0.1 VTU Question Papers

2020-JULY-6b, For the circuit shown in Figure 1 find i) $i(0+)$, $v(0+)$, $\frac{di(0+)}{dt}$, $\frac{dv(0+)}{dt}$, $i(\infty)$, $v(\infty)$.

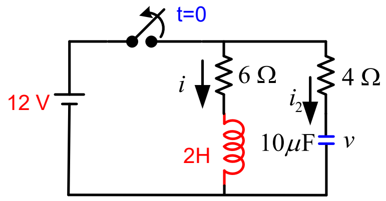

Figure 1: Example

Solution:

When the switch is closed at $t = 0+$ the capacitor acts as short circuit and inductor acts as open circuit which is as shown in Figure 3.

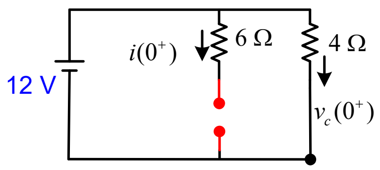

Figure 2: Example

$$
\begin{aligned}
i(0+) &= 0\\
v(0+) &= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{di(0+)}{dt} &= 0\\
\frac{dv(0+)}{dt} &= 0
\end{aligned}
$$

When the switch is closed at $t = \infty$ the capacitor acts as open circuit and inductor acts as short circuit which is as shown in Figure **??**.

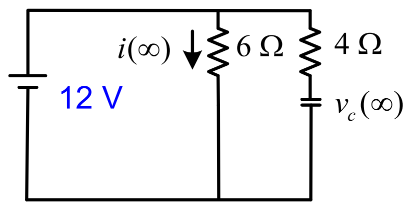

Figure 3: Example

$$
\begin{aligned}
i(\infty) &= \frac{12}{6} = 2A\\
v(\infty) &= 12V
\end{aligned}
$$

2020-Aug JAN-2014) For the circuit shown in Figure 4 switch K is changed from position a to b at t=0, steady state condition having been reached before switching. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ b at $t = 0^+$.

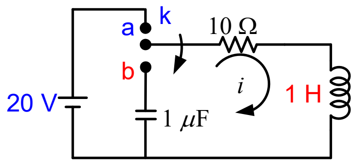

Figure 4: Example

Solution:

When switch is at position a and reached steady state, which is as shown in Figure 5(a).

$$
\begin{aligned}
Ri(0^-) &= V\\
i(0^-) &= \frac{V}{R} = \frac{20}{10} = 2\ A
\end{aligned}
$$

When the switch is at position b, the circuit is as shown in Figure 5 (b)

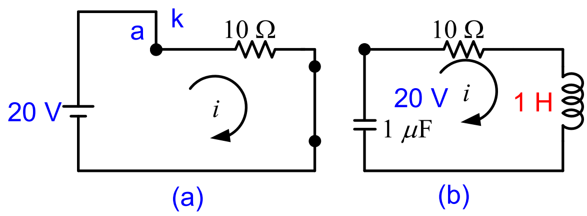

Figure 5: Example

$$
Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0 \tag{1}
$$

At $t = 0^+$

$$
Ri(0^+) + L\frac{di}{dt}(0+) + \frac{1}{C}\int i(0^+)dt = 0
$$

It is given that capacitor is initially uncharged

$$
\begin{aligned}
\frac{1}{C}\int i(0^+)dt = v_c(0^+)) &= 0\\
Ri(0^+) + L\frac{di}{dt}(0^+) &= 0\\
10 \times 2 + 1\frac{di}{dt}(0+) &= 0\\
\frac{di}{dt}(0^+) &= -20\ A/sec
\end{aligned}
$$

Differentiating equation 1

$$
R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0
$$

Substituting initial conditions

$$
\begin{aligned}
R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{C} &= 0\\
10 \times (-20) + 1\frac{d^2i}{dt^2}(0^+) + \frac{2}{1 \times 10^{-6}} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{d^2i}{dt^2}(0^+) &= 200 - 2 \times 10^6\\
&= -1.9998 \times 10^6 A/sec^2
\end{aligned}
$$

JULY-2019-CBCS In the circuit shown in Figure 6 switch S is changed from position a to b at $t = 0$. Solve for i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$ if $R = 100\Omega$, $L = 0.1H$ *and* $C = 0.25\mu F$ and V=100 V. Assume that the capacitor is initially uncharged.

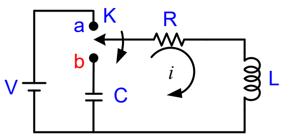

Figure 6: JULY-2019

Solution:

When the switch is at position a at $t = 0^-$, inductor acts as a short circuit which is as shown in Figure 7.

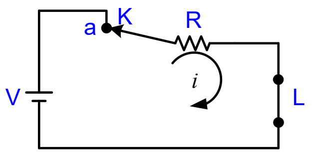

Figure 7: JULY-2019

$$\begin{aligned} Ri(0^-) &= V \\ i(0^-) &= \frac{V}{R} = \frac{100}{100} = 1\ A \end{aligned}$$

Current through inductor cannot change instantaneously.

$$i(0^+) = i(0^-) = 1\ A$$

It is given that capacitor is initially uncharged.

$$v_c(0^-) = 0 = v_c(0^+)V$$

When the switch is at position b, and at $t = 0^+$, the circuit is as shown in Figure 8.

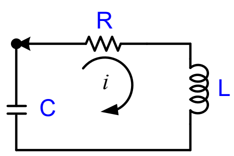

Figure 8: JULY-2019

By KVL around the loop

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0 \qquad (2)$$

At $t = 0^+$

$$Ri(0^+) + L\frac{di}{dt}(^0+) + \frac{1}{C}\int i(0^+)dt = 0$$

It is given that capacitor is initially uncharged

$$\frac{1}{C}\int i(0^+)dt = v_c(0^+) = 0$$

$$\begin{aligned} Ri(0^+) + L\frac{di}{dt}(0^+) &= 0 \\ 100 \times (1) + L\frac{di}{dt}(0^+) &= 0 \\ L\frac{di}{dt}(0+) &= -100 \\ \frac{di}{dt}(0+) &= \frac{-100}{0.1} \\ &= -1000\ A/sec \end{aligned}$$

Differentiating equation 2

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Substituting initial conditions

$$\begin{aligned} L\frac{d^2i}{dt^2}(0^+) &= -R\frac{di}{dt}(0^+) - \frac{i(0^+)}{C} \\ 0.1\frac{d^2i}{dt^2}(0^+)+ &= -100 \times (-1000) - \frac{1}{0.25 \times 10^{-6}} \\ 0.1\frac{d^2i}{dt^2}(0^+) &= 0.1 \times 10^6 + 4 \times 10^6 \\ \frac{d^2i}{dt^2}(0^+) &= 39 \times 10^6 A/sec^2 \end{aligned}$$

JULY-2018-CBCS The switch in the network shown in Figure 9 is closed at $t = 0$. Determine the voltage across the capacitor.

