Diversity[1]

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May 11, 2015

Diversity: [1]

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- All the slides are prepared based on the reference material
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- This material is prepared based on Wireless Communication for M Tech in DECS course as per Visvesvaraya Technological University (VTU) syllabus (Karnataka State, India).

Diversity Syllabus

Diversity

- Introduction
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  2. Definition of the Correlation Coefficient
- Microdiversity
  1. Spatial Diversity
  2. Temporal Diversity
  3. Frequency Diversity
  4. Angle Diversity
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- Macrodiversity and Simulcast
- Combination of Signals
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- Error Probability in Fading Channels with Diversity Reception
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- Transmit Diversity
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Introduction

- **Diversity:** To improve the performance over fading channel, the information symbols are passed through **multiple signal paths**, each of which fades independently, to make reliable communication.
- Diversity technique is used to improve the **Bit Error Rate (BER)**
- The principle of diversity is to ensure that the same information reaches the receiver (RX) on **statistically independent channels**.

- Diversity is most efficient when the different transmission channels (also called diversity branches) carry independently fading copies of the same signal.
- The correlation coefficient characterizes the correlation between signals on different diversity branches.
- The most important one is the correlation coefficient of signal envelopes x and y :

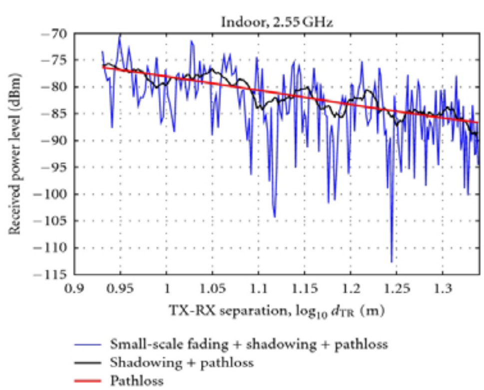
$$\rho_{xy} = \frac{E\{x.y\} - E\{x\}.E\{y\}}{\sqrt{(E\{x^2\} - E\{x\}^2).(E\{y^2\} - E\{y\}^2)}} \quad (1)$$

- For two statistically independent signals, the relationship $E\{x.y\} = E\{x\}.E\{y\}$ holds. therefore, the correlation coefficient becomes zero.
- Signals are said to be "effectively" de-correlated if ρ is below a certain threshold (typically 0.5 or 0.7).

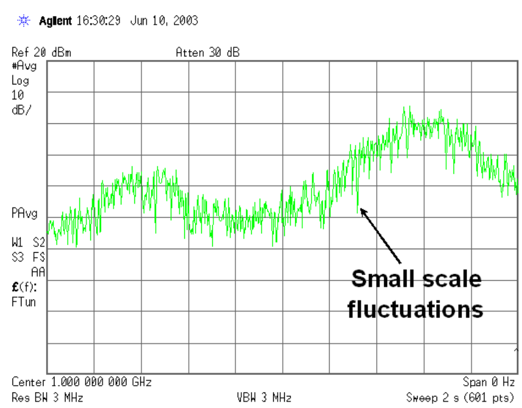
Microdiversity

Microdiversity

- Diversity methods used to combat **small-scale fading** are called "microdiversity".
- Small-scale fading, or simply fading describes the rapid fluctuation of the amplitude of a radio signal over a **short period of time or travel distance**
- It is caused by **interference** between two or more versions of the transmitted signal which arrive at the receiver at **different times.**
- The **five** most common methods are as follows:
  1. **Spatial diversity: several antenna elements separated in space.**
  2. **Temporal diversity: transmission of the transmit signal at different times.**
  3. **Frequency diversity: transmission of the signal on different frequencies.**
  4. **Angular diversity: multiple antennas (with or without spatial separation) with different antenna patterns.**
  5. **Polarization diversity: multiple antennas with different polarizations (e.g., vertical and horizontal).**



(a) Fading



(b) Small-scale fading

Figure 1: Small-scale fading

Spatial Diversity

- Spatial diversity is the **oldest, simplest and most widely** used form of diversity.
- The transmit signal is received at **several** antenna elements, and the signals from these antennas are then further processed.
- The performance is influenced by **correlation** of the signals between the antenna elements.
- A **large correlation** between signals at antenna elements is **undesirable**, as it decreases the effectiveness of diversity.
- In antenna diversity a relationship between **antenna spacing and the correlation coefficient** are important parameters.
- The relationship is different for Base station (BS) and mobile station (MS) antennas.
- These classification are as follows

1. **MS in cellular and cordless systems**
2. **BS in cordless systems and WLANs**
3. **BSs in cellular systems**

1. MS in cellular and cordless systems:

- It is a standard assumption that waves are incident from **all directions** at the MS.
- The **constructive and destructive interference** of Multi Path Components (MPCs) i.e., points where high and low received power, respectively are spaced approximately $\lambda/4$ apart.
- Therefore the distance that is required for decorrelation of received signals is $\lambda/4$.
- The plot of correlation coefficient and distance is as shown in Figure 6

