Chapter 1

Transient behavior and Initial Conditions

1.0.1 Introduction:

Most of the transmission lines, electrical circuits and communication networks are made up of network elements like resistor R, inductor L, and Capacitor C. These networks are connected by voltage and current sources. It is most useful to understand the behavior of the network when we switched on the network by supplying voltage source. It is most important to determine the transient response of R-L, R-C, R-L-C series circuits for d.c and a.c excitations.

Assuming that at reference time $t = 0$, the switch in the circuit is closed and also assuming that switch act in zero time. To differentiate between the time immediately before and immediately after the operation of a switch, is represented as $t = 0^-$ and $t = 0^+$ signs are used. The condition existing just before the switch is operated will be designated as $i(0^-)$, $v(0^-)$, $q(0^-)$ and the conditions existing after closing of a switch is designated as as $i(0^+)$,$v(0^+)$, $q(0^+)$. Also initial conditions of a network depend on the past history of the network prior the closing of the network at $t = 0^-$ and the network structure at $t = 0^+$, after switching.

The evaluation of voltages and currents and their derivatives at $t = 0+$, are known as **initial conditions** and evaluation of condition at $t = \infty$ are known as **final conditions**.

The following are the objectives of studying the behavior of the circuit for Initial-Conditions:

- The most important reason is that the initial and final conditions must be known to evaluate the arbitrary constants that appear in the general solution of a differential equation.
- The initial conditions give knowledge of the behavior of the circuit elements at the instant of switching
- The final conditions give knowledge of the behavior of the circuit elements after the settling of circuit at $t = \infty$

1.1 Initial Conditions

Resistor

Consider a circuit which consists of resistor R connected as shown in Figure 1.1. The circuit resistor R is connected by a voltage source V in series with switch K as shown in Figure.

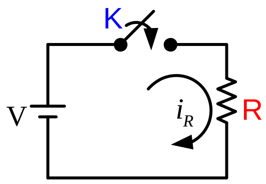


Figure 1.1: Series resonance circuit

When the switch K is closed at t=0 the current I is flowing in a circuit and is given by

$$I = \frac{V}{R}$$

Inductor

Consider a circuit which consists of inductor L connected as shown in Figure **??** (a). The inductor L is connected by a voltage source V in series with switch K as shown in Figure. When the switch K is closed at t=0 the current flowing in a inductor at $t = 0^+$ is zero, the inductor acts as a open circuit at $t = 0^+$ which is as shown in Figure **??** (b).

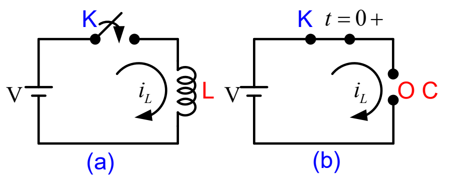


Figure 1.2: Inductor circuit

The final-condition of an inductor circuit is derived from the following relationship.

$$v = L\frac{di}{dt}$$

Under steady state condition, rate of change of current flowing in inductor is $\frac{di}{dt} = 0$. This means, v = 0 and hence L acts as short at $t = \infty$. The equivalent circuits of an inductor at $t = \infty$ is as shown in Figure 1.3

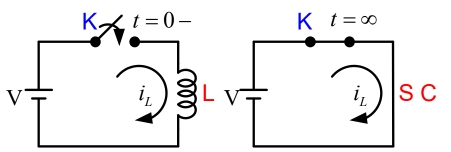


Figure 1.3: Inductor circuit at $t = 0+$

Capacitor

Consider a circuit which consists of capacitor C connected as shown in Figure 1.4. The capacitor is connected by a voltage source V in series with switch K as shown in Figure 1.4. When the switch K is closed at t=0 capacitor C acts as short circuit and current flows in a capacitor instantaneously.