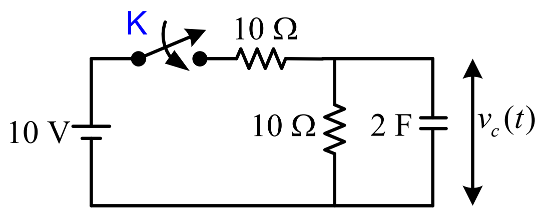

Figure 9: 2018-CBCS-Question Paper

Solution:

Before closing the switch at $t = 0^-$ the voltage across the capacitor is

$$v_c(0^-) = i(0^-)R_2 \qquad (3)$$

The switch is closed at $t = 0$at $t = 0^+$

$$\begin{aligned} v_c(0^+) &= i(0^+)10 \\ v_c(0^+) &= \frac{10}{10+10} \times 10 \\ &= 5V \end{aligned}$$

JAN-2017-CBCS In the circuit shown in Figure 10 V=10 v $R = 10\ \Omega$ $L = 1\ H$ and $C = 10\ \mu F$ and $v_c(0) = 0$, find $i(0^+)$, $\frac{di}{dt}(0^+)$ and $\frac{d^2i}{dt^2}(0^+)$.

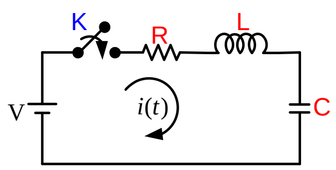

Figure 10: 2017-CBCS-Question Paper

Solution:

The switch is closed at $t = 0^+$ the inductor acts as open circuit and capacitor acts as short circuit which is as shown in Figure 11. At $t = 0^-$, $i(0^-) = i(0^+) = 0$

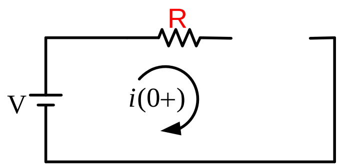

Figure 11: Example

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 10 \qquad (4)$$

At $t = 0^+$

$$\begin{aligned} Ri(0^+) + L\frac{di}{dt}(0^+) + \frac{1}{C}\int i(0^+)dt &= V \\ \frac{di}{dt}(0+) &= \frac{V}{L} \\ &= 10A/sec \end{aligned}$$

Differentiating equation 4

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0 \qquad (5)$$

Substituting initial conditions

$$R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{C} = 0 \qquad (6)$$

$$\begin{aligned} L\frac{d^2i}{dt^2}(0^+) &= -10 \times 10 - \frac{i(^0+)}{C} \\ 1\frac{d^2i}{dt^2}(0^+) &= 100 - \frac{0}{C} \\ \frac{d^2i}{dt^2}(0^+) &= -100A/sec^2 \end{aligned}$$

2020-July-5b-CBCS JAN-2017-6b-CBCS In the network shown in Figure 12 a steady state is reached with the switch K open. At t=0, the switch is closed. For the given element values, determine $v_a(0^-)$ *and* $v_a(0^+)$

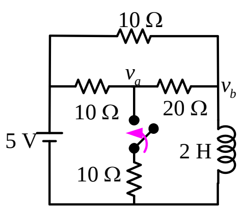

Figure 12: 2017-CBCS-Question Paper

Solution:

When switch is opened and steady state reached inductor acts as short circuit which is as shown in Figure 13 (a)

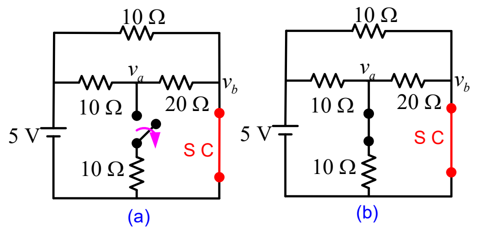

Figure 13: 2017-CBCS-Question Paper

$$\begin{aligned} \frac{V_a(0^-) - 5}{10} + \frac{V_a(0-) - V_b(0^-)}{20} &= 0 \\ V_a(0^-)\left[\frac{1}{10} + \frac{1}{20}\right] &= \frac{5}{10} \\ V_a(0^-) = \frac{0.5}{0.15} &= 3.333V \end{aligned}$$

$$i_L(0^-) = i_L(0^+) = [0.15] = \frac{V_a(0-)}{20} + \frac{5}{10} = 0.667A$$

When switch is closed, 10 Ω resistor is connected to a point v_a steady state reached inductor acts as short circuit which is as shown in Figure 13 (b)

$$\begin{aligned} \frac{v_a - 5}{10} + \frac{v_a - v_b}{20} + \frac{v_a}{10} &= 0 \\ v_a(0^+)\left[\frac{1}{10} + \frac{1}{10} + \frac{1}{20}\right] - \frac{v_b}{20} &= \frac{5}{10} \\ 0.25v_a(0^+) - 0.05v_b(0+) &= 0.5 \end{aligned}$$

$$\begin{aligned} \frac{v_b - v_a}{20} + \frac{v_b - 5}{10} + i_L &= 0 \\ -0.05v_a(0^+) + 0.15v_b(0+) &= 0.5 - 0.667A \\ -0.05v_a(0^+) + 0.15v_b(0+) &= -0.1667 \end{aligned}$$

JAN-2017-NON-CBCS For the circuit shown in Figure 14 the switch S is changed from position 1 to 2 at $t = 0$. The circuit was under steady state before this action. Determine the value v and i at $t = 0^+$ and their first and second derivatives.