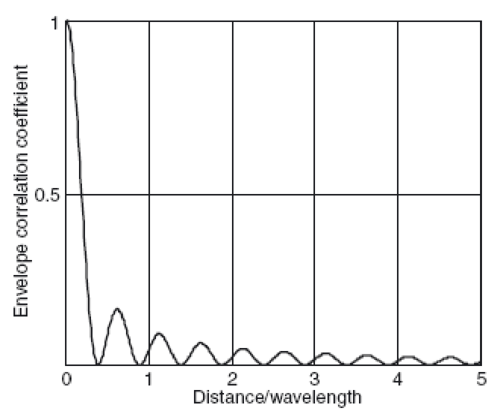


Figure 2: Envelope correlation coefficient as a function of antenna separation.

- The minimum distance for antenna elements in GSM (at 900 MHz) is about 8 cm, and for various cordless and cellular systems at the 1,800-MHz band it is about **4 cm**.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = 0.333\ m = 33.3cm \quad \therefore \quad \lambda/4 = 8cm$$

- For Wireless Local Area Networks (WLANs) (at 2.4 and 5 GHz), the distances are **even smaller**. It is thus clearly possible to place two antennas on an MS of a cellular system.

2. BS in cordless systems and WLANs:
   - It is assumed that, the angular distribution of incident radiation at indoor BSs is **uniform** i.e., radiation is incident with **equal strength from all directions.**
   - It is also same for MSs.

3. BSs in cellular systems:
   - For a cellular BS, the incidence radiation is **not uniform** in all directions due to **Interacting Objects** (IOs) are concentrated around the MS is as shown in Figure 3.
   - Since all waves are incident essentially from one direction, the correlation coefficient is much higher.
   - Consider a case when there are only two MPCs whose wave vectors are at an angle α with respect to each other.

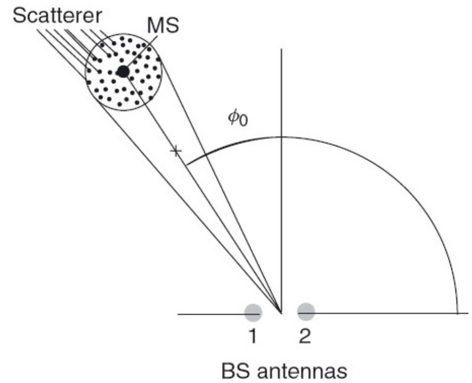


Figure 3: Scatterers concentrated around the mobile station.

- The distance between the **maxima and minima** of the interference pattern is **larger** for the **smaller value** of α.
- For very small α, the connection line between antenna elements lies on a “ridge” of the interference pattern and antenna elements are **completely correlated**.

Numerical evaluations of the correlation coefficient as a function of antenna spacing are shown in Figure

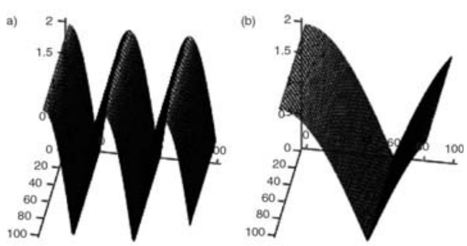


Figure 4: Interference pattern.

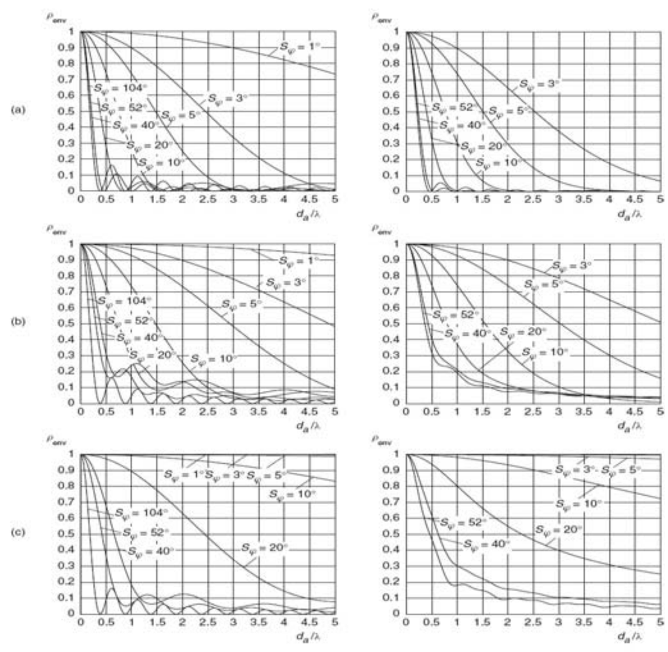


Figure 5: Envelope correlation coefficient at the BS for uniform and Gaussian probability density function (pdf) of the directions of arrival (a) $\phi_0 = 90°$ (b) $\phi_0 = 45°$ (c) $\phi_0 = 10°$

Temporal Diversity

- **Transmission of the transmit signal at different times.**
- Temporal diversity can be realized in different ways:

1. Repetition coding:
   - The signal is **repeated** several times, where the repetition intervals are long enough to achieve decorrelation.
   - This achieves diversity, but is also highly **bandwidth inefficient**.
   - **Spectral efficiency decreases** by a factor that is equal to the number of repetitions.
2. Automatic Repeat reQuest (ARQ):
   - The RX sends a message to the TX to indicate whether it **received the data with sufficient quality**, if not, then the transmission is **repeated**.
   - The spectral efficiency of ARQ is **better** than that of repetition coding, since it requires multiple transmissions only when the first transmission occurs in a bad fading state, while for repetition coding, retransmissions occur always.
   - On the downside, **ARQ requires a feedback** channel.
3. Combination of interleaving and coding:
   - It is **repetition coding in which a forward error correction coding with interleaving** is used.
   - The different symbols of a codeword are transmitted at different times, which increases the probability that at least some of them arrive with a good SNR.

Frequency Diversity

- The same signal is transmitted at two **different frequencies**.
- If the frequencies are spaced apart by more than the **coherent bandwidth** of the channel, then the probability is low that the signal is in **deep fade** at both frequencies simultaneously.

The correlation between two frequencies can be obtained by the equation

$$\rho = \frac{1}{1 + (2\pi)^2 S_\tau^2 (f_2 - f_1)^2}$$

- The figure shows ρ as a function of the spacing between the two frequencies.
- It is **not actually repeat** the same information at two different frequencies, the **small parts of the information** are conveyed by **different frequency components**.
- The RX can then **sum** over the different frequencies to recover the original information.

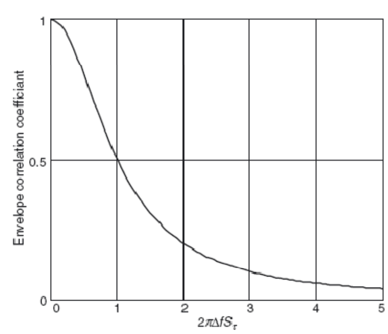


Figure 6: Correlation coefficient of the envelope as a function of normalized frequency spacing.

The spreading can be done by different methods

- **Compressing the information in time:** i.e., sending short bursts that occupy a large bandwidth- TDMA
- **Code Division Multiple Access(CDMA)**
- **Multicarrier CDMA and coded orthogonal frequency division multiplexing.**
- **Frequency hopping in conjunction with coding**:different parts of a codeword are transmitted on different carrier frequencies.

Angle Diversity

- Angular diversity is usually used in **conjunction** with **spacial diversity,** it enhances **decorrelation** of signals at closely spaced antennas.
- Different types of antennas have **different patterns**, but even identical antennas can have different patterns when mounted close to each other,this effect is called mutual coupling.
- Consider two antennas A and B as shown in Figure 7 where antenna B acts as a reflector for antenna A, whose pattern is **skewed** to the left.
- The pattern of antenna B is **skewed** to the right due to reflections from antenna A. Thus,the two patterns are different

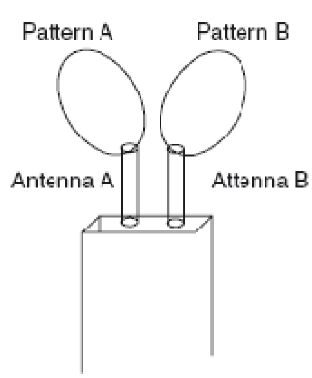


Figure 7: Angle diversity for closely spaced antennas.

Configurations of diversity antennas at a mobile station

- The different patterns are created when the antennas are located on different parts of the **casing of the mobile station**.
- **Patch antennas** and inverted-F antennas can be placed on all parts of the casing as shown in Figure 8
- In all of these cases, **decorrelation** is good even if the antennas are placed very closely to each other.

