Communication Through Band Limited Linear Filter Channels

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[1, 2, 3, 4, 5, 6]

Note:

- Slides are prepared to use in class room purpose, may be used as a reference material
- All the slides are prepared based on the reference material
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- This material is not for commercial purpose.
- This material is prepared based on Advanced Digital Communication for DECS M Tech course as per Visvesvaraya Technological University (VTU) syllabus (Karnataka State, India).

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2. Linear Equalization

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Intersymbol Interference [1, 2, 3, 4, 5, 6]

Fourier transform of the rectangular Pulse

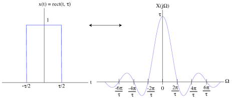

Figure: Fourier transform pair

$$x(t) = \begin{cases} A & -\frac{T}{2} < t < +\frac{T}{2} \\ 0 & elsewhere \end{cases}$$

The fourier transform is expressed as

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

$$\int_{-T/2}^{T/2} Ae^{-j2\pi ft}dt = \frac{A}{-j2\pi f}\left[e^{j\pi fT} - e^{-j\pi fT}\right]_{-T/2}^{T/2}$$

$$X(f) = \frac{TA}{\pi fT}\left[\frac{e^{j\pi fT} - e^{-j\pi fT}}{2j}\right] = \frac{AT\sin(\pi fT)}{\pi fT}$$

$$X(f) = AT\sin c(fT)$$

Since $\sin x = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$ and $\sin\left(\frac{\pi\theta}{\pi\theta}\right) = \sin c(\theta)$

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   - Inter-symbol-interference, takes place when a given transmitted symbol is distorted by other transmitted symbols.
   - Intersymbol interference (ISI) occurs when a pulse spreads out in such a way that it interferes with adjacent pulses at the sample instant.
2. Causes of ISI
   1. Due to band-limiting effect of practical channel
   2. Due to the multi-path effects (delay spread).

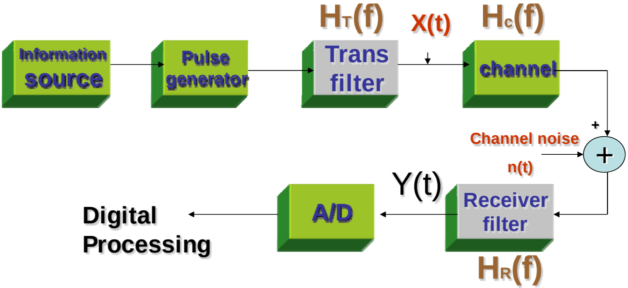
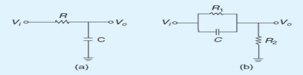


Figure: Basic Digital Communication System



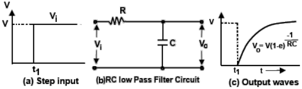


Figure 2

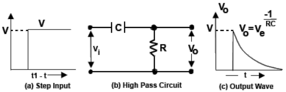


Figure 2

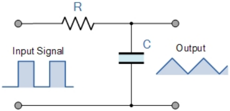


Figure: Physical Model for the wired network

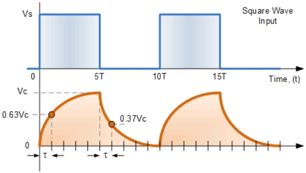
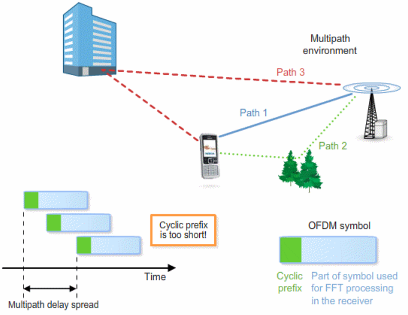


Figure: Effect of Low Pass Filter

- The input impedance of an electrical network is the equivalent impedance "seen" by a power source connected to that network..
- Transmission medium, disperse the signal and has effect of loss of high-frequency content.



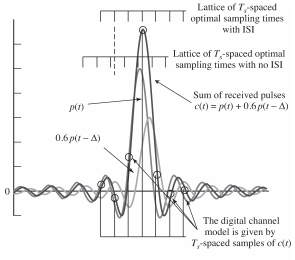
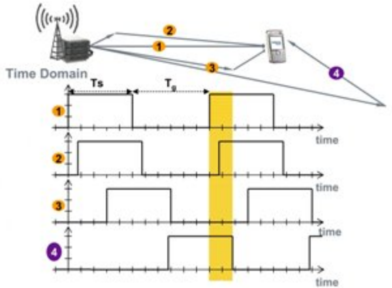


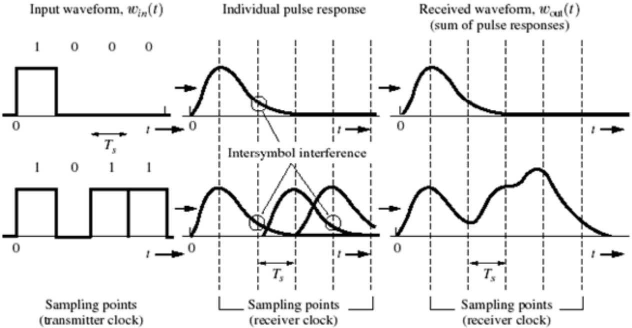
Figure: Effect of multipath



- The multi-path propagation in the channel will cause a delay-spread and inter-symbol interference (ISI).
- Radio channel: model as FIR filter.
-

ISI will degrade performance of our receiver, unless mitigated by some mechanism. This mechanism is called an equalizer.

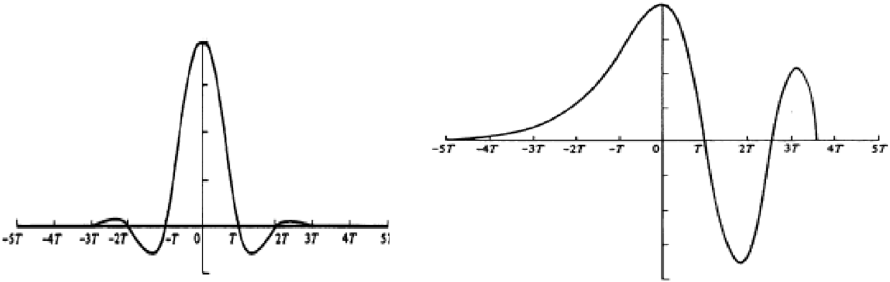
1. If the rectangular multilevel pulses are filtered improperly as they pass through a communications system, they will spread in time, and the pulse for each symbol may be smeared into adjacent time slots and cause Intersymbol Interference.