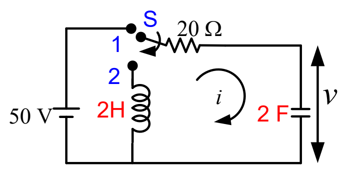

Figure 14: Example

Solution:

When switch was at position 1 at $t = 0^-$, under steady state condition capacitor charges with voltage of $v(0^-) = 50 = v(0^+)$ and after that it acts as an open circuit which is as shown in Figure 15

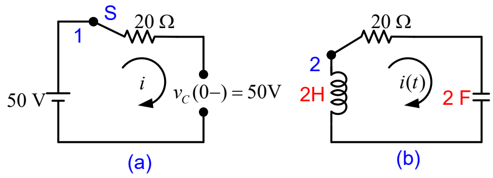

Figure 15: Example

At $t = 0^-$, inductor is in open circuit and capacitor is after fully charging it is also in open circuit state. That is

$$i(0^-) = 0 \quad and\ also \quad i(0^+) = 0 \tag{7}$$

When switch is at position 2, and at $t = 0^+$ the circuit diagram is as shown in Figure 15

Applying KVL for the circuit we have

$$\begin{aligned} Ri + L\frac{di}{dt} + \frac{1}{C}\int idt &= 0 \\ Ri + L\frac{di}{dt} + v_c(t)dt &= 0 \end{aligned}$$

At $t = 0^+$ and $v_c(0^+) = 50$

$$\begin{aligned} Ri(0^+) + L\frac{di}{dt}(0^+) + v_c(0^+) &= 0 \\ 20 \times 0 + 2\frac{di}{dt}(0^+) + 50 &= 0 \\ \frac{di}{dt}(0^+) &= \frac{-50}{2} = -25 \end{aligned}$$

Differentiating equation (1)

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0 \tag{8}$$

Substituting initial conditions

$$\begin{aligned} R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{C} &= 0 \\ 20 \times (-25) + 2\frac{d^2i}{dt^2}(0+) + \frac{0}{C} &= 0 \\ \frac{d^2i}{dt^2}(0^+) &= \frac{500}{2} = 250A/sec^2 \end{aligned}$$

July-2016 For the circuit shown in Figure 16 the switch K is changed from position A to B at t=0. After having reached steady state in position A. Find i $\frac{di}{dt}, \frac{d^2i}{dt^2}, \ and \frac{d^3i}{dt^3}$ at t=0+.

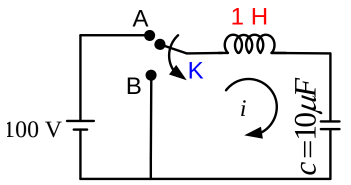

Figure 16: Example

Solution:

Before connecting to position B switch was at position A at t=0- under steady state condition capacitor charges with voltage of $v(0-) = 100 = v(0+)$ and after that it acts as an open circuit which is as shown in Figure 17(a)

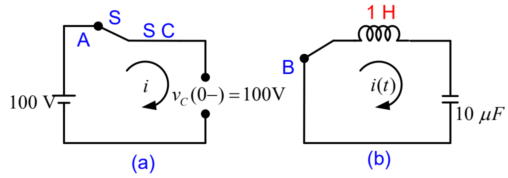

Figure 17: Example

At t = 0- , inductor is in short circuit and capacitor is after fully charged and it is also in open circuit state.

$$i(0-) = 0 \quad and\ also \quad i(0+) = 0 \tag{9}$$

When switch is at position B, and at t=0+ the circuit diagram is as shown in Figure 15 (b)

Applying KVL for the circuit we have

$$\begin{aligned} L\frac{di}{dt} + \frac{1}{C}\int idt &= 0 \\ L\frac{di}{dt} + v_c(t)dt &= 0 \end{aligned}$$

At t = 0+ and $v_c(0+) = 100$

$$\begin{aligned} L\frac{di}{dt}(0+) + v_c(0+) &= 0 \\ 1\frac{di}{dt}(0+) + 100 &= 0 \\ \frac{di}{dt}(0+) &= -100 \end{aligned}$$

Differentiating equation (1)

$$L\frac{d^2i}{dt^2} + \frac{i}{C} = 0 \tag{10}$$

Substituting initial conditions

$$\begin{aligned} \frac{d^2i}{dt^2}(0+) + \frac{i(0+)}{C} &= 0 \\ 1\frac{d^2i}{dt^2}(0+) + \frac{0}{C} &= 0 \\ \frac{d^2i}{dt^2}(0+) &= 0A/sec^2 \\ \frac{d^3i}{dt^3}(0+) &= 0A/sec^2 \end{aligned}$$

July-2016-2 For the circuit shown in Figure 18 the switch K is opened at t=0. Find i $\frac{di}{dt}, v_c, \frac{dv_c}{dt}$ at t=0+.

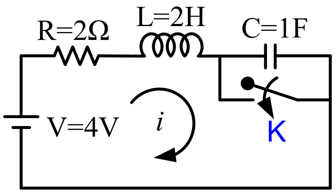

Figure 18: Example

Solution: When switch is at position a at t=0- circuit which is as shown in Figure 19

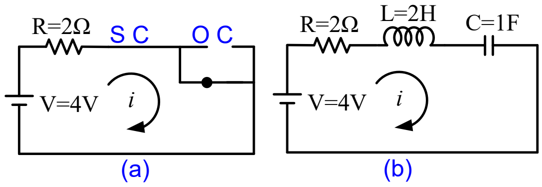

Figure 19: Example

When switch is at opened, at t=0+ circuit which is as shown in Figure 19 (a)

$$V = Ri(0-) \tag{11}$$

$$i(0-) = \frac{V}{R} = \frac{4}{2} = 2\ A \tag{12}$$

When switch is at opened, at t=0+ circuit which is as shown in Figure 19 (b)

Applying KVL

$$V = \frac{1}{C}\int idt + L\frac{di}{dt} + Ri \tag{13}$$

At t=0+

$$2 = 0 + 1 \times \frac{di(0+)}{dt} + 1 \times i(0+) \tag{14}$$

$$\begin{aligned} 2 &= 0 + 1 \times \frac{di(0+)}{dt} + 1 \times i(0+) \\ \frac{di}{dt}(0+) &= 2 - 1 \times 2 = 0 \end{aligned}$$

The voltage across capacitor is

$$v_c(t) = \frac{1}{C}\int i \tag{15}$$

$$\begin{aligned} \frac{dv_c(t)}{dt} &= \frac{1}{C}i \\ \frac{dv_c(t)}{dt}(0+) &= \frac{1}{C}i(0+) \\ \frac{dv_c(t)}{dt}(0+) &= \frac{1}{1}2 = 2V \end{aligned}$$

July-2015-6-b For the circuit shown in Figure 20 the switch K is opened at $t = 0$ after reaching the steady state condition. Determine voltage across switch and its first and second derivatives at $t = 0^+$.