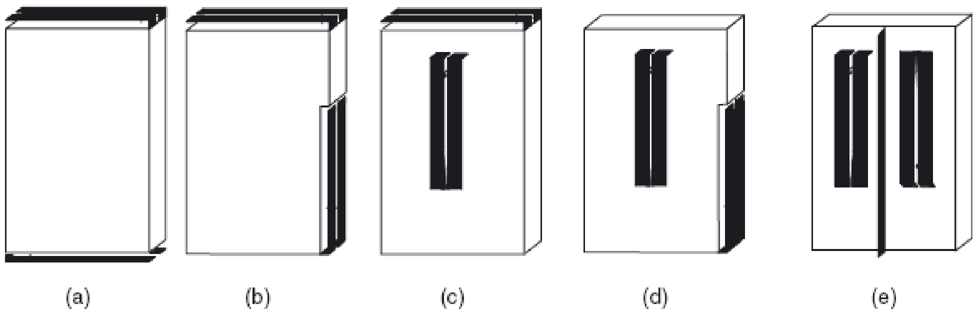
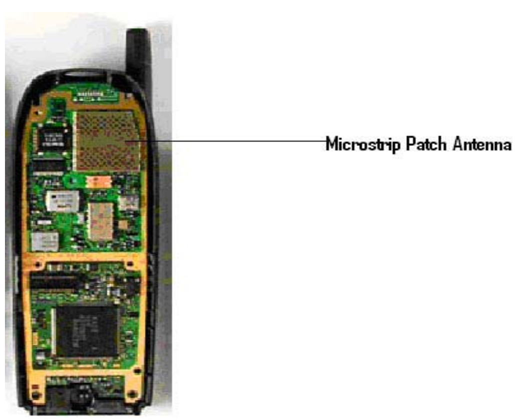
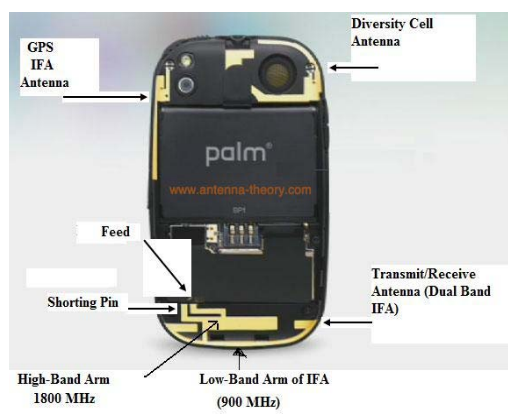


Figure 8: Configurations of diversity antennas at a mobile station.



(a) Patch antenna



(b) Inverted F antenna

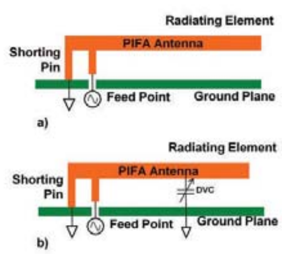


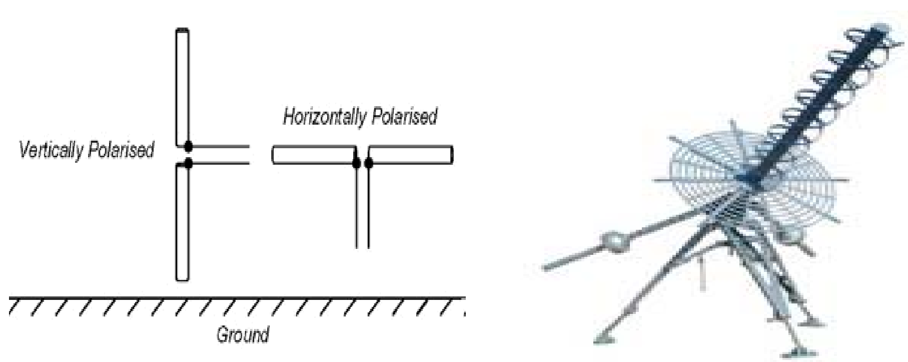
Figure 9: Configurations of diversity antennas at a mobile station.

Polarization Diversity

- An electromagnetic wave consists of two components, a **magnetic field and an electric field** and these two waves are **perpendicular** to each other.
- Polarization (also polarisation) is a property of waves that can oscillate with more than one orientation
- Polarization is of two types
  1. **Linear polarisation: Horizontal, Vertical**
  2. **Circular polarisation: left-hand circular, right-hand circular**
- Vertically polarized antennas have their electric field **perpendicular to the Earths surface**.
- Horizontally polarized antennas have their electric field **parallel to the Earths surface**.

Figure 10: Plot of Horizontal/Vertical Polarization

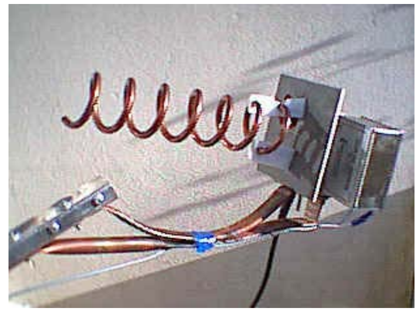
- Horizontal and vertical Polarization propagate **differently** in wireless channel, as the **reflection and diffraction** process depends on Polarization
- Even if the transmitted antenna only sends signals with a single Polarization, the propagation effects in the channel lead to depolarization so that both polarizations arrive at the RX.
- The **fading** of signals with **different polarizations** is statistically **independent**.
- Thus receiving both polarizations using a dual-polarized antenna, and processing the signals separately, **offers diversity**.
- This diversity can be obtained without any requirement for a **minimum distance** between antenna elements.



(a) Horizontal/Vertical (b) Circular

Figure 11: Types of antennas

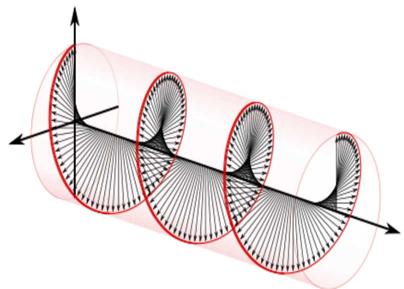
(a) Circular



(b) Circular

Figure 12: Types of antennas

(a) Circular



(b) Circular

Macrodiversity

- Diversity methods used for small scale fading are **not suitable** for combating large-scale fading, which is created by shadowing effects.
- **Shadowing** is independent of transmit frequency and polarization.
- Macrodiversity is a kind of space diversity scheme using several transmitter antennas for transferring the **same signal**.
- The distance between the transmitters is much **longer** than the wavelength and are typically situated in different base station sites or access points.
- If there is a hill between the TX and RX, adding antennas on either the BS or the MS does not help to eliminate the shadowing caused by this hill.
- Use a separate base station (BS2) that is placed in such a way that the hill is not in the connection line between the MS and BS2.
- This in turn implies a large distance between BS1 and BS2, which gives rise to the word **macrodiversity**.

- **Receiver Macrodiversity** is a form of antenna combining and requires an infrastructure that mediates the signals from the local antennas or receiver or decoder.
- Transmitter Macrodiversity may be a form of **simulcasting**.
- **Simulcasting:** Where the same signal is transmitted simultaneously from different BSs.
- Simulcast is also widely used for broadcast applications, especially digital TV.
- In this case, the exact synchronization of all possible RXs is not possible each RX would require a different timing advance from the TXs.
- The simplest method for Macrodiversity is the use of on-frequency repeaters that receive the signal and retransmit an amplified version of it.
- A disadvantage of simulcast is the large amount of signaling information that has to be carried on landlines, Synchronization information as well as transmit data have to be transported on landlines (or microwave links) to the BSs.
- The use of **on-frequency repeaters** is simpler than that of simulcast, as no synchronization is required.
- On the other hand, delay dispersion is larger, because
  1. The runtime from BS to repeater, and repeater to MS is larger (compared with the runtime from a second BS).
  2. The repeater itself introduces additional delays due to the group delays of electronic components, filters, etc.

Combination of Signals

Combination of Signals

There are **two ways** of exploiting signals from the multiple diversity branches:

1. **Selection diversity: Selecting the best signal** among all the signals received from different branches at the receiving end.
2. **Combining diversity:** Where **all copies of the signal are combined**, and the combined signal is decoded.

- Combining diversity leads to better performance, as **all available** information is exploited. but it requires a **more complex RX** than selection diversity.
- In most RXs, all processing is done in the baseband.
- An RX with combining diversity needs to downconvert all available signals, and combine them appropriately in the baseband.
- It requires **Nr antenna elements** as well as **Nr complete Radio Frequency (RF) chains**.
- An RX with selection diversity requires **only one RF chain**, as it processes only a single received signal at a time.

In these considerations,the gain of multiple antennas is due to two effects:

1. **Diversity gain:**
   It reflects the fact that it is improbable that several antenna elements are in a fading dip simultaneously.
   The probability for very low signal levels is thus decreased by the use of multiple antenna elements.
2. **Beamforming gain:**
   It reflects the fact that the combiner performs an averaging over the noise at different antennas.
   Thus, even if the signal levels at all antenna elements are identical, the combiner output SNR is larger than the SNR at a single-antenna element.

Selection diversity

Received-Signal-Strength-Indication-Driven Diversity

- In this method, the RX selects the signal with the largest instantaneous power or Received Signal Strength Indication RSSI.
- It requires **Nr antenna elements, Nr RSSI sensors, and a Nr-to-1 multiplexer (switch)**, but only one RF **chain** is as shown in Figure 14.
- The method allows simple tracking of the selection criterion even in fast-fading channels.

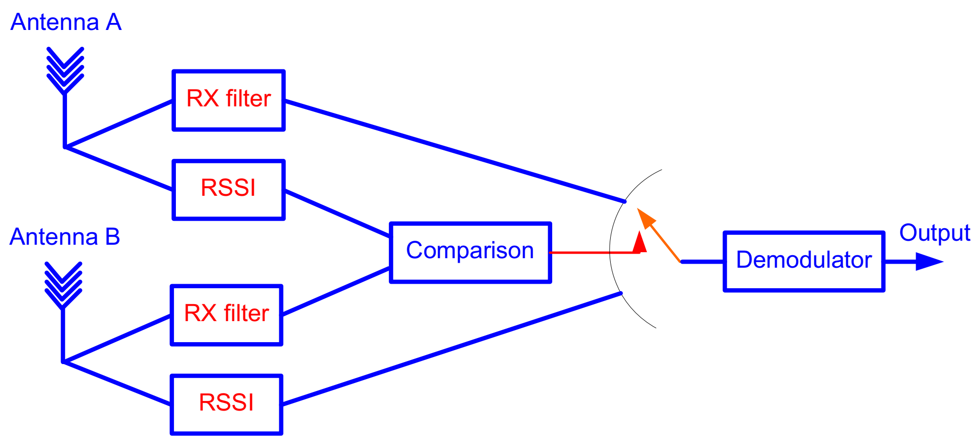


Figure 14: RSSI-driven selection diversity.