[redacted] Examples of ISI on received pulses in a binary communication system.

Figure: ISI on received data

- In the case non-ideal channel the received pulse does not have zero crossings at $\pm T, \pm 2T$, and so on.



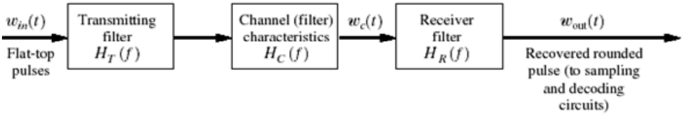
Channel input **Channel output**

Figure: ISI on received data

1. Equivalent impulse response:

$$h_e(t) = h(t) * h_T(t) * h_C(t) * h_R(t)$$

2. where $h_e(t)$ is the pulse shape that will appear at the output of the receiver filter.



[redacted] Baseband pulse-transmission system.

Figure: ISI on received data

3. Equivalent transfer function:

$$\mathrm{H}_e(f) = H(f)\, H_T(f)\, H_C(f)\, H_R(f) \quad \text{Where} \quad \mathrm{H}(f) = F\left[\prod\left(\frac{t}{T_s}\right)\right]$$

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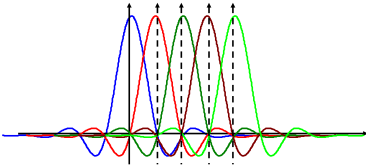
   - Equalizer.

Nyquists First Method for Zero ISI

- ISI can be eliminated by using an equivalent transfer function, He(f), such that the impulse response satisfies the condition:

$$h_e\left(kT_s + \tau\right) = \begin{cases} C, k = 0 \\ 0, k \neq 0 \end{cases}$$

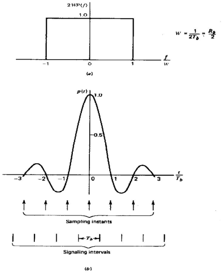


- where k is an integer, T_s is the symbol (sample) period τ is the offset in the receiver sampling clock times C is a nonzero constant
- Now choose the $\frac{\sin x}{x}$ function for $h_e(t)$

$$w_{out}\left(t\right) = \sum_n a_n h_e\left(t - nT_s\right)$$

- where h_e is a

$$h_e(t) = \frac{\sin \pi f_s t}{\pi f_s t}$$



[redacted] (a) Ideal amplitude response. (b) Ideal basic pulse shape.

- Since pulses are not possible to create due to:
- Infinite time duration.
- Sharp transition band in the frequency domain.
- The Sinc pulse shape can cause significant ISI in the presence of timing errors.
- If the received signal is not sampled at exactly the bit instant (Synchronization Errors), then ISI will occur.

Raised Cosine-Rolloff Nyquist Filtering for Zero ISI

- The Raised Cosine Nyquist filter is defined by its rollof factor number $r = f_\Delta / f_o$

$$H_e(f) = \begin{cases} 1, \ |f| < f_1 \\ \frac{1}{2}\left\{1 + \cos\left[\frac{\pi(|f| - f_1)}{2f_\Delta}\right]\right\}, \ f_1 < |f| < B \\ 0, |f| > B \end{cases}$$

  B is the Absolute Bandwidth

- where $f_\Delta = B - f_0$ $f_1 \equiv f_0 - f_\Delta$ Where f_o is the 6 dB bandwidth of the filter
  Rolloff factor: $r = \frac{f_\Delta}{f_0}$ Bandwidth: $B = \frac{R_b}{2}(1 + r)$

$$h_e(t) = F^{-1}[H_e(f)] = 2f_0 \left(\frac{\sin 2\pi f_0 t}{2\pi f_0 t}\right)\left[\frac{\cos 2\pi f_\Delta t}{1 - (4f_\Delta t)^2}\right]$$

- The tails of he(t) are now decreasing much faster than the Sa function (As a function of t2).
- ISI due to synchronization errors will be much lower.

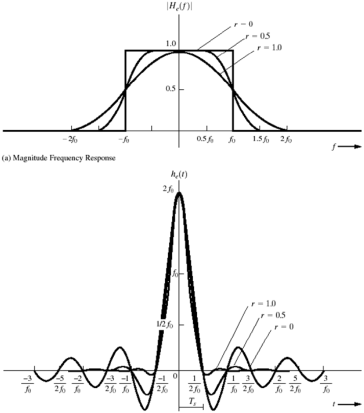


Figure: Frequency response and impulse responses of Raised Cosine pulses for various values of the roll off parameter.

$$\int_0^T r(t)f_k(t)dt = \int_0^T [s_m(t) + n(t)]f_k(t)dt$$

$$r_k = s_{mk} + n_k$$

$$s_{mk} = \int_0^T s_m(t)f_k(t)dt$$

$$n_k = \int_0^T n(t)f_k(t)dt$$

$$\begin{aligned} r(t) &= \sum_{k=1}^{N} s_{mk} f_k(t) + \sum_{k=1}^{N} n_k f_k(t) \\ &= \sum_{k=1}^{N} r_k f_k(t) + \sum_{k=1}^{N} n_k f_k(t) \\ &= \sum_{k=1}^{N} r_k f_k(t) + n'(t) \end{aligned}$$

$n'(t)$ is a zero-mean Gaussian noise process that represents the difference between original noise process n(t) and the part corresponding to the projection of n(t) onto the basis functions $f_k(t)$.
The correlator output rk conditioned on the mth signal being transmitted are Gaussian random variables with mean

$$E(r_k) = E(s_{mk} + n_k) = s_{mk}$$

and Variance is

$$\sigma_r^2 = \sigma_n^2 = N_0/2$$

Optimum receiver for channels with ISI and AWGN

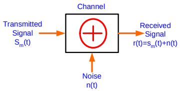


Figure: Receiver model with AWGN

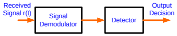


Figure: Receiver model

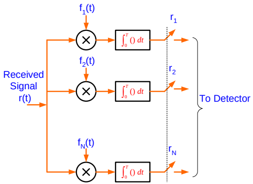


Figure: Correlation type demodulator

 Optimum receiver for channels with ISI and AWGN

The conditional probability density functions of the random variables $r = [r_1, r_2, \dots r_N]$ are:

$$p(r|s_m) = \prod_{k=1}^{N} p(r_k|s_{mk}) \qquad m = 1, 2, \dots M$$

where

$$p(r_k|s_{mk}) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(r_k - s_{mk})^2}{N_0}\right] \qquad k = 1, 2, \dots N$$