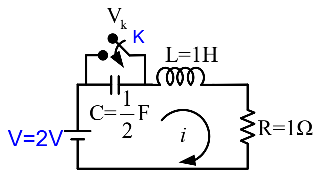

Figure 20: Example

Solution: Before opening switch and at t=0- circuit diagram is as shown in Figure 21 (a)

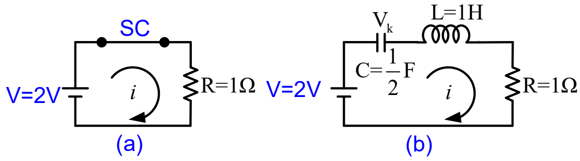

Figure 21: Example

$$V = Ri(0-)$$

$$i(0-) = \frac{V}{R} = \frac{2}{1} = 2\ A = i(0+)$$

When switch is opened at t=0+ circuit diagram is as shown in Figure 21 (b)

Applying KVL

$$V = \frac{1}{C}\int idt + L\frac{di}{dt} + Ri \tag{16}$$

At t=0+

$$2 = 0 + 1 \times \frac{di(0+)}{dt} + 1 \times i(0+) \tag{17}$$

$$\begin{aligned} 2 &= 0 + 1 \times \frac{di(0+)}{dt} + 1 \times i(0+) \\ \frac{di}{dt}(0+) &= 2 - 1 \times 2 = 0 \end{aligned}$$

Differentiating Equation 16

$$\begin{aligned} 0 &= \frac{i}{C} + L\frac{d^2i}{dt^2} + R\frac{di}{dt} \\ 1 \times \frac{d^2i_1}{dt^2}(0+) &= -R\frac{di}{dt}(0+) - \frac{i(0+)}{C} \\ \frac{d^2i(0+)}{dt^2} &= -1 \times 0 - \frac{2}{1/2}(0+) = -4\ A/sec^2 \end{aligned}$$

Differentiating Equation 16

$$\begin{aligned} 0 &= \frac{1}{C}\frac{di}{dt} + L\frac{d^3i}{dt^3} + R\frac{d^2i}{dt^2} \\ 1 \times \frac{d^3i_1}{dt^3}(0+) &= -R\frac{d^2i(0+)}{dt^2} - \frac{1}{C}\frac{d(0+)i}{dt} \\ \frac{d^3i(0+)}{d3^2} &= -1 \times (-4) - 0 = 4\ A/sec^2 \end{aligned}$$

The voltage across capacitor is

$$V_k + L\frac{di}{dt} + R\times i = 2 \tag{18}$$

$$\begin{aligned}\frac{dV_k}{dt} + L\frac{d^2i}{dt^2} + R\times\frac{di}{dt} &= 0\\ \frac{dv_c(t)}{dt}(0+) &= \frac{1}{C}(5-5) = 0\end{aligned}$$

Differentiating equation (1)

$$\frac{d^2V_k}{dt^2} + L\frac{d^3i}{dt^3} + R\times\frac{d^2i}{dt^2} = 0 \tag{19}$$

$$\begin{aligned}\frac{d^2V_k(0+)}{dt^2} &= -L\frac{d^3i(0+)}{dt^3} + R\times\frac{d^2i(0+)}{dt^2}\\ \frac{d^2V_k(0+)}{dt^2} &= -1\times 4 - 1\times(-4) = 0\ V/sec^2\end{aligned}$$

DEC-2015 6-a For the circuit shown in Figure 22 the switch K is changed from position A to B at t=0, the steady state having been reached before switching. Calculate i, $\frac{di}{dt}$, *and* $\frac{d^2i}{dt^2}$ at t=0+.

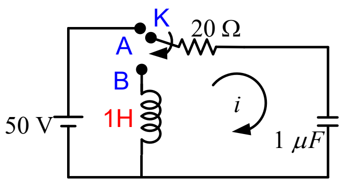

Figure 22: Example

Solution:

Before connecting to position 2 switch was at position 1 at t=0- under steady state condition capacitor charges with voltage of $v(0-) = 50 = v(0+)$ and after that it acts as an open circuit which is as shown in Figure 23 (a)

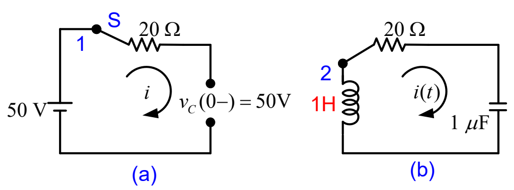

Figure 23: Example

At t = 0- , inductor is in open circuit and capacitor is after fully charging it is also in open circuit state. That is

$$i(0-) = 0 \ \ and\ also\ \ i(0+) = 0 \tag{20}$$

When switch is at position 2, and at t=0+ the circuit diagram is as shown in Figure 23 (b)

Applying KVL for the circuit we have

$$\begin{aligned}Ri + L\frac{di}{dt} + \frac{1}{C}\int idt &= 0\\ Ri + L\frac{di}{dt} + v_c(t)dt &= 0\end{aligned}$$

At t = 0+ and $v_c(0+) = 50$

$$\begin{aligned}Ri(0+) + L\frac{di}{dt}(0+) + v_c(0+) &= 0\\ 20\times 0 + 1\frac{di}{dt}(0+) + 50 &= 0\\ \frac{di}{dt}(0+) &= \frac{-50}{1} = -50\ A/sec\end{aligned}$$

Differentiating equation (1)

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0 \tag{21}$$

Substituting initial conditions

$$\begin{aligned}R\frac{di}{dt}(0+) + L\frac{d^2i}{dt^2}(0+) + \frac{i(0+)}{C} &= 0\\ 20\times(-50) + 1\frac{d^2i}{dt^2}(0+) + \frac{0}{C} &= 0\\ \frac{d^2i}{dt^2}(0+) &= \frac{1000}{1} = 1000A/sec^2\end{aligned}$$

Q DEC-2015 6b) In the circuit shown in Figure 24 the steady state is reached with switch K is open. The switch K is closed at $t = 0$. Solve for $i_1, i_2, \frac{di_1}{dt}, \frac{di_2}{dt}$ at $t = 0+$.