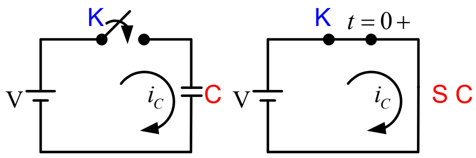


Figure 1.4: Capacitor circuit at $t = 0+$

If the capacitor is initially charged with charge q_0 coulombs at t=0-, then at t=0+ the capacitor is equivalent to voltage source $v_0 = \frac{q_0}{c}$ which is as shown in Figure 1.5

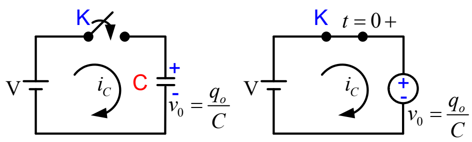


Figure 1.5: Capacitor circuit at $t = 0+$

The final condition of capacitor circuit is derived from the following relationship. The voltage across capacitor is

$$v = C\frac{dv}{dt}$$

Under steady state condition, rate of change of capacitor voltage is $\frac{dv}{dt} = 0$. This means, v = 0 and hence C acts as open circuit at $t = \infty$. The equivalent circuits of a capacitor at $t = \infty$ is as shown in Figure 1.6

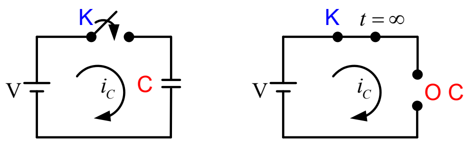


Figure 1.6: Capacitor circuit at $t = \infty$

If the capacitor is initially charged with voltage v_0 then the final condition at $t = \infty$ of a capacitor circuit is replaced with voltage source v_0 with open circuit which is as shown in Figure 1.7

$$v = C\frac{dv}{dt}$$

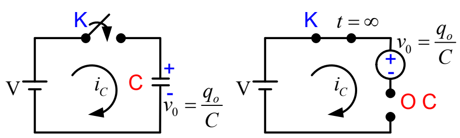
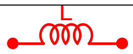
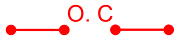
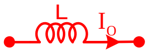
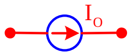
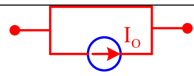
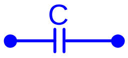
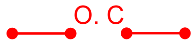
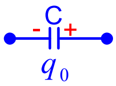
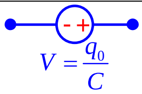
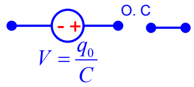


Figure 1.7: Capacitor circuit at $t = \infty$

Table 1.1: Initial and Final Conditions

at t=0-	at t=0+	at $t = \infty$
		
		
		
		
		

Procedure for Evaluating Initial Conditions:

1. Before closing or opening the switch at t=0- find the history of the network, at t=0- find i(0-), v(0-), i.e., current through inductor and voltage across the capacitor before switching
2. Draw the circuit after switching operation at t=0+.
3. Replace inductor with open circuit or by current source having source
4. Replace capacitor with short circuit or with a voltage source $v_c = \frac{q_0}{c}$ if it has an initial charge q_0.
5. Find i(0+), and v(0+) at t=0+
6. Obtain an expression for $\frac{di}{dt}$ and find $\frac{di}{dt}$ at t=0+
7. Obtain an expression for $\frac{d^2i}{dt^2}$ and find $\frac{d^2i}{dt^2}$ at t=0+
8. Similarly determine voltages across circuit elements and its derivatives.

1.2 Solutions

Q 1) In the circuit shown in Figure 1.8 the switch K is closed at $t = 0$, with capacitor uncharged. Find the values i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$, for element values as follows V=100 V $R = 1000\ \Omega$ and $C = 1\ \mu F$.