1. If the BER is determined by noise, then RSSI-driven diversity is the best of all the selection diversity methods, as maximization of the RSSI also maximizes the SNR.
2. If the BER is determined by co-channel interference, then RSSI is no longer a good selection criterion.
3. High receive power can be caused by a high level of interference, such that the RSSI criterion makes the system select branches with a low signal-to-interference ratio.
4. This is especially critical when interference is caused mainly by one dominant interferer a situation that is typical for Frequency Division Multiple Access or TDMA systems.
5. Similarly, RSSI-driven diversity is suboptimum if the errors are caused by the frequency selectivity of the channel.
6. The RSSI-driven selection diversity have a BER that is higher by a constant factor compared with optimum diversity.

- Consider instantaneous signal amplitude is Rayleigh distributed, and the SNR of the nth diversity branch, γ_n
- Then the SNR distribution of the output of the selector is.

$$pdf_{\gamma_n}(\gamma_n) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma_n}{\bar{\gamma}}\right) \quad (2)$$

- Where $\bar{\gamma}$ is the mean branch SNR.
- The cumulative distribution function (cdf) is then

$$cdf_{\gamma_n}(\gamma_n) = 1 - \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma_n}{\bar{\gamma}}\right) \quad (3)$$

- **cdf:** The probability that the instantaneous SNR lies below a given level, As the RX selects the branch with the largest SNR, the probability that the chosen signal lies below the threshold is the product of the probabilities that the SNR at each branch is below the threshold.
- The cdf of the selected signal is the product of the cdfs of each branch:

$$cdf_{\gamma_n}(\gamma_n) = \left[1 - exp\left(-\frac{\gamma_n}{\bar{\gamma}}\right)\right]^{N_r} \quad (4)$$

Bit-Error-Rate-Driven Diversity

- In this method first a **training sequence** (a bit sequence that is known at the RX) is transmitted, then RX demodulates the signal from each receive antenna element and **compares** it with the transmitted signal.
- The **smallest BER antenna** is selected for subsequent reception of data signals.
- A similar approach is the use of the mean square error of the "soft-decision" demodulated signal, or the correlation between transmit and receive signal.
- If the channel is **time variant**, the training sequence has to be **repeated at regular intervals** and selection of the best antenna has to be done anew.
- The repetition rate of training sequence depends on the **coherence time** of the channel.

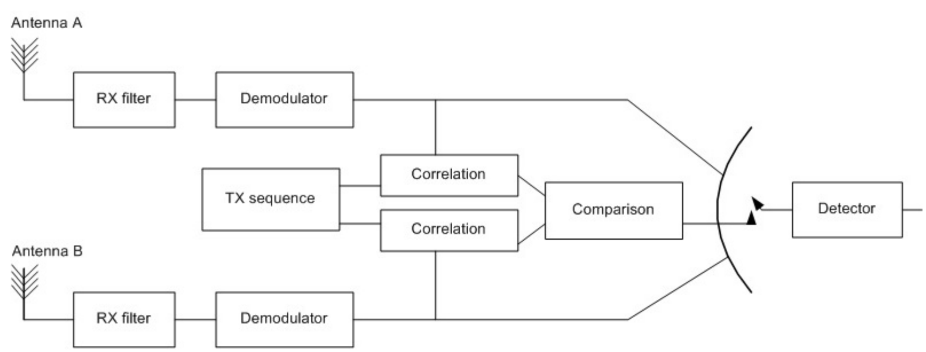


Figure 15: BER-driven selection diversity.

BER-driven diversity has several drawbacks:

1. The RX needs Nr number RF chains and demodulators and also the training sequence has to be repeated which decreases spectral efficiency
2. If the RX has only one demodulator, then it is not possible to continuously monitor the selection criterion (i.e., the BER) of all diversity branches. This is especially critical if the channel is time variant.
3. Since the duration of the training sequence is finite, the BER probability cannot be determined exactly.
4. The variance of the BER around its true mean decreases as the duration of the training sequence increases.
5. Spectral efficiency decreases as the longer training sequences used

Switched Diversity

- In selection diversity, **all diversity branches have to be monitored** in order to know when to select a different antenna, this leads to either **increased hardware or reduced spectral efficiency**.
- In switched diversity, use **current antenna as long as SNR is acceptable**, if it falls below a certain **threshold**, then the **RX switches to a different antenna**.
- Switching only depends on the quality of the active diversity branch; it does not matter whether the other branch actually provides a better signal quality or not.
- Switched diversity runs into problems when both branches have **signal quality below the threshold**: the RX just switches back and forth between the branches.
- This problem can be avoided by introducing a **hysteresis or hold time**, so that the new diversity branch is used for a certain amount of time, independent of the actual signal quality.
- There are two free parameters: **switching threshold and hysteresis time**. These parameters have to be selected very carefully:
- If the threshold is chosen too low, then a diversity branch is used even when the other antenna might offer better quality; if it is chosen too high, then it becomes probable that the branch the RX switches to actually offers lower signal quality than the currently active one.
- If hysteresis time is chosen too long, then a “bad” diversity branch can be used for a long time; if it is chosen too short, then the RX spends all the time switching between two antennas.

Combining Diversity

Basic Principle

- Selection diversity wastes signal energy by discarding(Nr -1)copies of the received signal.
- In combining diversity each signal copy is multiplied by a (complex) weight and added up.
- Each complex weight w_n^* can be consisting of a phase correction, plus a (real) weight for the amplitude:
- Phase correction causes the signal amplitudes to add up, while,on the other hand, noise is added incoherently, so that noise powers add up.
- For amplitude weighting, two methods are widely used:
  1. **Maximum Ratio Combining(MRC):** weighs all signal copies by their amplitude.
  2. **Equal Gain Combining(EGC):** where all amplitude weights are the same (in other words, there is no weighting, but just a phase correction).

Maximum Ratio Combining

- Combining all the signals in a co-phased and weighted manner so as to have the highest achievable SNR at the receiver at all times.
- MRC compensates for the phases, and weights the signals from the different antenna branches according to SNR.
- Let us assume a propagation channel is slow fading and flat fading.The only disturbance is AWGN.
- Each channel realization can be written as a time-invariant filter with impulse response:

$$h_n(\tau) = \alpha_n \delta(\tau) \tag{5}$$

- where α_n is the (instantaneous) gain of diversity branch n.

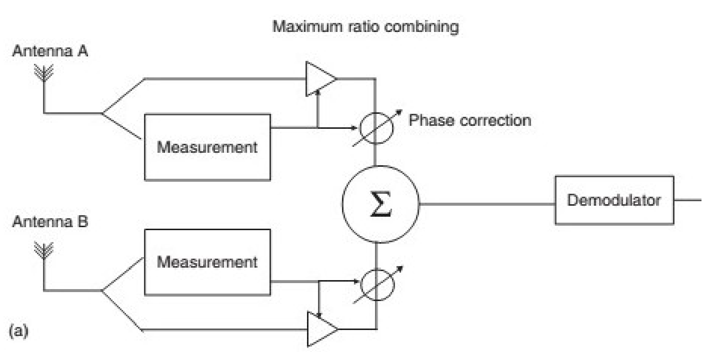


Figure 16: Maximum Ratio Combining.

- The signals at the different branches are multiplied with weights w_n^* and added up, so that the SNR is given by

$$\gamma_n \frac{\left|\sum_{n=1}^{N} w_n^* \alpha_n\right|^2}{p_n \sum_{n=1}^{N} |w_n|^2} \quad (6)$$

- where Pn is the noise power per branch(assumed to be the same in each branch).
- The SNR is maximized by choosing the weights is given by

$$w_{MRC} = \alpha_n \quad (7)$$

- the signals are phase-corrected(remember that the received signals are multiplied with w^*)and weighted by the amplitude.
- The output SNR of the diversity combiner is the sum of the branch SNRs:

$$\gamma_{MRC} = \sum_{n=1}^{N} \gamma_n \quad (8)$$

- The SNR distribution in each branch is exponential (corresponding to Rayleigh fading),and all branches have the same mean SNR $\overline{\gamma}_n = \overline{\gamma}$ then pdf is

$$pdf_\gamma(\gamma) = \frac{1}{(N_r - 1)} \frac{\gamma^{N_r - 1}}{\overline{\gamma^{N_r}}} \exp\left(-\frac{\gamma}{\overline{\gamma}}\right) \quad (9)$$

- The mean SNR of the combiner output is the mean branch SNR, multiplied by the number of diversity branches:

$$\overline{\gamma}_{MRC} = N_r \overline{\gamma} \quad (10)$$

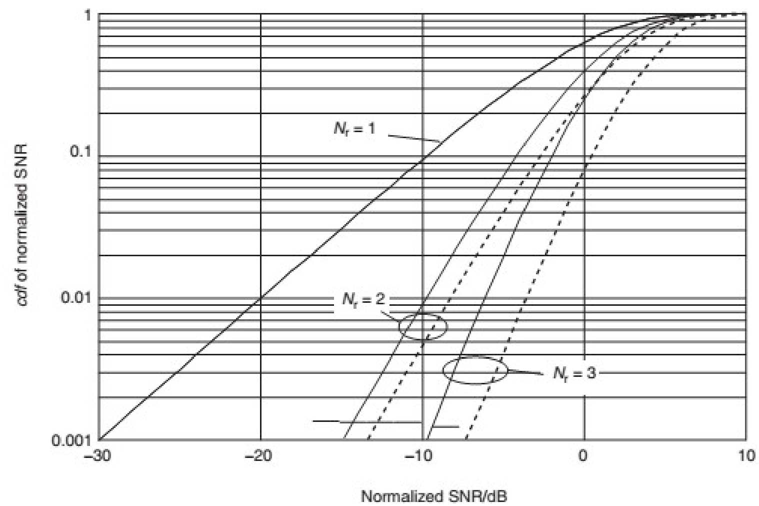


Figure 17: cdf of selection diversity (solid), and MRC (dashed)

- The slope of the distribution is the same for MRC and selection diversity,but that the difference in the mean values increases with increasing Nr.
- The selection diversity discards Nr -1 signal copies.