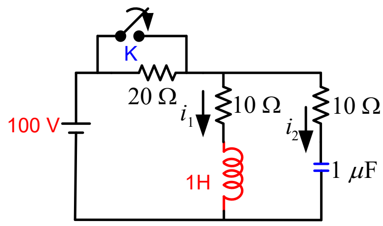

Figure 24: Example

Solution:

When switch is opened and when steady state is reached capacitor acts as open circuit and inductor acts as short circuit which is as shown in Figure 25(a).

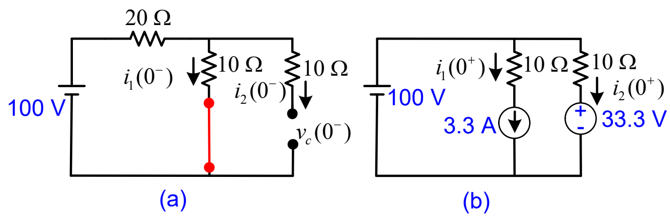

Figure 25: Example

$$\begin{aligned}i_1(0-) &= \frac{100}{20+10} = 3.33A\\ i_2(0-) &= 0\end{aligned}$$

Voltage across capacitor is voltage across R_2

$$v_c(0-) = i_1(0-) \times 10 = 3.33 \times 10 = 33.33V$$

When switch is closed at $t = 0$ 20 Ω is short circuited. Inductor acts as current source with a value of 3.33 A and capacitor acts as voltage source with a value of 33.33 V which is as shown in Figure 25(b).

$$i_2(0+) = \frac{100 - 33.33}{10} = 6.667$$

Apply KVL for the inductor branch

$$100 = 10i_1 + L\frac{di_1}{dt}$$

At $t = 0^+$

$$\begin{aligned} 10i_1(0^+) + L\frac{di_1(0^+)}{dt} &= 100 \\ 1\frac{di_1(0^+)}{dt} &= 100 - 10 \times 3.33 \\ \frac{di_1(0^+)}{dt} &= 66.7A/sec \end{aligned}$$

Apply KVL for the capacitor branch.

$$100 = 10i_2 + \frac{1}{C}\int i_2 dt$$

At $t = 0^+$

$$100 = 10i_2(0+) + \frac{1}{C}\int i_2(0+)dt$$

Differentiating we get

$$\begin{aligned} 0 &= 10\frac{di_2(0+)}{dt} + \frac{1}{C}i_2(0+) \\ \frac{di_2(0+)}{dt} &= -\frac{1}{10 \times C}i_2(0+) \\ &= -\frac{1}{10 \times 1 \times 10^{-6}} \times 6.667 \\ &= -0.667 \times 10^6 \ A/sec \end{aligned}$$

2014-JULY) In the circuit shown in Figure 26 the switch S is changed from position a to b at $t = 0$, steady state condition having been reached before switching. Find the values of i, $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0^+$.

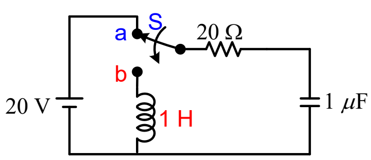

Figure 26: 2014-JULY

Solution:

When the switch is at position a, and at $t = 0^-$ the circuit is as shown in Figure ??. When the steady state is reached, capacitor is fully charged with a capacitor voltage $v_c(0^-) = 40V = v_c(0^+)$ and current in the circuit $i_L(0^-) = 0 = i_L(0^+)$.

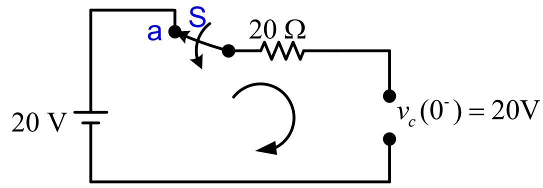

Figure 27: 2014-JULY

At $t = 0^+$ the circuit is redrawn and is as shown in Figure 28 (a).

$$\frac{1}{C}\int i(0^+)dt = v_c(0^+) = 20V$$

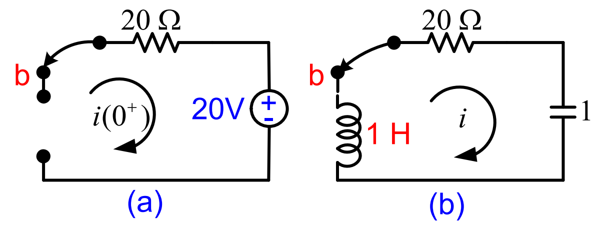

Figure 28: 2014-JULY

At $t > 0^+$ the circuit is redrawn and is as shown in Figure ?? (b).

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0 \tag{22}$$

$$\begin{aligned} Ri(0^+) + L\frac{di}{dt}(0^+) + \frac{1}{C}\int i(0^+)dt &= 0 \\ 20 \times 0 + 1\frac{di}{dt}(0^+) + 20 &= 0 \\ \frac{di}{dt}(0^+) &= -20A/sec \end{aligned}$$

Differentiating equation 22

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

Substituting initial conditions

$$\begin{aligned} R\frac{di}{dt}(0^+) + L\frac{d^2i}{dt^2}(0^+) + \frac{i(0^+)}{C} &= 0 \\ 20 \times (-20) + 1\frac{d^2i}{dt^2}(0^+) + \frac{0}{C} &= 0 \\ \frac{d^2i}{dt^2}(0^+) &= 400A/sec^2 \end{aligned}$$

Q 2012-JULY-6b and 2013-DEC-6a) In the circuit shown in Figure 29 the switch K is changed from position 1 to 2 at $t = 0$, steady state condition having reached before switching. Find i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t = 0^+$.