By substituting Equation (A) into Equation (B), we obtain the joint conditional PDFs

$$p(r|s_m) = \frac{1}{(\pi N_0)^{N/2}} \exp\left[-\sum_{k=1}^{N} \frac{(r_k - s_{mk})^2}{N_0}\right] m = 1, 2, \dots M$$

$$\ln p(r|s_m) = -\frac{1}{2} \ln(\pi N_0) - \frac{1}{N_0} \sum_{k=1}^{N} (r_k - s_{mk})^2$$

Optimum receiver for channels with ISI and AWGN

1. For many physical channels, such as telephone lines, not only are they bandlimited, but they also introduce distortions in their passbands. Such a channel can be modeled by an LTI filter followed by an AWGN source as shown in Figure.
2. The model consists of sequence of information symbols I_k and $H_T(f)$, $H_C(f)$, and $H_R(f)$ are the transfer functions of the transmission (pulse-shaping) filter, the dispersive channel and the receiving filter, respectively.

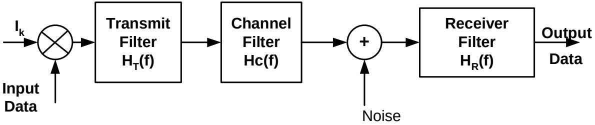


Figure: ISI on received data

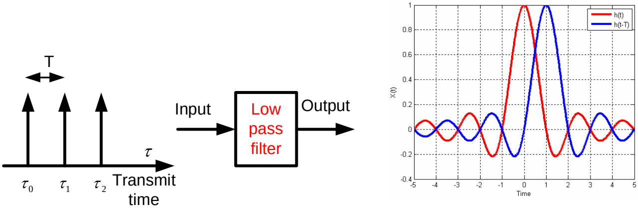


Figure: Filtered impulse sequence output versus input

The transmitted signal is given by

$$V(t) = \sum_{n=0}^{\infty} I_{\mathrm{n}} g(t - nT) \tag{1}$$

where, In $\Rightarrow$ discrete information bearing sequence and g(t) $\Rightarrow$ signal pulse
The received signal is expressed as

$$r_l(t) = \sum_{n} I_n h(t - nT) + z(t) \tag{2}$$

where, h(t) $\Rightarrow$ response of the channel to the input signal pulse g(t) and z(t) $\Rightarrow$ AWGN
Expand the received signal as

$$r_l(t) = \lim_{N \to \infty} \sum_{k=1}^{N} r_k f_k(t) \tag{3}$$

where, $f_k(t) \Rightarrow$ complete set of orthonormal functions
$r_k \Rightarrow$ random variables obtained by projecting r(t) onto the set $f_k(t)$

$$r_k = \sum_{n} I_n h_{kn} + z_k \tag{4}$$

where, k = 1,2,...
$h_{kn} \Rightarrow$ value obtained from projecting $h(t - nT)$ onto $f_k(t)$
$z_k \Rightarrow$ value obtained from projecting $z(t)$ onto $f_k(t)$

The joint probability density function of the random variables $r_N = [r_1,\ r_2 \ldots, r_N]$ conditioned on the transmitted sequence $I_P = [I_1,\ I_2 \ldots, I_P]$ is given by

$$p(r_N/I_p) = \left(\frac{1}{2\pi N_0}\right)^N \exp\left(-\frac{1}{2N_0}\sum_{k=1}^{N}\left|r_k - \sum_n I_n h_{kn}\right|^2\right) \quad (5)$$

As the number N of observable random variables approaches infinity, the logarithm of equation 5 is **Proportional to the Metrics** $PM(I_P)$:

$$PM(I_p) = -\int_{-\infty}^{\infty}\left|r_l(t) - \sum_n I_n h(t-nT)\right|^2 dt \quad (6)$$

$$PM(I_p) = -\int_{-\infty}^{\infty}|r_l(t)|^2 dt + 2\mathrm{Re}\left[\sum_n I_n^* \int_{-\infty}^{\infty} r_l(t)h^*(t-nT)dt\right] - \sum_n\sum_m I_n^* I_m \int_{-\infty}^{\infty} h^*(t-nT)h(t-mT)dt \quad (7)$$

The maximum likelihood estimates of the symbols $I_1,\ I_2 \ldots, I_N$ are those that maximize this quantity. The integral $\int_{-\infty}^{\infty}|r_l(t)|^2$ is common to all metrics, it may be discarded. The integral involving r(t) gives rise to the variables

$$y_n = y(nT) = \int_{-\infty}^{\infty} r_l(t)h^*(t-nT)dt \quad (8)$$

The correlation metrics is given as

$$CM(I_p) = 2Re\left(\sum_n I_n^* y_n\right) - \sum_n \sum_m I_n^* I_m x_{n-m} \quad (9)$$

x(t) is the response of the matched filter to h(t),which represents the autocorrelation function of h(t)

$$x_n = x(nT) = \int_{-\infty}^{\infty} h^*(t)h(t+nT)dt \quad (10)$$

If we substitute r(t) from equation 2 into equation 7,then we obtain i.e.,

$$y_n = y(nT) = \int_{-\infty}^{\infty} r_l(t)h^*(t-nT)dt \quad and \quad r_l(t) = \sum_n I_n h(t-nT) + z(t) \quad (11)$$

$$y_k = \sum_n I_n x_{k-n} + v_k \quad (12)$$

where additive noise sequence of the output of the matched filter is

$$v_k = \int_{-\infty}^{\infty} Z(t)h^*(t-kT)dt \quad (13)$$

The output of the demodulator at the sampling instants is corrupted by ISI as indicated by equation 12.