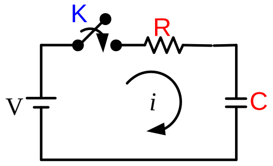


Figure 1.8: Example

Solution:

KVL for the given circuit is

$$Ri + \frac{1}{C}\int idt = V \tag{1.1}$$

At $t = 0^+$ the capacitor acts as short circuit which is as shown in Figure 1.9

$$\begin{aligned} Ri(0^+) &= V \\ i(0^+) &= \frac{V}{R} = \frac{100}{1000} = 0.1A \end{aligned}$$

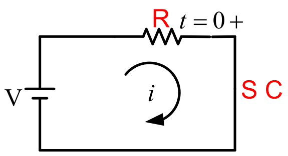


Figure 1.9: Example

Differentiating equation 1.1

$$R\frac{di}{dt} + \frac{i}{C} = 0$$

Substituting initial conditions

$$\begin{aligned} R\frac{di}{dt}(0^+) + \frac{i(0^+)}{C} &= 0 \\ \frac{di}{dt}(0^+) &= -\frac{i(0^+)}{RC} \\ \frac{di}{dt}(0^+) &= -\frac{0.1}{1000 \times 1 \times 10^{-6}} \\ &= -100A/sec \\ \frac{d^2i}{dt^2}(0^+) &= -\frac{1}{RC}\frac{di}{dt}(0^+) \\ &= -\frac{.1}{1000 \times 1 \times 10^{-6}}(-100) \\ &= -\frac{.1}{1000 \times 1 \times 10^{-6}}(-100) \\ &= 1 \times 10^5 A/sec^2 \end{aligned}$$

Q 2) In the circuit shown in Figure 1.10 the switch K is closed at $t = 0$, with zero current in the conductor. Find the values i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$, for element values as follows V=100 V $R = 10\ \Omega$ and $L = 1\ H$.

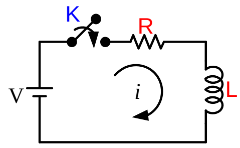


Figure 1.10: Example

Solution:

$$Ri + L\frac{di}{dt} = V \tag{1.2}$$

At $t = 0^+$ the inductor acts as open circuit which is as shown in Figure 1.11

$$i(0^+) = 0$$

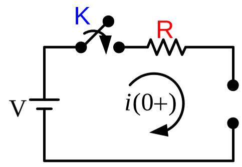


Figure 1.11: Example

From equation 1.2 and substituting initial conditions

$$\begin{aligned} L\frac{di}{dt} &= V - Ri \\ L\frac{di}{dt}(0^+) &= V - Ri(0^+) = 100 - 0 \\ \frac{di}{dt}(0^+) &= \frac{V}{L} = \frac{100}{1} = 100A/sec \\ \frac{d^2i}{dt^2}(0^+) &= -\frac{R}{L}\frac{di}{dt}(0^+) = -\frac{10}{1} \times 100 \\ &= -1000A/sec^2 \end{aligned}$$

Q 3- 2014-JAN) In the circuit shown in Figure 1.12 the switch K is closed at $t = 0$, with capacitor uncharged. Find the values of $i(0^+)$, $\frac{di}{dt}(0^+)$ and $\frac{d^2i}{dt^2}(0^+)$ for element values as follows, V=10 V $R = 10\ \Omega$ $L = 1\ H$ and $C = 10\ \mu F$ and $v_c(0) = 0$.

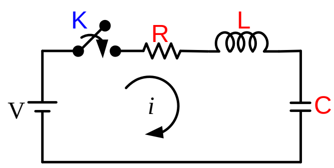


Figure 1.12: Example

Solution:

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0 \tag{1.3}$$

At $t = 0^+$ the inductor acts as open circuit and capacitor acts as short circuit which is as shown in Figure 1.13

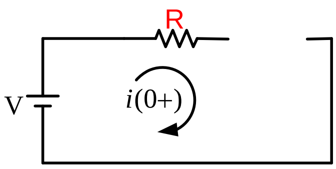


Figure 1.13: Example

$$i(0^-) = i(0^+) = 0$$

$$\begin{aligned}
Ri(0^+) + L\frac{di}{dt}(0^+) + \frac{1}{C}\int i(0+)dt &= V \\
R \times 0 + L\frac{di}{dt}(0+) + 0 &= V \\
L\frac{di}{dt}(0+) &= V \\
\frac{di}{dt}(0+) &= \frac{V}{L} = \frac{10}{1} \\
&= 10A/sec
\end{aligned}$$

Differentiating equation 1.3

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Substituting initial conditions

$$R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{C} = 0$$

$$\begin{aligned}
10 \times 10 + L\frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{C} &= 0 \\
\frac{d^2i}{dt^2}(0^+) &= \frac{-100}{L} = \frac{-100}{1} \\
&= -100A/sec^2
\end{aligned}$$

Q 4) In the circuit shown in Figure 1.14 switch S is changed from position a to b at $t = 0$. Steady state conditions have been reached at $t = 0^-$. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$ with capacitor is initially uncharged.