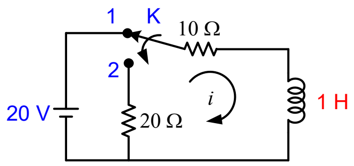

Figure 29: 2013-JULY-6a-1

Solution:

When switch is at position 1 and when steady state is reached inductor acts as short circuit which is as shown in Figure 30. At $t = 0^-$

$$i(0^-) = \frac{20}{10} = 2A$$

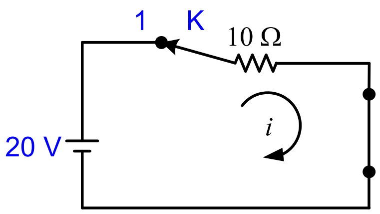

Figure 30: 2013-JULY-6a-1

Current cannot change instantaneously $i(0^+) = i(0^-) = 2A$, at $t = 0^+$ the circuit is as shown in Figure 31

$$\begin{aligned} 30i + L\frac{di}{dt} &= 0 \\ 30i(0^+) + L\frac{di(0^+)}{dt} &= 0 \\ \frac{di(0^+)}{dt} &= -30i(0^+) = -30 \times 2 \\ &= -60\ A/sec \end{aligned}$$

Differentiating

$$\begin{aligned} \frac{d^2i(0+)}{dt^2} &= -\frac{30di(0^+)}{dt} \\ &= -30 \times (-60) \\ &= 1800\ A/sec^2 \end{aligned}$$

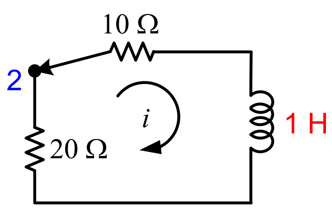

Figure 31: 2013-JULY-6a-1

Q 2013-DEC-6b) In the circuit shown in Figure 32 the switch K is opened at $t = 0$. Find v, $\frac{dv}{dt}$, $\frac{d^2v}{dt^2}$ at $t = 0^+$.

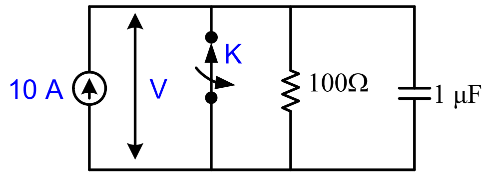

Figure 32: 2013-JULY-6a-1

Solution:

The switch is closed at $t = 0^-$. When switch is opened at $t = 0$ and which is as shown in Figure ??. At $t = 0^-$

$$v(0^-) = 0 = v(0^+)$$

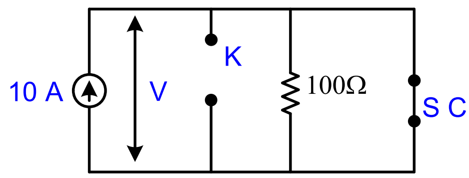

Figure 33: 2013-JULY-6a-1

At $t \geq 0^+$ the circuit is as shown if Figure 34

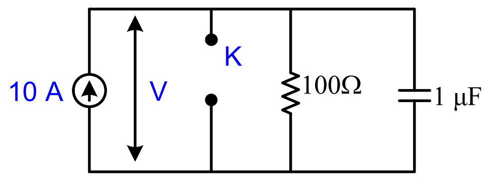

Figure 34: 2013-JULY-6a-1

$$\begin{aligned} \frac{V(t)}{R} + C\frac{dV(t)}{dt} &= I \\ \frac{V(0^+)}{R} + C\frac{dV(0^+)}{dt} &= 10 \\ \frac{dV(0^+)}{dt} &= \frac{10}{1 \times 10^{-6}} \\ &= 10 \times 10^6 V/sec \end{aligned}$$

Differentiating

$$\begin{aligned} \frac{1}{R}\frac{dV(t)}{dt} + C\frac{d^2V(t)}{dt^2} &= 0 \\ \frac{1}{R}\frac{dV(0^+)}{dt} + C\frac{d^2V(0^+)}{dt^2} &= 0 \\ \frac{d^2V(0^+)}{dt^2} &= -\frac{1}{RC}\frac{dV(0^+)}{dt} \\ &= -\frac{10 \times 10^6}{100 \times 1 \times 10^{-6}} \\ &= -1 \times 10^{12} V/sec^2 \end{aligned}$$

Q 2013-JULY-6c) In the circuit shown in Figure 35 the switch K is closed at $t = 0$, all capacitor voltages and inductor currents are zero. Find $v_1, v_2, v_3, \frac{dv_1}{dt}, \frac{dv_2}{dt}, \frac{dv_3}{dt}$ at $t = 0^+$.

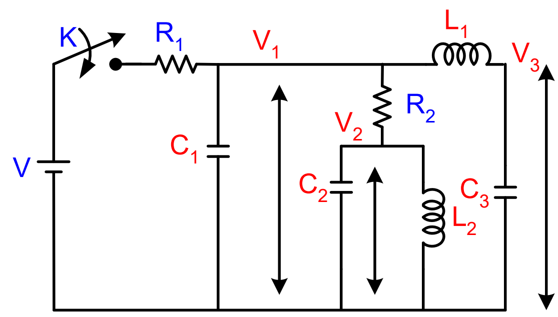

Figure 35: 2013-JULY-6c

Solution:

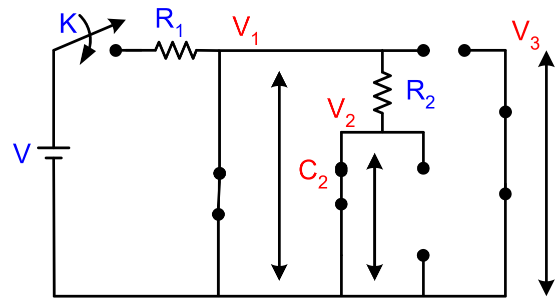

Figure 36: 2013-JULY-6c

When switch K is closed at $t = 0$ Capacitors C_1, C_2 *and* C_3 acts as a short circuit and inductors L_1 *and* L_3 acts as a open circuit.