The metrics that are computed for MLSE of the sequence I are given by equation 8. It can be seen that these matrics can be computed recursively in the Viterbi algorithm,according to the relation

$$CM_n(I_n) = CM_{n-1}(I_{n-1}) + \mathrm{Re}\left[I_n^* \left(2y_n - x_0 I_n - 2\sum_{m=1}^{L} x_m I_{n-m} \right) \right] \text{.........}(12)$$

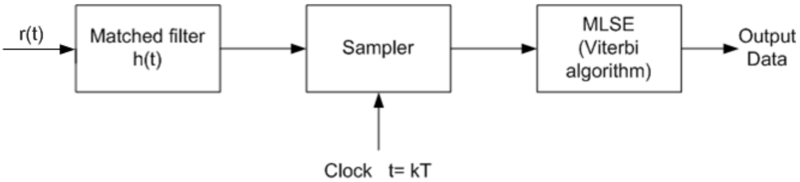


Figure: Optimum Receiver for an AWGN Channel with ISI

A Discrete-Time Model for a Channel with ISI

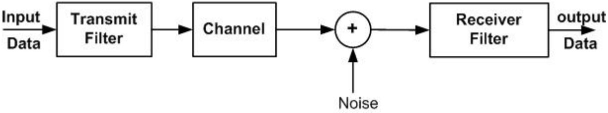


Figure: Block diagram of baseband transmission system

The output of the receiving filter is

$$y_k = \sum_{n=0}^{n=\infty} I_n x_{k-n} + v_k$$

$$\mathrm{y_k} = I_k + \sum_{\substack{n=0 \\ n \neq k}}^{\infty} I_n x_{k-n} + \nu_k \tag{14}$$

The term I_n represents the information bearing sequence of symbols
The term y_k represents the output and I_k represents the desired symbol at k^{th} sampling instant
The term x_n represents the the sampled value of x(t),where x(t) is the pulse function received from the channel.The term,

$$\sum_{\substack{n=0 \\ n \neq k}}^{\infty} I_n x_{k-n}$$

represents the ISI and ν_k is the additive Gaussian random varaible at the k^{th} sanpling instant.

- Transmitter sends discrete time symbols at a rate of $1/T$ symbols/second and the sampled output of the matched filter at the receiver is also a discrete time signal with samples at a rate of $1/T$ symbols/second.
- The cascade of the transmit filter at the input with impulse response g(t), the channel with impulse response c(t), the matched filter with the impulse response $h^*(t)$ and the sampler can be represented by an equivalent discrete time transversal filter having tap coefficients x_k and spans a time interval of 2LT.
- Its input is the sequence of information symbols I_k and its output is the discrete time sequence y_k given by an equation (14).
- The equivalent discrete time model is as shown in Figure where z^{-1} delay of T.

$$\mathrm{y_k} = \sum_n I_n x_{k-n} + \nu_k \qquad (15)$$

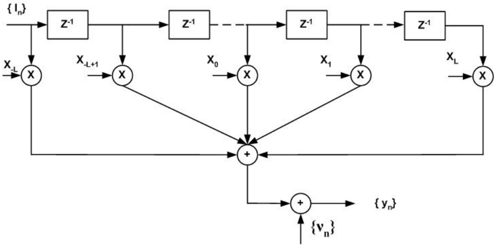


Figure: Equivalent discrete time model of channel with ISI

It is desirable to whiten the noise sequence by further filtering the sequence y_k. This is accomplished by passing the sequence y_k through a noise whitening filter $1/F(1/z)$.The output sequence v_k can be expressed as

$$v_k = \sum_{n=0}^{L} f_n I_{k-n} + \eta_k \tag{16}$$

where η_k is a white Gaussian noise sequence and f_k is a set of tap co efficents of a digital filter F(z) representing the cascade of transmitting filter,channel,matched filter,sampler and noise whitening filter.From this we can write the model of discrete time system with white noise as shown below

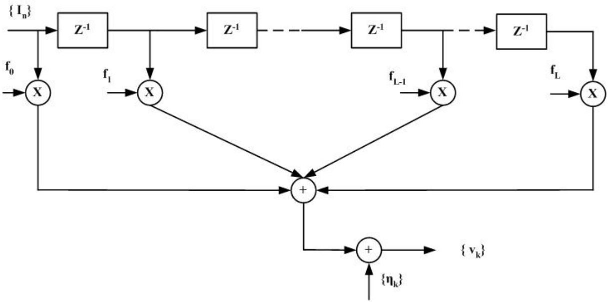


Figure: Equivalent discrete time model of ISI channel with AWGN

Linear Equalization

Equalizers

An equalizer is a digital filter that is used to mitigate the effects of intersymbol interference that is introduced by the channel. Equalization can be done using

- MLSE (Maximum-Likelihood Sequence Estimation)
- Filtering
  - Linear Transversal filter
  - Decision Feedback filter
  - Adaptive equalizer

Linear Transversal Filter

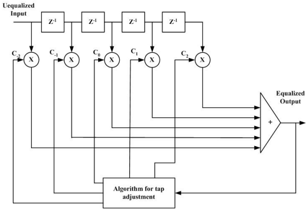
- The linear filter most often used is linear transversal filter.
- Its input is sequence is v_k expressed as .

$$v_k = \sum_{n=0}^{L} f_n I_{k-n} + \eta_k \tag{17}$$

- where η_k is a white Gaussian noise sequence and f_n is a set of tap coefficients
- Its output is the estimate of the information sequence I_k and estimate of the k^{th} symbol is expressed as

$$\hat{I}_k = \sum_{j=-k}^{k} c_j v_{k-j} \tag{18}$$

- where c_j are the 2K+1 tap weight coefficients of the filter.
- If $\hat{I}_k$ is not identical to the transmitted information symbol I_k, an error has occurred.