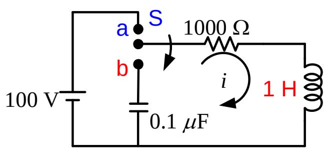


Figure 1.14: Example

Solution:

At $t = 0^-$ the inductor acts as a short circuit which is as shown in Figure 1.15.

$$\begin{aligned}
Ri(0^-) &= V \\
i(0^-) &= \frac{V}{R} = \frac{100}{1000} = 0.1\ A
\end{aligned}$$

Current through inductor cannot change instantaneously, $i(0^+) = i(0^-) = 0.1\ A$, and it is given that capacitor is initially uncharged $v_c(0^-) = 0$ and also $v_c(0^+) = 0$.

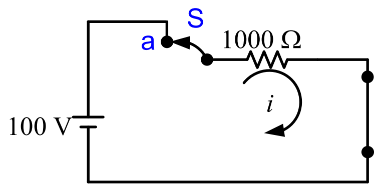


Figure 1.15: Example

When switch is at position b, and at $t = 0^+$, the circuit is as shown in Figure 1.16.

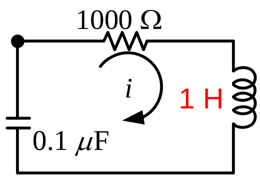


Figure 1.16: Example

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0 \qquad (1.4)$$

At $t = 0^+$

$$Ri(0^+) + L\frac{di}{dt}(0^+) + \frac{1}{C}\int i(0^+)dt = 0$$

It is given that capacitor is initially uncharged

$$\frac{1}{C}\int i(0^+)dt = v_c(0^+) = 0$$

$$\begin{aligned}
Ri(0^+) + L\frac{di}{dt}(0^+) &= 0 \\
1000 \times (0.1) + L\frac{di}{dt}(0^+) &= 0 \\
L\frac{di}{dt}(0^+) &= -100 \\
\frac{di}{dt}(0^+) &= \frac{-100}{1} \\
&= -100\ A/sec
\end{aligned}$$

Differentiating equation 1.4

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Substituting initial conditions

$$\begin{aligned}
R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{C} &= 0 \\
1000 \times (-100) + 1\frac{d^2i}{dt^2}(0^+) + \frac{0.1}{0.1 \times 10^{-6}} &= 0
\end{aligned}$$

$$\frac{d^2i}{dt^2}(0^+) = -9 \times 10^5 A/sec^2$$

Q 4-2 JAN-2014) In the circuit shown in Figure 1.17 switch K is changed from position a to b at t=0, steady state condition having been reached before switching. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ b at $t = 0^+$.

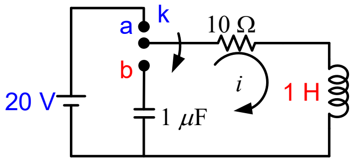


Figure 1.17: Example

Solution:

When switch is at position a and reached steady state, which is as shown in Figure 1.18(a).

$$\begin{aligned} Ri(0^-) &= V \\ i(0^-) &= \frac{V}{R} = \frac{20}{10} = 2\ A \end{aligned}$$

When the switch is at position b, the circuit is as shown in Figure 1.18 (b)