Equal Gain Combining

- For EGC, we find that the SNR of the combiner output is

$$\gamma_{ECG} = \frac{\left(\sum_{n=1}^{N_r} \sqrt{\gamma_n}\right)^2}{N_r}$$

- The mean SNR of the combiner output can be found to be if all branches suffer from Rayleigh fading with the same mean SNR $\overline{\gamma}$

$$\overline{\gamma}_{ECG} = \overline{\gamma}\left(1 + (N_r - 1)\frac{\pi}{4}\right)$$

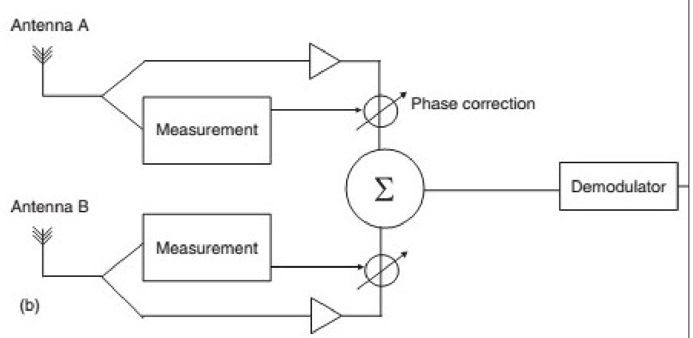


Figure 18: Equal Gain Combining.

- It is quite remarkable that EGC performs worse than MRC by only a factor p/4 (in terms of mean SNR).
- The performance difference between EGC and MRC becomes bigger when mean branch SNRs are also different.

Optimum Combining

- If the signal is affected by **AWGN and interference** then to maximize the Signal-to-Interference-and-Noise Ratio (SINR), the **weights** should be determined according to a strategy called **optimum combining**.
- The first step is the determination of the correlation matrix of noise and interference at the different antenna elements:

$$R = \delta_n^2 I + \sum_{k=1}^{k} E\left\{r_k r_k^{\dagger}\right\}$$

- $r_k^{\dagger}$ is Hermitian transpose, E expectation is over a time period when the channel remains constant, and r_k is the received signal vector of the kth interferer.
- The complex gain of the flat-fading channels of the Nr diversity branches are written into the vector h_d.
- The vector containing the optimum receive weights is then:

$$w_{opt} = R^{-1} h_d$$

- These weights have to be adjusted as the channel changes.

- It is easy to see that for a noise-limited system the correlation matrix becomes a(scaled) identity matrix, and optimum combining reduces to MRC.
- The minimizing the Mean Square Error is equivalent to maximizing the SINR, i.e.,

$$SINR = \frac{w^* h_d h_d^* w}{w^* R w} \tag{11}$$

- The SINR is a generalized Rayleigh quotient, and is maximized by the generalized eigenvalue corresponding to the maximum generalized eigenvector of

$$h_d h_d^* w = \lambda R w \tag{12}$$

- Optimum combining of signals fromNr diversity branches givesNr degrees of freedom.
- Ns =Nr -1 interferers can be eliminated, while the remaining Nr-Ns antennas behave like normal diversity antennas that can be used for noise reduction.
- The **Hermitian transpose or conjugate transpose** of an m-by-n matrix A with complex entries is the n-by-m matrix $A^{\dagger}$ obtained from A by taking the **transpose** and then taking the **complex conjugate** of each entry (i.e., negating their imaginary parts only).

$$A = \begin{bmatrix} 1 & -2-i \\ 1+i & i \end{bmatrix} \qquad A^{\dagger} = \begin{bmatrix} 1 & -2-i \\ 1+i & i \end{bmatrix}$$

Hybrid Selection - Maximum Ratio Combining

- In hybrid selection scheme, the **best L out of Nr antenna** signals are chosen, down converted, and processed.
- The number of required RF chains are **reduced from Nr to L**, hence savings hardware circuitry.
- The approach is called **Hybrid Selection/Maximum Ratio Combining (H-S/MRC), or sometimes also Generalized Selection Combining(GSC)**.
- The instantaneous output SNR of H-S/MRC is:

$$\gamma_{H-S/MRC} = \sum_{n=1}^{N} \gamma(n)$$

- It provides good diversity gain, as they select the best antenna branches for combining.
- If the signals at all antenna elements are completely correlated, then the SNR gain of H-S/MRC is only L, compared with Nr for an MRC scheme.
- The mean and variance of the output SNR for H-S/MRC is

$$\overline{\gamma}_{H-S/MRC} = L\left(1 + \sum_{n=L+1}^{N_r} \frac{1}{n}\right)\overline{\gamma} \qquad \overline{\sigma^2}_{H-S/MRC} = L\left(1 + \sum_{n=L+1}^{N_r} \frac{1}{n^2}\right)\overline{\gamma^2}$$

References

A. F. Molisch, *Wireless Communications*, 2nd ed. A John Wiley and Sons, Ltd.,, 2010.