Linear Transversal Filter

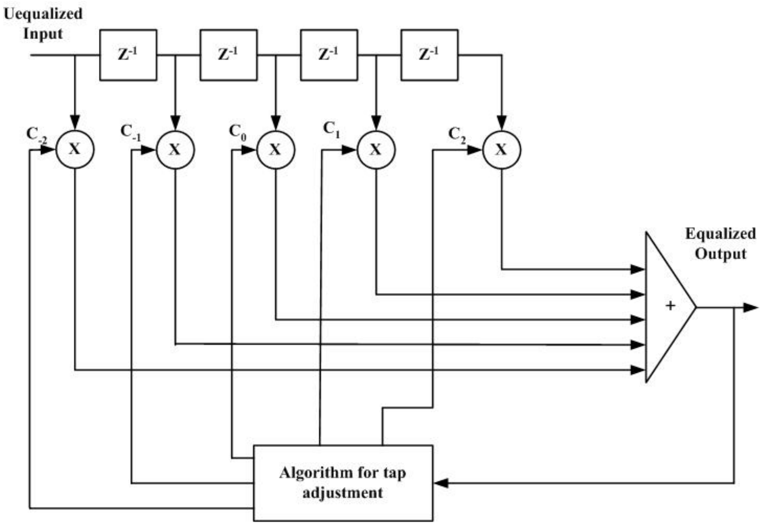


Figure: Linear Transversal Filter

- Considerable research has been performed on the criterion for optimizing the filter coefficients $\{c_j\}$.
- The most meaningful measure of performance for a digital communication system is the average probability of error, it is desirable to choose the coefficients to minimize this performance index.
- There are two criteria have been used in optimizing the equalizer coefficients. Those are
  1. Peak distortion criteria
  2. Mean Square error criteria.

Peak Distortion Criteria

The peak distortion criteria is simply defined as the worst case inter symbol interference at the output of the equalizer. The minimization of this performance criteria is called the peak distortion criteria.
Let the cascade of discrete time linear filer model having an impulse response f_n and the equalizer having an impulse response c_n can be represented by a single equivalent filter having an impulse response

$$q_n = \sum_{j=-\infty}^{\infty} c_j f_{n-j} \tag{19}$$

That is q_n is simply the convolution of c_n and f_n The equalizer output at the k^{th} sampling instant can be expressed in the form

$$\hat{I}_k = q_0 I_k + \sum_{n \neq k} I_n q_{k-n} + \sum_{j=-\infty}^{\infty} c_j \eta_{k-j} \tag{20}$$

The first term represents a scaled version of the desired symbol.

The second term is the intersymbol interference. *The peak values of this interference which is called the peak distortion*

Peak Distortion Criteria

The peak distortion is given by

$$D(c) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} |q_n| \tag{21}$$

$$= \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left| \sum_{j=-\infty}^{\infty} c_j f_{n-j} \right| \tag{22}$$

Inter symbol interference can be completely eliminated by making D(c)=0, i.e.,$q_n = 0$ for all n except n=0

$$q_n = \sum_{j=-\infty}^{\infty} c_j f_{n-j} = \begin{cases} 1, & \text{if } n = 0; \\ 0, & \text{if } n \neq 0; \end{cases} \tag{23}$$

By taking Z transform of eqn 23 we obtain

$$Q(z) = C(z)F(z) \tag{24}$$

$$C(z) = \frac{1}{F(z)} \tag{25}$$

Peak Distortion Criteria

From eqn 24, the transfer function of the equalizer C(z) is simply the reciprocal of F(z).

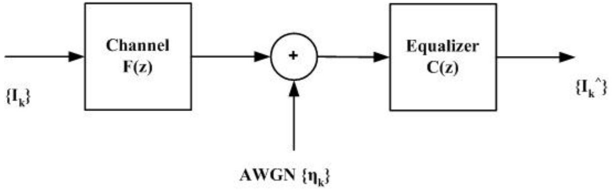


Figure: Zero forcing equalizer

So to completly eliminate ISI,requires the use of an inverse filter to F(z).Such a filter(equalizer) is known as zero forcing equalizer.

Peak Distortion Criteria

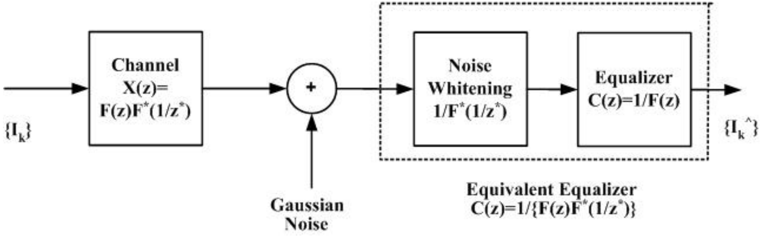


Figure: Equivalent Zero forcing equalizer

The cascade of the noise whitening filter having the transfer function $1/F(1/z)$ and the zero forcing equalizer having the transfer function $1/F(z)$ results in an equivalent zero forcing equalizer having a transfer function

$$C(z) = \frac{1}{F(z)F(1/z)} = \frac{1}{X(z)} \tag{26}$$

Peak Distortion Criteria

The performance of the equalizer that completely eliminates the ISI can be expressed in terms of SNR at its output. For convenience let the received signal energy is normalized to unity. The signal to noise ratio is defined as the ratio of the power of the signal part to the variance of the noise part.

$$\gamma = \frac{1}{\sigma_n^2} \tag{27}$$

where γ is Signal to noise ratio and σ_n^2 is the variance of the noise at the output of the equalizer. Let $S_{nn}(\omega)$ be the power spectral density of the noise sequence at the output of the equalizer.

$$S_{nn}(\omega) = \frac{N_0}{X(e^{jwT})}, \qquad |\omega| \leqslant \pi/T \tag{28}$$

The variance of the noise variable can be obtained using Power spectral density as follows

$$\sigma_n^2 = \frac{T}{\pi} \int_{-\pi/T}^{\pi/T} S_{nn}(\omega) d\omega \tag{29}$$

Peak Distortion Criteria

Therefore the SNR for the zero forcing equalizer is given by

$$\gamma = \left[\frac{TN_0}{2\pi} \int_{-\pi/T}^{\pi/T} \frac{1}{X(e^{jwT})} \right]^{-1} \tag{30}$$

Let $H(\omega)$ indicate the spectrum of the channel response.

$$X(e^{j\omega T}) = \frac{1}{T} \sum_{-\infty}^{\infty} \left| H\left(\omega + \frac{2\pi n}{T}\right) \right|^2 \tag{31}$$

Therefore eqn 30 becomes

$$\gamma = \left[\frac{T^2 N_0}{2\pi} \int_{-\pi/T}^{\pi/T} \frac{1}{\sum_{-\infty}^{\infty} \left| H\left(\omega + \frac{2\pi n}{T}\right) \right|^2} \right]^{-1} \tag{32}$$

Signal To noise ratio

$$\gamma = \left[\frac{T^2 N_0}{2\pi} \int_{-\pi/T}^{\pi/T} \frac{1}{\sum_{-\infty}^{\infty} \left| H\left(\omega + \frac{2\pi n}{T}\right) \right|^2} \right]^{-1}$$

We observe that, if the $H(\omega)$ possesses any zeroes ,the integrand becomes infinite and the SNR goes to zero. So we can say that the performance of the equalizer is poor when the $H(\omega)$ possesses any nulls or takes small values. This happens because, if the channel contains a null in its frequency response, the zero forcing equalizer attempts to compensate this by introducing infinite gain at that frequency.