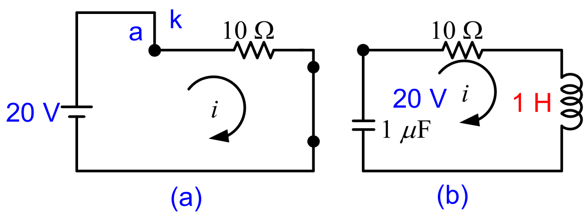


Figure 1.18: Example

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0 \tag{1.5}$$

At $t = 0^+$

$$Ri(0^+) + L\frac{di}{dt}(0+) + \frac{1}{C}\int i(0^+)dt = 0$$

It is given that capacitor is initially uncharged

$$\begin{aligned} \frac{1}{C}\int i(0^+)dt = v_c(0^+)) &= 0 \\ Ri(0^+) + L\frac{di}{dt}(0^+) &= 0 \\ 10 \times 2 + 1\frac{di}{dt}(0+) &= 0 \\ \frac{di}{dt}(0^+) &= -20\ A/sec \end{aligned}$$

Differentiating equation 1.5

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Substituting initial conditions

$$\begin{aligned} R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{C} &= 0 \\ 10 \times (-20) + 1\frac{d^2i}{dt^2}(0^+) + \frac{2}{1 \times 10^{-6}} &= 0 \end{aligned}$$

$$\begin{aligned} \frac{d^2i}{dt^2}(0^+) &= 200 - 2 \times 10^6 \\ &= -1.9998 \times 10^6 A/sec^2 \end{aligned}$$

Q 4-3) In the circuit shown in Figure ?? K is changed from position a to b at $t = 0$, steady state condition having been reached before switching. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ b at $t = 0^+$.

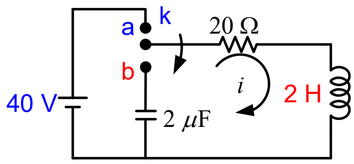


Figure 1.19: Example

Solution:

Before connecting to position b 1.20(a)

$$\begin{aligned} Ri(0^-) &= V \\ i(0^-) &= \frac{V}{R} = \frac{40}{20} = 2\ A = i(0^+) \end{aligned}$$

When switch is at position b,and at t=0+ circuit which is as shown in Figure 1.20 (b)

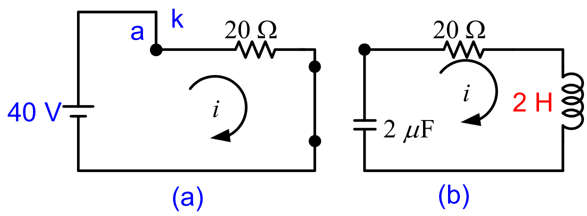


Figure 1.20: Example

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0 \tag{1.6}$$

$$Ri(0^+) + L\frac{di}{dt}(0^+) + \frac{1}{C}\int i(0^+)dt = 0$$

It is given that capacitor is initially uncharged

$$\frac{1}{C}\int i(0^+)dt = v_c(0^+) = 0$$

$$\begin{aligned} Ri(0^+) + L\frac{di}{dt}(0^+) &= 0 \\ 20 \times 2 + 2\frac{di}{dt}(0^+) &= 0 \\ \frac{di}{dt}(0^+) &= \frac{-40}{2} = -20\ A/sec \end{aligned}$$

Differentiating equation 1.6

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Substituting initial conditions

$$R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{C} = 0$$

$$20 \times (-20) + 2\frac{d^2i}{dt^2}(0^+) + \frac{2}{2 \times 10^{-6}} = 0$$

$$2\frac{d^2i}{dt^2}(0^+) = 400 - 1 \times 10^6 = -0.9996 \times 10^6 A/sec^2$$

$$\frac{d^2i}{dt^2}(0^+) = \frac{-0.9996 \times 10^6}{2} = -0.4998 \times 10^6 A/sec^2$$

Q 4-4-1- 2018-JULY) In the circuit shown in Figure 1.21 the switch S is changed from position a to b at $t = 0$, steady state condition having been reached before switching. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.

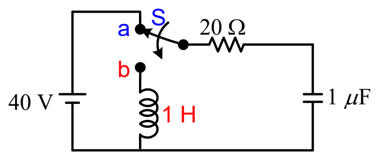


Figure 1.21: Example

Solution:

When the switch is at position a, and at $t = 0^-$ the circuit is as shown in Figure 1.21. When the steady state is reached, capacitor is fully charged with a capacitor voltage $v_c(0^-) = 40V$ and current in the circuit $i_c(0^-) = 0$.