KCL for node V_1 and at $t = 0^+$

$$\begin{aligned}
\frac{V_1 - v(t)}{R_1} + C_1\frac{dV_1}{dt} + \frac{V_1 - V_2}{R_2} &= 0 \\
\frac{V_1 - v(0^+)}{R_1} + C_1\frac{dV_1(0^+)}{dt} + \frac{V_1(0^+) - V_2(0^+)}{R_2} &= 0 \\
\frac{0 - v(0^+)}{R_1} + C_1\frac{dV_1(0^+)}{dt} + \frac{0 - 0}{R_2} &= 0
\end{aligned}$$

$$\frac{dV_1(0^+)}{dt} = \frac{v(0^+)}{C_1 R_1}$$

KCL for node V_2 and at $t = 0^+$

$$\begin{aligned}
\frac{V_2 - V_1}{R_2} + C_2\frac{dV_2}{dt} + \frac{1}{L_2}\int (V_2)dt &= 0 \\
0 + C_2\frac{dV_2(0^+)}{dt} + L_2\int (V_2(0^+))dt &= 0 \\
\frac{dV_2(0^+)}{dt} &= 0
\end{aligned}$$

KCL for node V_3 and at $t = 0^+$

$$\begin{aligned}
\frac{1}{L_1}\int (V_3 - V_1)dt + C_3\frac{dV_3}{dt} + &= 0 \\
\frac{1}{L_1}\int (V_3(0^+) - V_1(0^+))dt + C_3\frac{dV_3(0^+)}{dt} + &= 0 \\
\frac{dV_3(0^+)}{dt} &= 0
\end{aligned}$$

Q 2011-DEC-6b) In the circuit shown in Figure 37 the switch K is closed at $t = 0$. Find i, $\frac{di_1}{dt}, \frac{d_1^2}{dt^2}, \frac{di_2}{dt}, \frac{d_2^2}{dt^2}$ at $t = 0^+$.

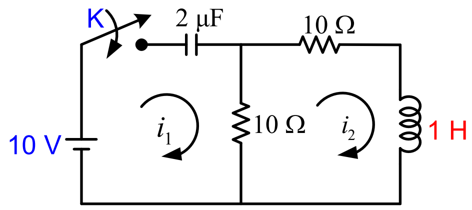

Figure 37: 2013-JULY-6a-1

Solution:

At $t = 0^-$

$$v_c(0^-) = 0 = v(0^+)$$

$$i_1(0^-) = i_2(0^-) = i_2(0^+) = 0$$

The switch is closed at $t = 0$ and at $t = 0^+$ the circuit is as shown in Figure 37.

$$i_1(0^+) = \frac{10}{10} = 1A \quad i_2(0^+) = 0A$$

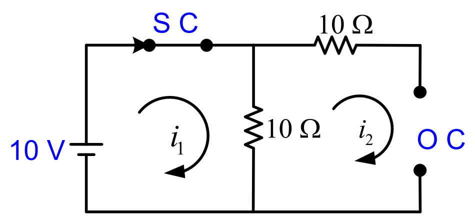

Figure 38: 2013-JULY-6a-1

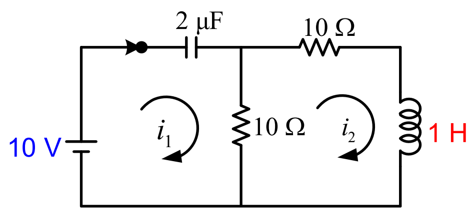

Figure 39: 2013-JULY-6a-1

Applying KVL for the loops

$$\frac{1}{C}\int i_1(t)dt + 10[i_1(t) - i_2(t)] = 10 \tag{23}$$

$$10[i_2(t) - i_1(t)] + 10i_2(t) + 1\frac{di_2}{dt} = 0 \tag{24}$$

At $t = 0^+$ the equation 24 becomes

$$10[i_2(0^+) - i_1(0^+)] + 10 i_2(0^+) + 1\frac{di_2(0^+)}{dt} = 0$$
$$10[0-1] + 10\times 0 + 1\frac{di_2(0^+)}{dt} = 0$$
$$\frac{di_2(0^+)}{dt} = 10\ A/sec$$

Differentiating Equation 23

$$\frac{i_1}{C} + 10[\frac{di_1}{dt} - \frac{di_2}{dt}] = 0 \qquad (25)$$

At $t = 0^+$

$$\frac{i_1(0^+)}{C} + 10[\frac{di_2(0^+)}{dt} - \frac{di_2(0^+)}{dt}] = 0$$

$$\frac{i_1(0^+)}{C} + 10[\frac{di_1(0^+)}{dt} - \frac{di_2(0^+)}{dt}] = 0$$
$$\frac{1}{2\times 10^{-6}} + 10[\frac{di_1(0^+)}{dt} - 10] = 0$$
$$10\frac{di_1(0^+)}{dt} = 100 - \frac{1}{2\times 10^{-6}}$$
$$\frac{di_1(0^+)}{dt} = -49990\ A/sec$$

Differentiating Equation 24

$$10\frac{di_2}{dt} - 10\frac{di_1}{dt} + 10\frac{di_2}{dt} + 1\frac{d^2 i_2}{dt^2} = 0 \qquad (26)$$

At $t = 0^+$

$$\frac{d^2 i_2(0^+)}{dt^2} = -10\times 10 + 10\times(-49990) - 10\times 10$$
$$\frac{d^2 i_2(0^+)}{dt^2} = -500100\ A/sec^2$$

Differentiating Equation 25

$$\frac{1}{C}\frac{di_1}{dt} + 10\frac{d^2 i_1}{dt^2} - 10\frac{d^2 i_2}{dt^2} = 0$$

At $t = 0^+$

$$10\frac{d^2 i_1(0^+)}{dt^2} = 10\frac{d^2 i_2(0^+)}{dt^2} - \frac{1}{2\times 10^6}\frac{di_1(0^+)}{dt}$$
$$\frac{d^2 i_1(0^+)}{dt^2} = -50010 - 0.5\times 10^5(-49990)$$
$$\frac{d^2 i_1(0^+)}{dt^2} = 2.499\times 10^9\ A/sec^2$$

Q 2020-JULY-6a, 2011-JULY-6c) Determine V, $\frac{dV}{dt}$, $\frac{d^2V}{dt^2}$ at $t = 0^+$ when the switch K is opened at $t = 0$ for the circuit shown in Figure 40.