Mean Square error criteria

In the MSE criteria, the tap weight coefficients c_j of the equalizer are adjusted to minimize the mean square value of the error

$$\varepsilon = I_k - \hat{I}_k \tag{33}$$

where I_k is the information symbol transmitted in the kth signalling interval and $\hat{I}_k$ is the estimate of that symbol at the output of the equalizer.The performance index for the mean square error (MSE) criteria denoted by J is

$$J = E\left|\varepsilon\right|^2 = E\left|I_k - \hat{I}_k\right|^2 \tag{34}$$

Infinite length equalizer

The tap coefficients of the equalizer should be selected to minimize the value of J when the equalizer has an infinite number of taps. The estimate $\hat{I}_k$ is expressed as

$$\hat{I}_k = \sum_{j=-k}^{k} c_j v_{k-j} \tag{35}$$

To minimize the mean square error, we have from the orthogonality principle as

$$E(\varepsilon_k v_{k-l}) = 0 \tag{36}$$

substituting eqn 33 in the above equation,

$$E\left[\left(I_k - \sum_{j=-\infty}^{\infty} c_j v_{k-j}\right) v_{k-l}\right] = 0 \tag{37}$$

$$\sum_{j=-\infty}^{\infty} c_j E(v_{k-j} v_{k-l}^*) = E(I_k v_{k-l}^*) \tag{38}$$

By substituting v_k value i.e., $v_k = \sum_{n=0}^{L} f_n I_{k-n} + \eta_k$ and, $x_k = \sum_{n=0}^{L-k} f_n^* f_{n+k}$

$$E(v_{k-j} v_{k-l}^*) = \sum_{n=0}^{L} f_n^* f_{n+l-j} + N_o \delta_{lj} \tag{39}$$

$$= \begin{cases} x_{l-j} + n_0 \delta_{lj} \\ 0 \end{cases} \tag{40}$$

$E(I_k v_{k-l}^*) = f_{-l}^*$

By substituting the values into equation 38 we have

$$\sum_{j=-\infty}^{\infty} c_j \sum_{n=0}^{L} f_n^* f_{n+l-j} + N_o \delta_{ij} = f_{-l}^* \tag{41}$$

Taking Z transform on both sides we have

$$C(z)[F(z)F^*(z^{-1}) + N_0] = F^*(z^{-1}) \tag{42}$$

$$C(z) = \frac{F^*(z^{-1})}{[F(z)F^*(z^{-1}) + N_0]} \tag{43}$$

When the noise whitening filter is incorporated into C(z) then

$$C(z) = \frac{1}{[F(z)F^*(z^{-1}) + N_0]} \tag{44}$$

On simplification we have

$$C(z) = \frac{1}{X(z) + N_0} \tag{45}$$

The expression for C(z) based on peak distortion criteria is

$$C(z) = \frac{1}{X(z)}$$

The expression for C(z) based on Mean square error criteria is

$$C(z) = \frac{1}{X(z) + N_0}$$

When N_0 is very small, the tap coefficients that minimize the peak distortion D(c) are approximately equal the coefficients that minimize the MSE performance index J.
When $N_0 \neq 0$,there is both residual interference and noise at the output of the equalizer.

Mean Square error criteria:Finite length equalizer

In this case transversal equalizer spans over a finite time duration. The output of the equalizer in the kth signaling interval

$$\hat{I}_k = \sum_{j=-k}^{k} c_j v_{k-j} \tag{46}$$

The mean square error (MSE) criteria for the equalizer having 2K+1 taps denoted by J(K) is

$$J(K) = E\left|\varepsilon\right|^2 = E\left|I_k - \hat{I}_k\right|^2 = E\left|I_k - \sum_{j=-k}^{k} c_j v_{k-j}\right|^2 \tag{47}$$

Minimization of J(K) with respect tap coefficients $|c_j|$ or equivalently forcing the error $I_k - \hat{I}_k$ to be orthogonal to the signal samples v^*_{j-l} $|l \leq K|$ yields the following equations.

$$\sum_{j=-k}^{k} c_j \Gamma_{ij} = \xi_l \tag{48}$$

where

$$\Gamma_{ij} = \begin{cases} x_{l-j} + N_0\delta_{ij} & (|l-j| \leq L) \\ 0 & (Otherwise) \end{cases} \tag{49}$$

$$\xi_l = \begin{cases} f^*_{-l} & (-Ll \leq l \leq 0) \\ 0 & (Otherwise) \end{cases} \tag{50}$$

It is convenient to express the set of linear equations in matrix form.

$$\Gamma C = \xi \tag{51}$$

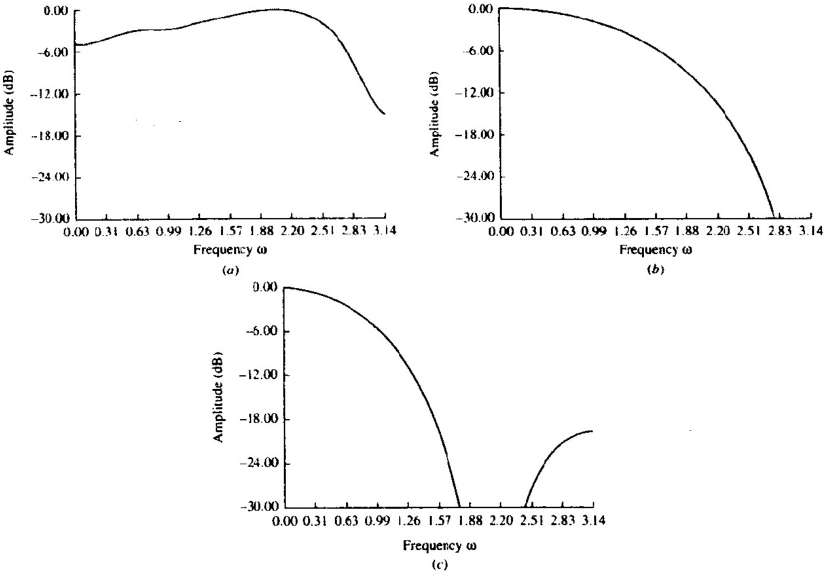
where C is denotes the column vector of 2K+1 tap weight coefficients, Γ denotes the $(2K+1) \times (2K+1)$ covariance matrix with elements Γ_{ij} and ξ is (2K+1) dimensional column vector with ξ_{ij} elements. The solution for this equation is

$$C_{opt} = \Gamma^{-1}\xi \tag{52}$$

The optimum tap weight coefficients will minimize the performance index J(K), with the result that the minimum value of J(K) is

$$J_{min}(K) = 1 - \sum_{j=-k}^{0} c_j f_{-l} \tag{53}$$

Channel Frequency response



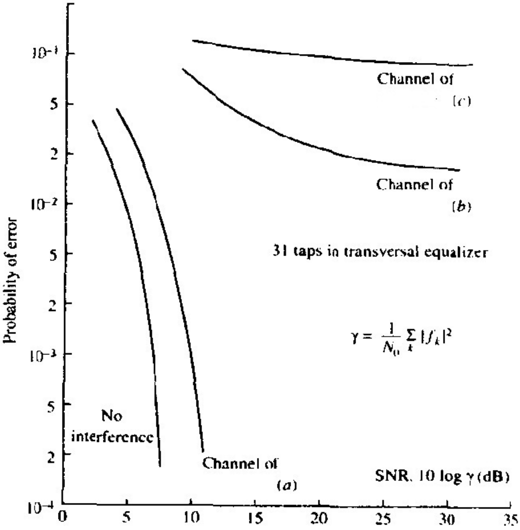


Figure: Error rate performance

Baseband and Passband Linear Equalization

- Linear equalizers discussed till now were described in terms of equivalent low-pass signals
- However in a practical implementation the linear equalizer can be realized either at baseband or at passband.
- The following block diagram illustrates the demodulation of QAM or multiphase PSK by first translating the signal to baseband and equalizing the baseband signal with an equalizer having complex valued coefficients.
- Equalizer with complex valued (inphase & quadraphase components) input is equal to four parallel equalizers with real-valued tap coefficients as shown in Fiugre

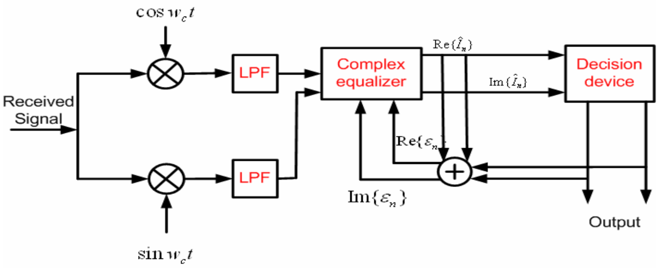


Figure: QAM and QPSK demodulator with baseband equalizer

Passband Linear Equalization

- The received signal is filtered and, in parallel, it is passed through a Hilbert transformer, called a phase splitting filter.
- In phase and quadrature components are fed to a passband complex equalizer.
- Following the equalization the signal is downconverted to a baseband and detected.

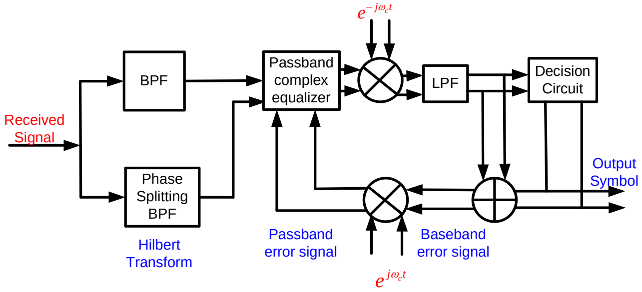


Figure: QAM or PSK signal equlization at passband

Decision Feedback Equalizer

Equalizer

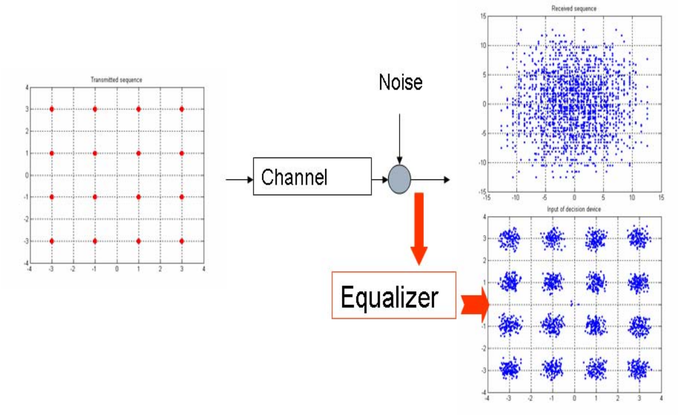


Figure: Equalizer

An equalizer is a digital filter that is used to mitigate the effects of intersymbol interference that is introduced by a time dispersive channel.

- Decision Feedback Equalization makes use of previous decisions in attempting to estimate the current symbol.
- Any trailing intersymbol interference caused by previous symbols is reconstructed and then subtracted. The DFE is inherently a nonlinear receiver.
- However, it can be analyzed using linear techniques, if one assumes all previous decisions are correct.
- The feedback filter accepts as input the decision from the previous symbol period; thus, the name decision feedback.
- The feedforward filter will try to shape the channel output signal so that it is a causal signal.
- The feedback section will then subtract (without noise enhancement) any trailing ISI.
- This principle of detecting symbols and using feedback to remove the ISI they cause (before detecting the next symbol), is called decision feedback equalization (DFE).

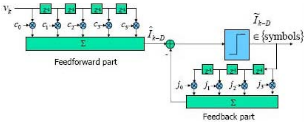


Figure: Feedback and Feedforward Filters

Decision Feedback Equalizer

- A DFE is a nonlinear equalizer that employs previous decisions to eliminate the ISI caused by previously detected symbols on the current symbol to be detected.
- The Decision Feedback Equalizer (DFE) depicted in Figure , consists of two filters, a feedforward filter and feedback filter.
- The input to the feedforward filter is received signal sequence v_k . The feedback filter has as its input the sequence of decisions on previously detected symbols.
- The feedback filter is used to remove that part of the ISI from the present estimate caused by previously detected symbols.

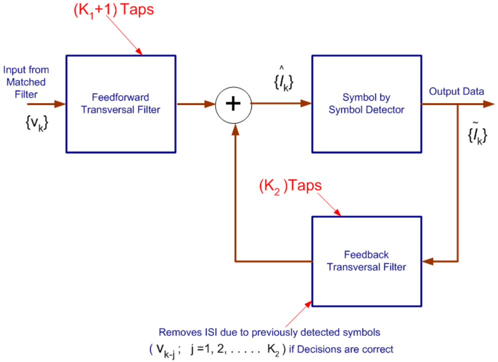


Figure: Structure of DFE

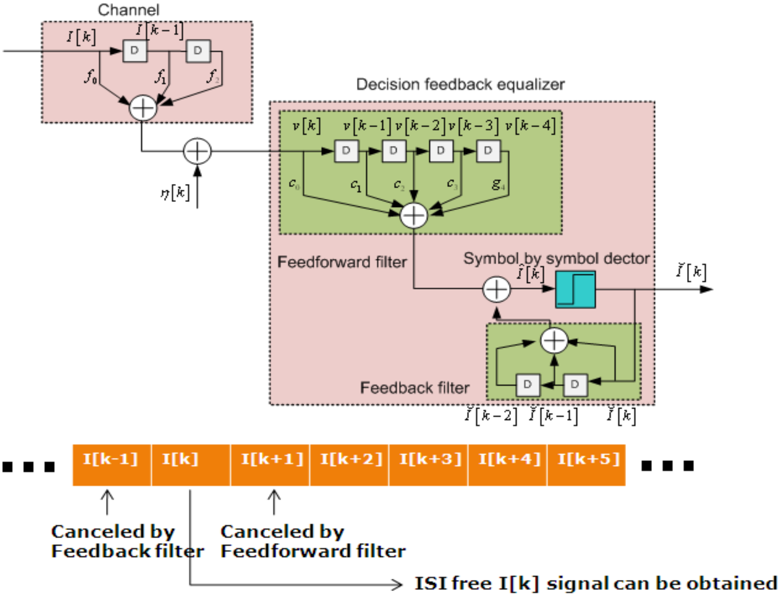


Figure: Feedback and Feedforward Filters

	Feedforward Filter	Feedback Filter
Tap	$c_j(j = -k_1 \ldots 0)$	$c_j(j = 1 \ldots k_2)$
Input	$v_{k-j}(j = -k_1 \ldots 0)$	$\tilde{I}_{k-j}(j = 1 \ldots k_2)$
Output	$\hat{I}_k = \sum_{j=-K_1}^{0} c_j v_{k-j} + \sum_{j=1}^{K_2} c_j \tilde{I}_{k-j}$	

1. A Feedforward filter, which is generally a fractionally-spaced FIR filter with adjustable tap coefficients.
2. a feedback filter, which is an FIR filter with symbol-spaced taps having adjustable coefficients
3. Input to the Feedback filter is the set of previously detected symbols.

1. Equalizer output can be expressed as :

$$\hat{I}_k = \sum_{j=-K_1}^{0} c_j v_{k-j} + \sum_{j=1}^{K_2} c_j \tilde{I}_{k-j}$$

2. where $\hat{I}_k$ is the estimate of the k th information symbol ,c_j are the Tap coefficients of the filter.
3. where $\tilde{I}_{k-1}......\tilde{I}_{k-k_2}$ are previously detected symbols.
4. The equalizer is assumed to have $(K_1 + 1)$ taps in the feedforward section and K_2 in its feedback section.
5. The equalizer is nonlinear because the feedback filter contains previously detected symbols $\tilde{I}_k$.

1. Based on the assumption that previously detected symbols in the feedback filter are correct , the minimization of MSE :

$$J(K_1, K_2) = E\left|I_k - \hat{I}_k\right|^2$$

Leads to the following set of linear equations for the coefficients of the feedforward filter

$$\sum_{j=-K_1}^{0} \psi_{lj} c_j = f^*_{-l}, \qquad l = -K_1, ... -1, 0$$

$$\psi_{lj} = \sum_{m=0}^{-l} f^*_m f_{m+l-j} + N_0 \delta_l j \qquad l, j = -K_1, ..., -1, 0$$

1. The coefficients of the feedback filter of the equalizer are given in terms of the coefficients of the feedforward section by the following expression:

$$C_k = \sum_{j=-K_1}^{0} C_j f_{k-j}, \qquad k = 1, 2, , ..., K_2$$

   provided that previous decisions are correct and that $K_2 \geq L$

2. The DFE determines the ISI from the previously detected symbols and subtracts it from the incoming symbols.
3. This equalizer does not suffer from noise enhancement because it estimates the channel rather than inverting it.
4. The DFE has better performance than the linear equalizer in a frequency-selective fading channel.The DFE is subject to error propagation if decisions are made incorrectly.

Predictive Decision Feedback Equalizer

- It is another DFE structure equivalent to one shown in DFE concept under the condition that the feedforwrd filter has an infinite number of taps.
- This structure consists of an fractional space equalizer (FSE) as a feedforward filter and a linear predictor as a feedback filter as shown in figure 31.
- If there is channel distortion the power in the error sequence at the output of the feedforward filter can be reduced by means of linear prediction based on the past values of the error sequence.

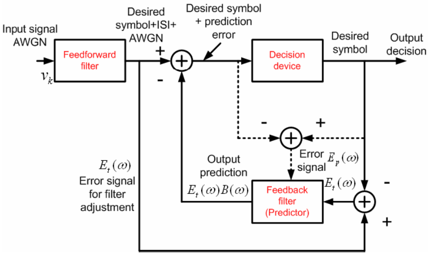


Figure: Block diagram of decision feedback equalizer

Predictive Decision Feedback Equalizer

- If B(W) represents the frequency response of the finite length feedback prediction

$$B(\omega) = \sum_{n=1}^{\infty} b_n e^{-j\omega nT}$$

- Then the error at the output of predictor is

$$E_p(\omega) = E_t(\omega) - E_t(\omega)B(\omega) = E_t(\omega)[1 - B(\omega)]$$

Thank You

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