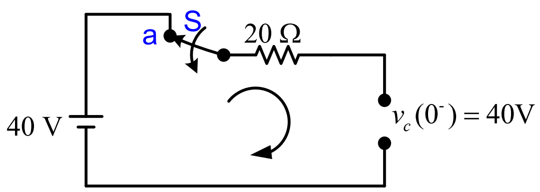


Figure 1.22: Example

At $t = 0^+$ the circuit is redrawn and is as shown in Figure 1.23 (a).

$$\frac{1}{C}\int i(0^+)dt = v_c(0^+) = 40V$$

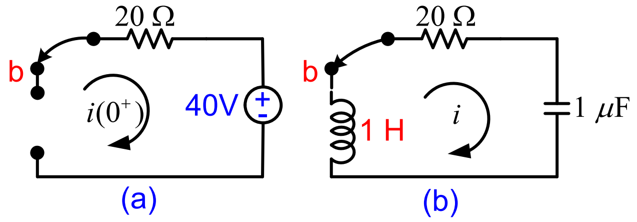


Figure 1.23: Example

At $t > 0^+$ the circuit is redrawn and is as shown in Figure 1.23 (b).

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0 \qquad (1.7)$$

$$\begin{aligned} Ri(0^+) + L\frac{di}{dt}(0^+) + \frac{1}{C}\int i(0^+)dt &= 0 \\ 20 \times 0 + 1\frac{di}{dt}(0^+) + 40 &= 0 \\ \frac{di}{dt}(0^+) &= -40A/sec \end{aligned}$$

Differentiating equation 1.7

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Substituting initial conditions

$$\begin{aligned} R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{C} &= 0 \\ 20 \times (-40) + 1\frac{d^2i}{dt^2}(0^+) + \frac{0}{C} &= 0 \\ \frac{d^2i}{dt^2}(0^+) &= 800A/sec^2 \end{aligned}$$

Q 5) In the circuit shown in Figure 1.27 the switch was in position a for sufficiently long time to have achieved steady state. At t=0, the switch was changed from a to b. Determine I_L and v_c their first and second derivatives at t=0+.

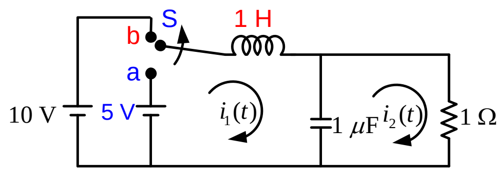


Figure 1.24: Example

Solution:

When switch is at position a and at t=0- the circuit is redrawn which is as shown in Figure 1.25

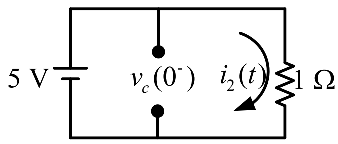


Figure 1.25: Example

$$\begin{aligned} i_1(0^-) &= \frac{5}{1} = 5\ A = i_1(0^+) \\ i_2(0^-) &= 5\ A = i_2(0^+) \\ v_c(0^-) &= 5\ V \quad v_c(0^+) = 5\ V \end{aligned}$$

When switch is at connected to b, and at $t = 0$ the circuit is redrawn which is as shown in Figure 1.26

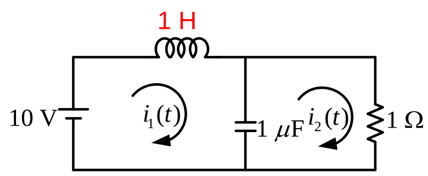


Figure 1.26: Example

$$1\frac{di_1}{dt} + v_c(t) = 10 \tag{1.8}$$

$$\frac{di_1}{dt}(0^+) \quad = \quad 10 - v_c(0^+) = 10 - 5 = 5A/sec$$

The voltage across capacitor is

$$v_c(t) = \frac{1}{C}\int (i_1 - i_2)dt \tag{1.9}$$

Differentiating above equation.

$$\frac{dv_c(t)}{dt} = \frac{1}{C}(i_1 - i_2) = \frac{1}{C}(5 - 5) = 0 \tag{1.10}$$

Differentiating equation 1.8

$$\frac{d^2 i_1}{dt} = \frac{1}{C}(i_1 - i_2) = \frac{1}{C}(5 - 5) = 0 \tag{1.11}$$

Differentiating equation 1.10

$$\frac{d^2 v_c(t)}{dt} = \frac{1}{C}\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right]$$

Substituting initial conditions

$$\begin{aligned}\frac{d^2 v_c(t)}{dt}(0^+) &= \frac{1}{C}\left[\frac{di_1}{dt}(0^+) - \frac{di_2(0+)}{dt}\right] \\ &= \frac{1}{1}[5 - 0] = 5V/sec^2\end{aligned}$$

Q 6) In the circuit shown in Figure 1.27 the switch is opened at t=0, after the network has attained the steady state with the switch closed. (a) Find an expression for the voltage across the switch at t=0+, (b) If the parameters are adjusted such that i(0+)=1A and $\frac{di}{dt}(0+) = -1A/sec$, what is the value of the derivative of the voltage across the switch.

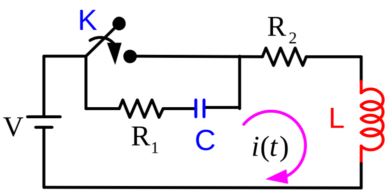


Figure 1.27: Example

Solution:

When switch is closed attains steady state is in which L acts as short circuit and capacitor acts as open circuit which is as shown in Figure 1.28

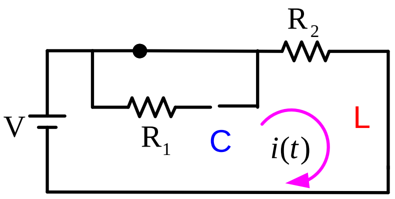


Figure 1.28: Example

$$\begin{aligned} R_2 i(0-) &= V \\ i(0-) &= \frac{V}{R_2} \end{aligned}$$

Also

$$v_c(0-) = 0\ V \qquad v_c(0+) = 0\ V$$

When switch is opened at t=0 circuit the voltage across the switch K is

$$\begin{aligned} v_k &= R_1 i(t) + \frac{1}{C}\int idt \\ v_k(0+) &= R_1 i(0+) + 0 = R_1\frac{V}{R_2} = V\frac{R_1}{R_2} \\ \frac{dv_k(t)}{dt} &= R_1\frac{di}{dt} + \frac{i}{C} \\ \frac{dv_k(t)}{dt}(0+) &= R_1(-1) + \frac{1}{C} = \frac{1}{C} - R_1\ V/s \end{aligned}$$

Q 7) In the circuit shown in Figure 1.29 the steady state is reached with switch is open. At t=0, the switch K is closed. Find the values of $V_a(0-)$ *and* $V_a(0+)$

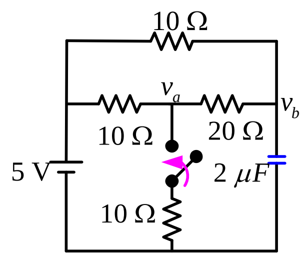


Figure 1.29: Example

Solution:

When switch is opened and when steady state is reached capacitor acts as an open circuit there is no current flows in a capacitor path, which is as shown in Figure 1.30

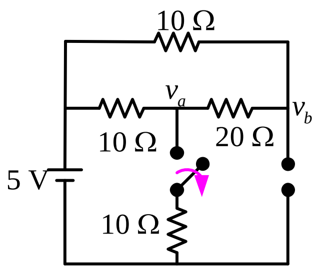


Figure 1.30: Example

$$v_a(0-) = 5\ V \qquad v_b(0-) = 5\ V$$

v_b is a Voltage across capacitor which is

$$v_b(0-) = 5\ V \qquad v_b(0+) = 5\ V$$

When switch is closed at t=0 apply KCL to node a and by initial conditions

$$
\begin{aligned}
\frac{v_a-5}{10}+\frac{v_a}{10}+\frac{v_a-v_b}{20} &= 0\\
\frac{v_a(0+)-5}{10}+\frac{v_a(0+)}{10}+\frac{v_a(0+)-v_b(0+)}{20} &= 0\\
\frac{v_a(0+)-5}{10}+\frac{v_a(0+)}{10}+\frac{v_a(0+)-5}{20} &= 0\\
\frac{v_a(0+)}{10}+\frac{v_a(0+)}{10}+\frac{v_a(0+)}{20}+\frac{-5}{10}+\frac{-5}{20} &= 0\\
\frac{5v_a(0+)}{20}+\frac{-15}{20} &= 0\\
\frac{5v_a(0+)}{20} &= \frac{15}{20}
\end{aligned}
$$

$$v_a(0+)=\frac{15}{5}=3Volts$$

Q 8-2019-JAN) In the circuit shown in Figure 1.31 the steady state is reached with switch is opened, at t=0 the switch k is closed. Find voltage across capacitor, initial values of $i_1, i_2, \frac{di_1}{dt}, \frac{di_2}{dt}$ at t=0+ and find $\frac{di_1}{dt}(\infty)$ where V=100V, $R_1 = 10\Omega, R_2 = 20\Omega, R_3 = 20\Omega, L = 1H, C = 1\mu F$

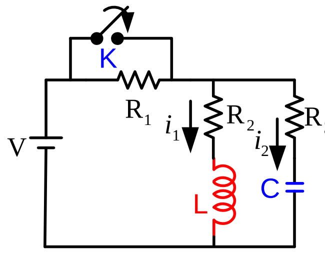


Figure 1.31: Example

Solution:

When switch is opened and when steady state is reached capacitor acts as open circuit and inductor acts as short circuit which is as shown in Figure 1.32(a).

$$
\begin{aligned}
i_2(0^-) &= 0A\\
i_1(0^-) &= \frac{V}{R_1+R_2}=\frac{100}{30}=3.33A
\end{aligned}
$$

Voltage across capacitor is same as the voltage across resistor R_2.

$$v_c(0^-)=i_1(0^-)\times R_2=3.33\times 20=66.67V$$

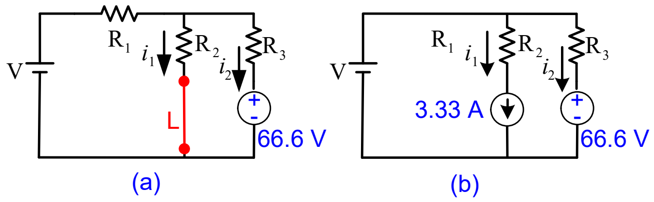


Figure 1.32: Example

When switch is closed at t=0 R_1 is short circuited. Inductor acts as current source with a value of 3.33 A and capacitor acts as voltage source with a value of 66.67 V which is as shown in Figure 1.32(b).

$$i_2(0^+)=\frac{100-66.67}{20}=1.67$$

For the inductor branch

$$
\begin{aligned}
R_2i_1(t)+L\frac{di_1}{dt} &= V\\
L\frac{di_1(0^+)}{dt} &= V-R_2i_1(0^+)\\
1\frac{di_1(0^+)}{dt} &= 100-20\times 3.33\\
\frac{di_1(0^+)}{dt} &= 33.3A/sec
\end{aligned}
$$

For the capacitor branch

$$R_3i_2(t)+\frac{1}{C}\int i_2(t)dt=V$$

Differentiating we get

$$
\begin{aligned}
R_3\frac{di_2(t)}{dt}+\frac{1}{C}i_2(t) &= 0\\
R_3\frac{di_2(0^+)}{dt}+\frac{1}{C}i_2(0^+) &= 0\\
\frac{di_2(0^+)}{dt} &= -\frac{1}{R_3C}i_2(0^+)\\
&= -\frac{1}{20\times 1\times 10^{-6}}1.678\\
&= 83500\ A/sec
\end{aligned}
$$

$$\frac{di_1(\infty)}{dt} = \frac{100}{20}=5A/sec$$