\ &= (1000110) \end{aligned}$$

- r is decoded into the code vector.

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- Since v* is not the actual code vector transmitted, a decoding error is committed.

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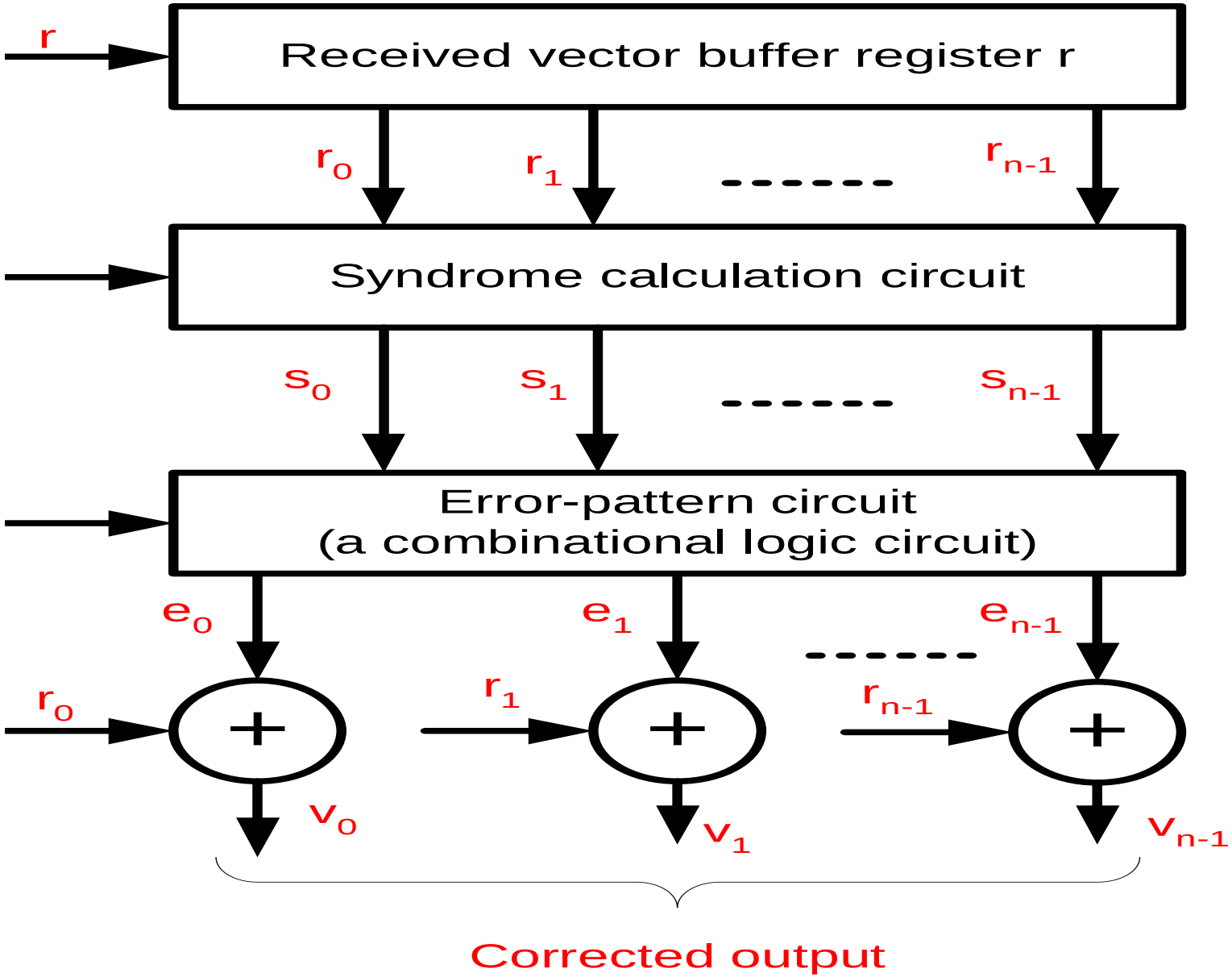
r
Received vector buffer register r
r0
r1
------
rn-1
Syndrome calculation circuit
s0
s1
------
sn-1
Error-pattern circuit
(a combinational logic circuit)
e0
e1
------
en-1
r0
r1
rn-1
v0
v1
vn-1
Corrected output

Example 3.9

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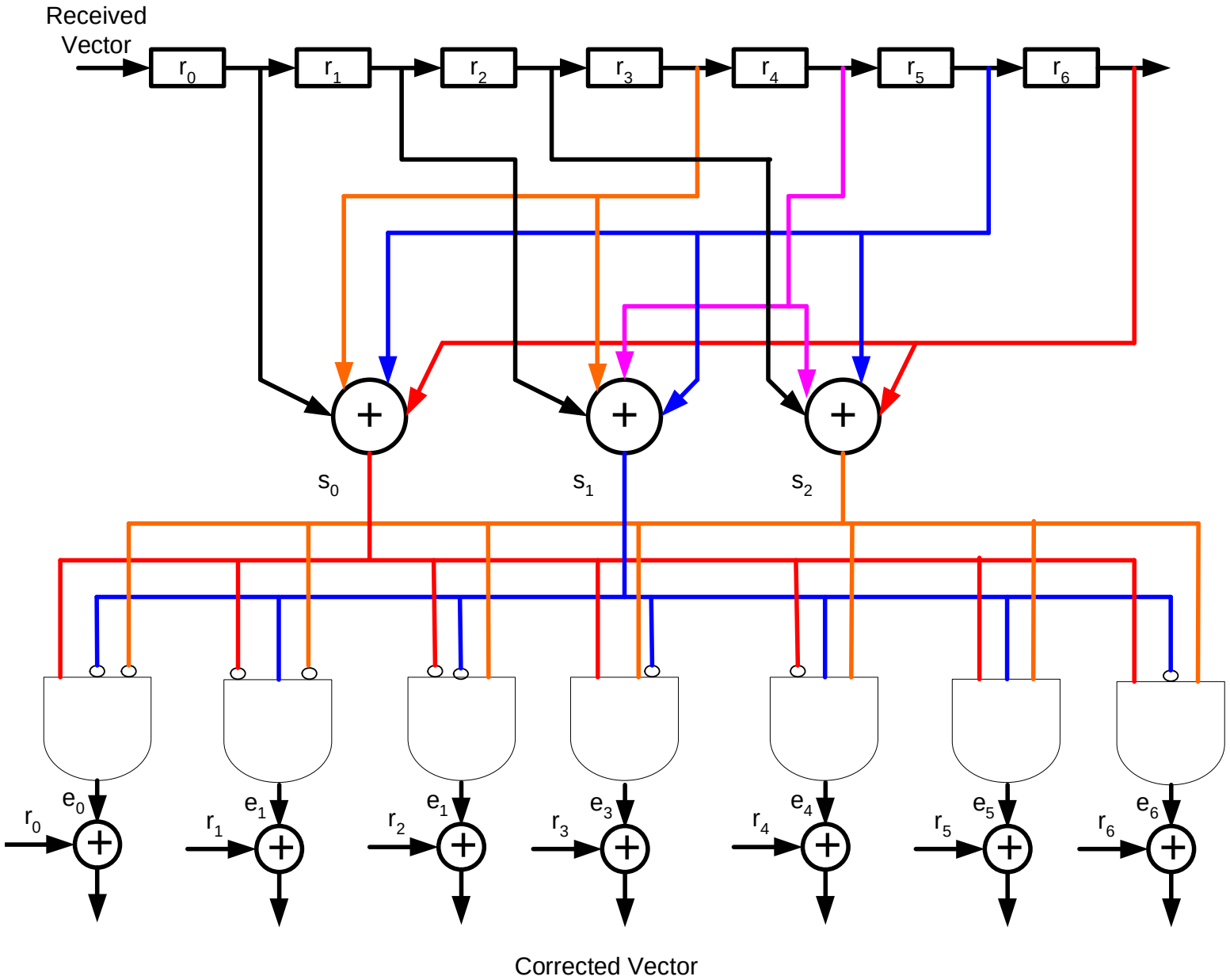
$$e_0 = s_0 \wedge s_1' \wedge s_2' \qquad e_1 = s_0' \wedge s_1 \wedge s_2' \qquad e_2 = s_0' \wedge s_1' \wedge s_2$$
$$e_3 = s_0 \wedge s_1 \wedge s_2' \qquad e_4 = s_0' \wedge s_1 \wedge s_2 \qquad e_5 = s_0 \wedge s_1 \wedge s_2$$
$$e_6 = s_0 \wedge s_1' \wedge s_2$$


Received
Vector
r0
r1
r2
r3
r4
r5
r6
s0
s1
s2
e0
e1
e1
e3
e4
e5
e6
r0
r1
r2
r3
r4
r5
r6
Corrected Vector