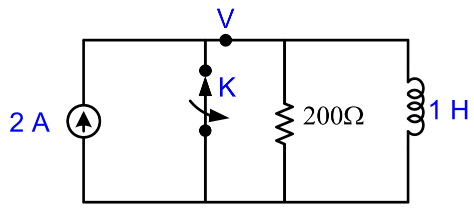

Figure 40: 2011-JULY-6C1

Solution:

At $t = 0^-$ the switch is closed

$$i_L(0^-) = 0$$

At $t = 0^+$ the switch is open which is as shown in Figure 41

$$V(0^+) = 2A\times 200 = 400V$$

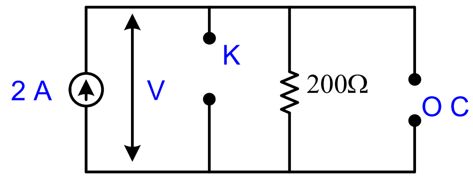

Figure 41: 2011-JULY-6C1

At $t \geq 0^+$ the s is as shown in Figure 42

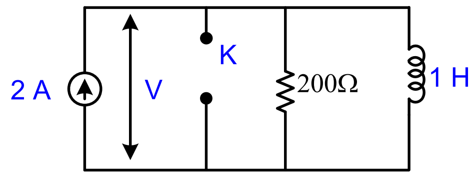

Figure 42: 2011-JULY-6C1

$$\frac{V(t)}{R} + \frac{1}{L}\frac{1}{C}\int V(t)(t)dt = 2 \qquad (27)$$

Differentiating Equation 27

$$\frac{1}{R}\frac{dV(t)}{dt} + \frac{1}{L}V(t) = 0$$

At $t = 0^+$

$$\frac{1}{R}\frac{dV(0^+)}{dt} + \frac{1}{L}V(0^+) = 0$$

$$\frac{1}{R}\frac{dV(0^+)}{dt} + \frac{1}{L}V(0^+) = 0$$
$$\frac{1}{200}\frac{dV(0^+)}{dt} = -\frac{1}{L}V(0^+) = -400V$$
$$\frac{dV(0^+)}{dt} = -\frac{1}{L}V(0^+) = -8\times 10^4 V/sec$$

Differentiating

$$\frac{1}{R}\frac{d^2 v(t)}{dt^2} = \frac{1}{L}\frac{dV(t)}{dt}$$

At $t = 0^+$

$$\frac{1}{R}\frac{d^2 v(0^+)}{dt^2} = \frac{1}{L}\frac{dV(0^+)}{dt}$$

$$\frac{d^2 v(0^+)}{dt^2} = 200\times 8\times 10^4 = 16\times 10^4 V/sec^2$$

Q Prep-Q1) In Figure 47 the switch was closed for a long time before it is opened at t=0. Find $i_1(0^-)$, $i_2(0^-)$ $i_1(0^+) i_2(0^+)$.

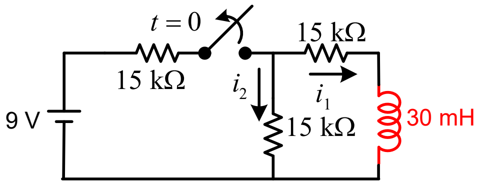

Figure 43: Prep-Q1

Solution:

At $t = 0^-$ the switch was closed and the modified circuit is as shown in Figure 44

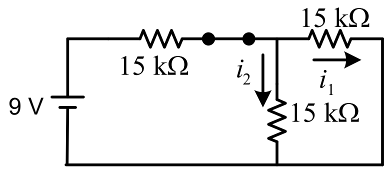

Figure 44: Prep-Q1

The impedance of the circuit is

$$\begin{aligned} R_{eq} &= 15 + \frac{15 \times 15}{15 + 15} k\Omega \\ &= 15 + 7.5 = 22.5k\Omega \end{aligned}$$

Total current supplied form the battery is

$$I = \frac{9}{R_{eq}} = \frac{9}{22.5k\Omega} 0.4mA$$

$$\begin{aligned} i_1(0^-) &= 0.4mA \frac{15}{15 + 15} 0.2mA \\ i_2(0^-) &= 0.4mA \frac{15}{15 + 15} 0.2mA \end{aligned}$$

At $t = 0^+$ the switch is open

$$\begin{aligned} i_1(0^+) &= 0.2mA \\ i_2(0^+) &= -0.2mA \end{aligned}$$

Q Prep-Q1) In Figure 47 the switch was closed for a long time before it is opened at t=0. Find $i_1(0^-)$, $i_2(0^-)$ $i_1(0^+) i_2(0^+)$.

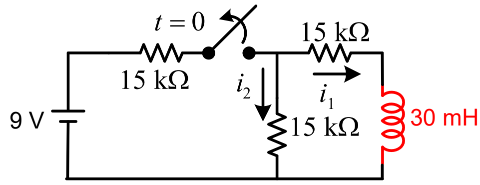

Figure 45: Prep-Q1

Solution:

Q Prep-Q1) In Figure 47 the switch was closed for a long time before it is opened at t=0. Find $i_1(0^-)$, $i_2(0^-)$ $i_1(0^+) i_2(0^+)$.

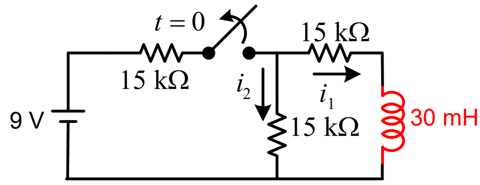

Figure 46: Prep-Q1

Solution:

Q Prep-Q1) In Figure 47 the switch was closed for a long time before it is opened at t=0. Find $i_1(0^-)$, $i_2(0^-)$ $i_1(0^+) i_2(0^+)$.

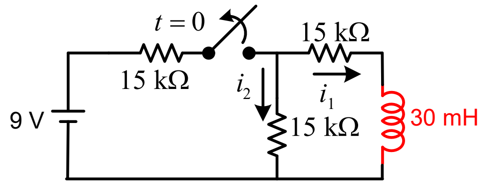

Figure 47: Prep-Q1

Solution: