

0.1 Mesh Analysis

Steps to find a current flowing in a circuit using Mesh Analysis

1. Identify loops or meshes in a circuit and Label a mesh current to N meshes
2. Apply KVL to each mesh with the corresponding mesh current to generate N equations.
3. Solve the resulting simultaneous linear equations for the unknown mesh currents using Cramer's Rule.

Q 1) In the circuit shown in Figure 53 determine all branch currents and the voltage across the 5Ω resistor by loop current analysis.

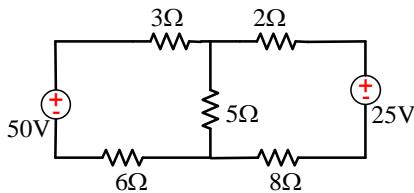


Figure 1

Solution:

In the given circuit there are two meshes and named as i_1 and i_2 as shown in Figure 2.

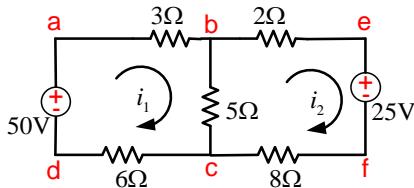


Figure 2

Applying the KVL for the loop abcda. The loop is passing through 3, 5 and 6Ω resistors and it touches negative terminal of the battery of 50 volts. The 5Ω resistor is common to loop currents i_1 and i_2 . For loop1 the current i_2 in 5Ω resistor is opposite to the i_1 current. Hence loop1 equation is.

$$\begin{aligned} 3i_1 + 5(i_1 - i_2) + 6i_1 - 50 &= 0 \\ 14i_1 - 5i_2 &= 50 \end{aligned}$$

Similarly for the loop befcb , The loop2 is passing through 2, 8 and 5Ω resistors and it touches positive terminal of the battery of 25 volts. The 5Ω resistor is common to loop currents i_1 and i_2 . For loop2 the current i_1 in 5Ω resistor is opposite to the i_2 current. Hence loop2 equation is.

$$\begin{aligned} 2i_2 + 8i_2 + 5(i_2 - i_1) + 25 &= 0 \\ -5i_1 + 15i_2 &= -25 \end{aligned}$$

The two simultaneous equations are:

$$14i_1 - 5i_2 = 50 \quad (1)$$

$$-5i_1 + 15i_2 = -25 \quad (2)$$

Multiply eqn 1 by 3 and adding with equation 2

$$\begin{aligned} 42i_1 - 15i_2 &= 150 \\ -5i_1 + 15i_2 &= -25 \\ \hline \hline &= \hline \\ 37i_1 &= 125 \end{aligned}$$

$$i_1 = 3.3784A \quad i_2 = -0.541A$$

$$i_{ab} = 3.3784A \quad i_{eb} = -i_2 = 0.541A$$

$$i_{bc} = i_1 - i_2 = 3.3784 - (-0.541) = 3.9194A$$

voltage across the 5Ω resistor is $5i_{bc} = 19.597V$

Q 2) In the circuit shown in Figure 3 determine the mesh currents i_1, i_2, i_3

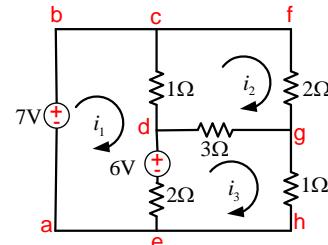


Figure 3

Solution:

Applying the KVL for the loop abcdea

$$\begin{aligned} 1(i_1 - i_2) + 2(i_1 - i_3) + 6 - 7 &= 0 \\ 3i_1 - i_2 - 2i_3 &= 1 \end{aligned}$$

For the loop cfgdc

$$\begin{aligned} 2i_2 + 3(i_2 - i_3) + 1(i_2 - i_1) &= 0 \\ -i_1 + 6i_2 - 3i_3 &= 0 \end{aligned}$$

For the loop dgħed

$$\begin{aligned} 3(i_3 - i_2) + 2(i_3 - i_1) + i_3 - 6 &= 0 \\ -2i_1 - 3i_2 + 6i_3 &= 6 \end{aligned}$$

The three mesh equations are,

$$\begin{aligned} 3i_1 - i_2 - 2i_3 &= 1 \\ -i_1 + 6i_2 - 3i_3 &= 0 \\ -2i_1 - 3i_2 + 6i_3 &= 6 \end{aligned}$$

Solving these equations Using Cramer's rule

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$ZI = V$$



$$I = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

$$i_1 = \frac{\begin{bmatrix} v_1 & -1 & -2 \\ v_2 & 6 & -3 \\ v_3 & -3 & 6 \end{bmatrix}}{\Delta}$$

where Δ is

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix}$$

$$3[6 \times 6 - (-3 \times -3)] + 1[-1 \times 6 - (-2 \times -3)] - 2[-1 \times -3 - (-2 \times 6)] = 3(36-9) + 1(-6-6) - 2(3+12) = 81-12-30=39$$

$$i_1 = \frac{\begin{vmatrix} 1 & -1 & -2 \\ 0 & 6 & -3 \\ 6 & -3 & 6 \end{vmatrix}}{\Delta} = \frac{1(36-9) + 1(18) - 2(-36)}{39} = \frac{27+18+72}{39} = 3A$$

$$i_2 = \frac{\begin{vmatrix} 3 & 1 & -2 \\ -1 & 0 & -3 \\ -2 & 6 & 6 \end{vmatrix}}{\Delta} = \frac{3(18) - 1(-6-6) - 2(-6)}{39} = \frac{54+12+12}{39} = 2A$$

$$i_3 = \frac{\begin{vmatrix} 3 & -1 & 1 \\ -1 & 6 & 0 \\ -2 & -3 & 6 \end{vmatrix}}{\Delta} = \frac{3(36) + 1(-6) + 1(3+12)}{39} = \frac{108-6+15}{39} = 3A$$

$$i_1 = 3A \quad i_2 = 2A \quad i_3 = 3A$$

Q 3) In the circuit shown in Figure 51 determine all branch currents by mesh current analysis.

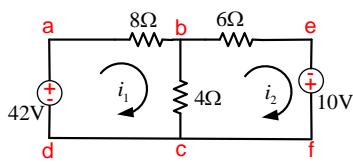


Figure 4

Solution:

Applying the KVL for the loop abcda

$$8i_1 + 4(i_1 - i_2) - 100 = 0$$

$$12i_1 - 4i_2 = 100$$

Similarly for the loop befcb

$$6i_2 + 4(i_2 - i_1) - 10 = 0$$

$$-4i_1 + 10i_2 = 10$$

Simultaneous equations are

$$12i_1 - 4i_2 = 100$$

$$-4i_1 + 10i_2 = 10$$

$$\Delta = \begin{vmatrix} 12 & -4 \\ -4 & 10 \end{vmatrix} = 12 \times 10 - (-4 \times -4) = 120 - 16 = 104$$

$$i_1 = \frac{\begin{vmatrix} 42 & -4 \\ 10 & 10 \end{vmatrix}}{\Delta} = \frac{420 + 40}{104} = \frac{460}{104} = 4.42A$$

$$i_2 = \frac{\begin{vmatrix} 12 & 42 \\ -4 & 10 \end{vmatrix}}{\Delta} = \frac{120 + 168}{104} = \frac{288}{104} = 2.769A$$

Q 4) In the circuit shown in Figure 57 determine the current I_x

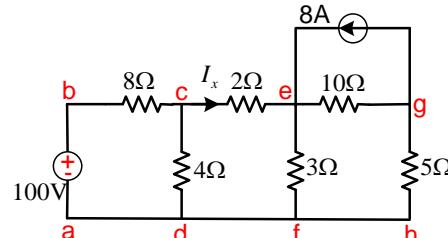


Figure 5

Solution: For the given circuit there is current source, to apply KVL current source has to be converted into voltage source, the modified circuit is as shown in Figure 58

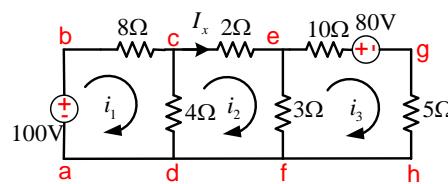


Figure 6

Applying the KVL for the loop abcda

$$8i_1 + 4(i_1 - i_2) - 100 = 0$$

$$12i_1 - 4i_2 = 100$$



For the loop cefdc

$$\begin{aligned} 2i_2 + 3(i_2 - i_3) + 4(i_2 - i_1) &= 0 \\ -4i_1 + 9i_2 - 3i_3 &= 0 \end{aligned}$$

For the loop eghfe

$$\begin{aligned} 10i_3 + 5i_3 + 3(i_3 - i_2) + 80 &= 0 \\ 0i_1 - 3i_2 + 18i_3 &= -80 \end{aligned}$$

The three mesh equations are,

$$\begin{aligned} 12i_1 - 4i_2 + 0i_3 &= 100 \\ -4i_1 + 9i_2 - 3i_3 &= 0 \\ 0i_1 - 3i_2 + 18i_3 &= -80 \end{aligned}$$

$$\Delta = \begin{vmatrix} 12 & -4 & 0 \\ -4 & 9 & -3 \\ 0 & -3 & 18 \end{vmatrix}$$

$$12[162 - 9] + 4(-72 - 0)] + 0[12 - 0] = 1836 - 288 = 1548$$

$$i_1 = \frac{\begin{vmatrix} 100 & -4 & 0 \\ 0 & 9 & -3 \\ -80 & -3 & 18 \end{vmatrix}}{\Delta}$$

$$= \frac{100(162 - 9) + 4(0 - 240)}{1548}$$

$$\frac{15300 - 960 + 0}{1548} = \frac{14340}{1548} = 9.26A$$

$$i_2 = \frac{\begin{vmatrix} 12 & 100 & 0 \\ -4 & 0 & -3 \\ 0 & -80 & 18 \end{vmatrix}}{\Delta}$$

$$= \frac{12(0 - 240) - 100(-72 - 0) + 0(0 + 720)}{1548}$$

$$\frac{-2880 + 7200 + 0}{1548} = \frac{4320}{1548} = 2.79A$$

$$i_3 = \frac{\begin{vmatrix} 12 & -4 & 100 \\ -4 & 9 & 0 \\ 0 & -3 & -80 \end{vmatrix}}{\Delta}$$

$$= \frac{12(-720 - 0) + 4(320 - 0) + 100(12 + 0)}{1548}$$

$$\frac{-8640 + 1280 + 1200}{1548} = \frac{-6160}{1548} = -3.97A$$

$$i_1 = 9.26A \quad i_2 = 2.79A \quad i_3 = -3.97A$$

Q 1) In the circuit shown in Figure 7 determine all branch currents by mesh current analysis.

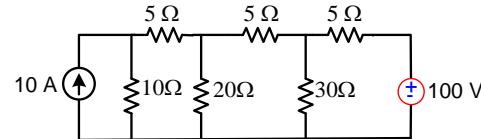


Figure 7

Solution:

Converting current source to voltage source and then $10\ \Omega$ is in series with $5\ \Omega$. The redrawn circuit is as shown in Figure 8

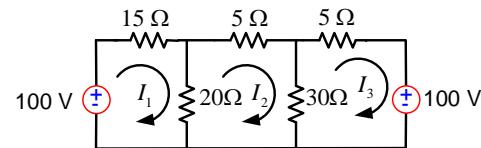


Figure 8

Applying the KVL for the loop

$$\begin{aligned} 35i_1 - 20i_2 + 0i_3 &= 100 \\ -20i_1 + 55i_2 - 30i_3 &= 0 \\ 0i_1 - 30i_2 + 35i_3 &= -100 \end{aligned}$$

$$\Delta = \begin{vmatrix} 35 & -20 & 0 \\ -20 & 55 & -30 \\ 0 & -30 & 35 \end{vmatrix} = 35(1925 - 900) + 20(-700)$$

$$\Delta = 25875 - 14000 = 21875$$

$$i_1 = \frac{\begin{vmatrix} 100 & -20 & 0 \\ 0 & 55 & -30 \\ -100 & -30 & 35 \end{vmatrix}}{\Delta}$$

$$= \frac{100(1925 - 900) + 20(-3000)}{\Delta}$$

$$= \frac{42500}{21875} = 1.9428A$$

$$i_2 = \frac{\begin{vmatrix} 35 & 100 & 0 \\ -20 & 0 & -30 \\ 0 & -100 & 35 \end{vmatrix}}{\Delta}$$

$$= \frac{-105000 + 2070000}{\Delta}$$

$$= \frac{-35000}{21875} = -1.6A$$

$$i_3 = \frac{\begin{vmatrix} 35 & -20 & 100 \\ -20 & 55 & -30 \\ 0 & -30 & -100 \end{vmatrix}}{\Delta}$$

$$= \frac{-192500 + 40000 + 60000}{\Delta}$$

$$= \frac{-92500}{21875} = -4.2285A$$



Q 2) In the circuit shown in Figure 9 determine all branch currents by mesh current analysis.

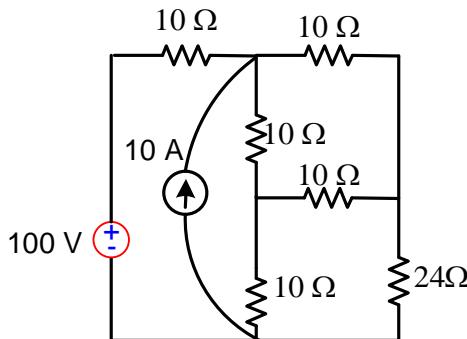


Figure 9

Solution:

Converting 100V voltage source to current source and then $10\ \Omega$ is in parallel with current source. The redrawn circuit is as shown in Figure 10

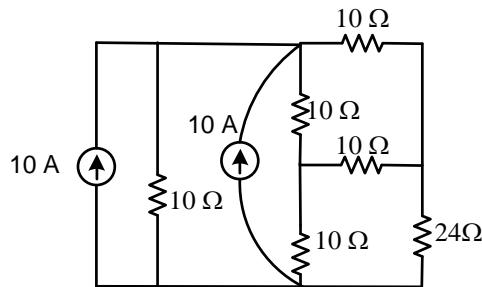


Figure 10

10 A current source and 10 A current source are in parallel which are added and become 20 A

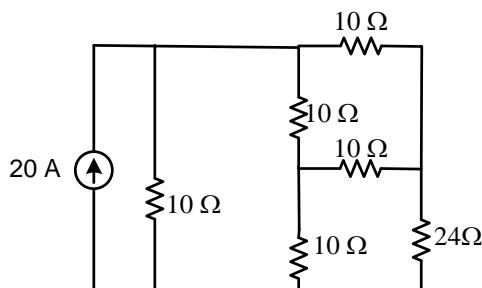


Figure 11

Converting 20 A current source to voltage source and then $10\ \Omega$ is in series with voltage source. The redrawn circuit is as shown in Figure 12

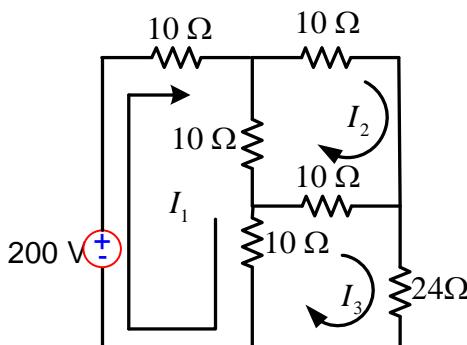


Figure 12
Applying the KVL for the loop

$$\begin{aligned} 30i_1 - 10i_2 - 10i_3 &= 200 \\ -10i_1 + 30i_2 - 10i_3 &= 0 \\ -10i_1 - 10i_2 + 44i_3 &= 0 \end{aligned}$$

$$i_1 = 8.997A \quad i_2 = 3.97A \quad i_3 = 2.941A$$

Q 3) In the circuit shown in Figure 13 determine the current I by mesh current analysis.

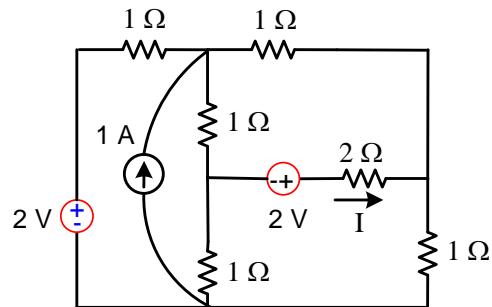


Figure 13

Solution:

Converting 2 V voltage source to current source and then $1\ \Omega$ is in parallel with current source. The redrawn circuit is as shown in Figure 14

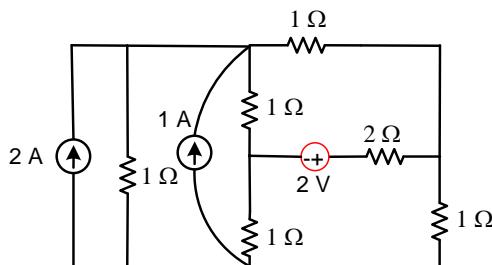


Figure 14

2 A current source and 1 A current source are in parallel which are added and become 3 A

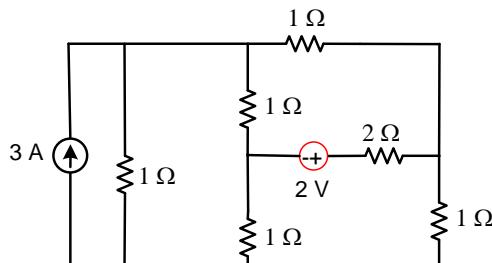


Figure 15

Converting 3 A current source to voltage source and then $1\ \Omega$ is in series with voltage source. The redrawn circuit is as shown in Figure 12

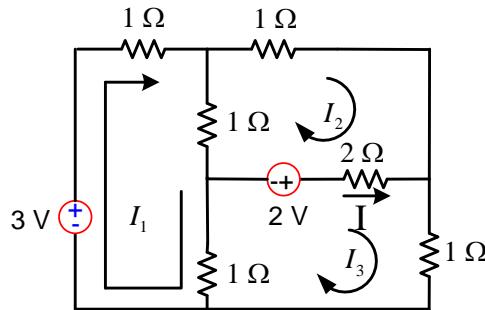


Figure 16

Applying the KVL for the loop

$$\begin{aligned} 3i_1 - i_2 - i_3 &= 3 \\ -i_1 + 4i_2 - 2i_3 &= -2 \\ -i_1 - 2i_2 + 4i_3 &= 2 \end{aligned}$$

fx-100MS/fx-115MS/fx-570MS/fx-991MS/

Press MODE MODE MODE MODE 1 EQN 3

Then $a1\ 3 = b1\ -1 = c1\ -1 = d1\ 3 =$ Then $a2\ -1 = b2\ 4 = c2\ -2 = d2\ -2 =$ Then $a3\ -1 = b3\ -2 = c3\ 4 = d1\ 2 =$

$$\begin{aligned} x &= 1.5 \\ y &= 0.4166 \\ z &= 1.0833 \end{aligned}$$

The current through $2\ \Omega$ resistor I is

$$I = I_3 - I_2 = 1.0833 - 0.4166 = 0.6667A$$

Q 5) Find the loop currents i_1, i_2, i_3 in the circuit shown in Figure 17

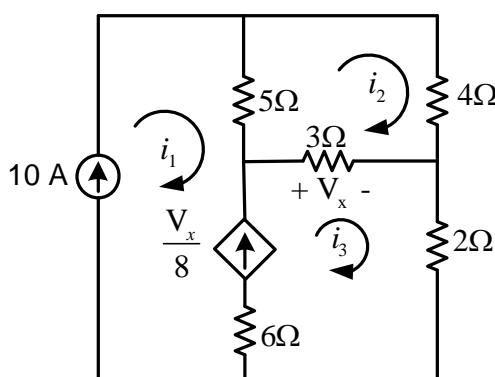


Figure 17

Solution:

$$\begin{aligned} V_x &= 3(i_3 - i_2) \\ i_1 &= 10A \\ i_3 - i_1 &= \frac{V_x}{8} = 0.125V_x \\ i_3 - 10 &= 0.125 \times 3(i_3 - i_2) \\ i_3 - 10 &= 0.375i_3 - 0.375i_2 \\ 0.375i_2 + 0.625i_3 &= 10 \end{aligned}$$

Applying KVL to mesh i_2

$$\begin{aligned} 4i_2 + 3(i_2 - i_3) + 5(i_2 - i_1) &= 0 \\ -5i_1 + 12i_2 - 3i_3 &= 0 \\ -5 \times 10 + 9i_2 - 4i_3 &= 0 \\ 12i_2 - 3i_3 &= 50 \end{aligned}$$

The simultaneous equations are

$$\begin{aligned} 0.375i_2 + 0.625i_3 &= 10 \\ 12i_2 - 3i_3 &= 50 \end{aligned}$$

$$\begin{aligned} \Delta &= \begin{vmatrix} 0.375 & 0.625 \\ 12 & -3 \end{vmatrix} = -1.125 - 7.5 = -8.6255 \\ i_2 &= \frac{\begin{vmatrix} 10 & 0.625 \\ 50 & -3 \end{vmatrix}}{\Delta} = \frac{-30 - 31.25}{-8.6255} = \frac{-61.25}{-8.6255} = 7.1A \end{aligned}$$

$$i_3 = \frac{\begin{vmatrix} 0.375 & 10 \\ 12 & 50 \end{vmatrix}}{\Delta} = \frac{18.75 - 120}{-8.6255} = \frac{-101.25}{-8.6255} = 11.74A$$

The three loop currents $i_1 = 10A, i_2 = 7.1A, i_3 = 11.74A$

0.2 Supermesh Analysis

- If there is a current source between meshes which is common for meshes, then we have to combine meshes which is called a *supermesh*.
- In supermesh write a KVL equation combining meshes.

Q 1) In the circuit shown in Figure 18 determine the current through 2Ω resistor.

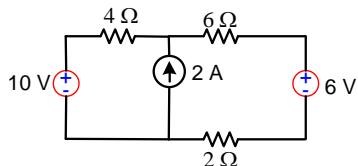


Figure 18

Solution:

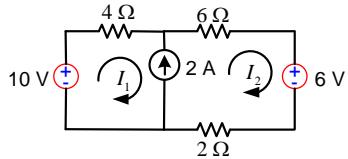


Figure 19

I_1 and I_2 are the two meshes in which 2 A current source is common to I_1 and I_2 meshes which is as shown in Figure 19.

Draw a supermesh by combining two meshes I_1 and I_2 as shown in Figure 20.

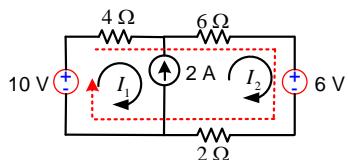


Figure 20

Apply KVL to the supermesh as

$$\begin{aligned} 4I_1 + 6I_2 + 6 + 2I_2 - 10 &= 0 \\ 4I_1 + 8I_2 &= 4 \end{aligned}$$

$$I_2 - I_1 = 2$$

$$\begin{aligned} 4I_1 + 8I_2 &= 4 \\ I_1 - I_2 &= -2 \end{aligned}$$

Solving the above equations

$$I_1 = -1A \quad I_2 = 1A$$

Current through 2Ω resistor is $I_2 = 1A$

Q 2) In the circuit shown in Figure 21 determine the current I_x

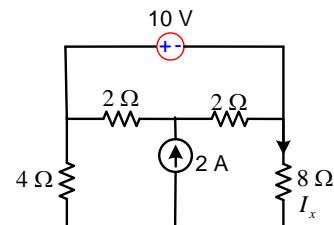


Figure 21

Solution:

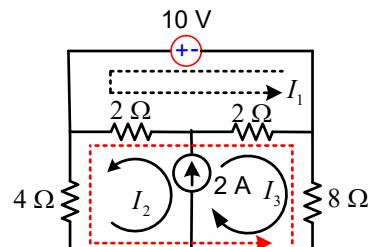


Figure 22

KVL for I_1 is

$$\begin{aligned} 2(I_1 - I_2) + 2(I_1 + I_3) - 10 &= 0 \\ 4I_1 - 2I_2 + 2I_3 &= 10 \end{aligned}$$

KVL for supermesh is

$$\begin{aligned} 4I_2 + 2(I_2 - I_1) + 2(-I_3 - I_1) - 8I_3 &= 0 \\ -4I_1 + 6I_2 - 10I_3 &= 0 \end{aligned}$$

For current source

$$I_2 + I_3 = 2$$

$$\begin{aligned} 4I_1 - 2I_2 + 2I_3 &= 10 \\ -4I_1 + 6I_2 - 10I_3 &= 0 \\ 0I_1 + I_2 + I_3 &= 2 \end{aligned}$$

Solving the above equations

$$I_1 = 3.667A \quad I_2 = 2.167A, \quad I_3 = 0.167A$$

Current through 8Ω resistor is $I_3 = 0.167A$

Q 3) In the circuit shown in Figure 23 determine the current I_x

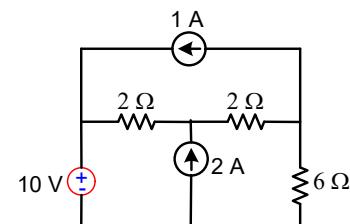


Figure 23

Solution:

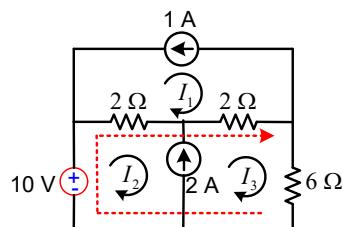


Figure 24

$$i_2 - i_1 = 0.5i_2$$

$$i_1 - 0.5i_2 = 0$$

$$4i_1 + 8i_2 + 2i_2 = 20$$

$$i_1 - 0.5i_2 = 0$$

$$I_1 = -1A$$

KVL for Supermesh is

$$\begin{aligned} 2(I_2 - I_1) + 2(I_3 - I_1) + 6I_3 - 10 &= 0 \\ -4I_1 + 2I_2 + 8I_3 &= 10 \\ -4(-1) + 2I_2 + 8I_3 &= 10 \\ 2I_2 + 8I_3 &= 10 - 4 = 6 \end{aligned}$$

$$I_3 - I_2 = 2$$

$$I_2 - I_3 = -2$$

$$\Delta = \begin{vmatrix} 4 & 8 \\ 1 & -0.5 \end{vmatrix} = -2 - 8 = -10$$

$$i_1 = \frac{\begin{vmatrix} 20 & 8 \\ 0 & -0.5 \end{vmatrix}}{\Delta} = \frac{-10}{-10} = 1A$$

$$i_1 - 0.5i_2 = 0$$

$$1 - 0.5i_2 = 0$$

$$i_2 = 2A$$

$$\begin{aligned} 2I_2 + 8I_3 &= 6 \\ I_2 - I_3 &= -2A \end{aligned}$$

Solving the above equations

$$I_1 = -1A, I_2 = -1A, I_3 = 1A$$

Q 4) In the circuit shown in Figure 25 determine the current I_x

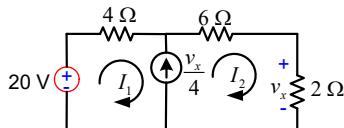


Figure 25

Solution:

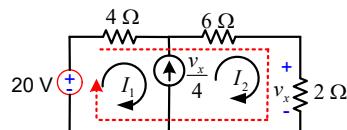


Figure 26: Example

$$\begin{aligned} 4i_1 + 6i_2 + 2i_x - 20 &= 0 \\ 4i_1 + 8i_2 + 2i_x &= 20 \end{aligned}$$

$$v_x = 2i_2$$

$$i_2 - i_1 = \frac{v_x}{4} = \frac{2i_2}{4} = \frac{i_2}{2}$$

Q 5) In the circuit shown in Figure 27 determine all the loop currents

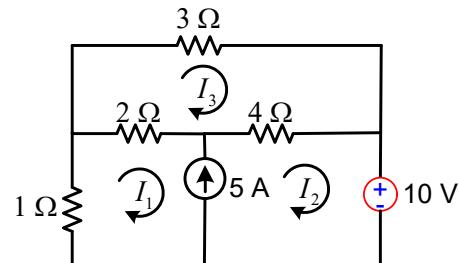


Figure 27

Solution:

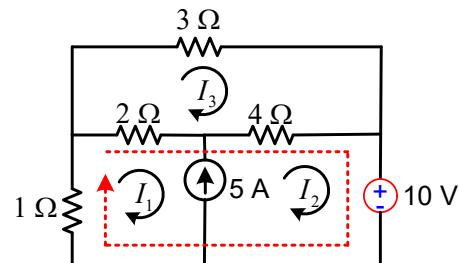


Figure 28

KVL for Supermesh

$$\begin{aligned} 1i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 &= 0 \\ 3i_1 + 4i_2 - 6i_3 &= -10 \end{aligned}$$

KVL for i_3

$$\begin{aligned} 3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) &= 0 \\ -2i_1 - 4i_2 + 9i_3 &= 0 \end{aligned}$$

$$i_2 - i_1 = 5$$

$$i_1 - i_2 = -5$$

$$\begin{aligned} 3i_1 + 4i_2 - 6i_3 &= -10 \\ -2i_1 - 4i_2 + 9i_3 &= 0 \\ i_1 - i_2 + 0i_3 &= -5 \end{aligned}$$

Solving the above equations

$$I_1 = 5.556A, I_2 = -0.556A, I_3 = -1.4814A$$

Q 6) In the circuit shown in Figure 29 determine the all the mesh currents

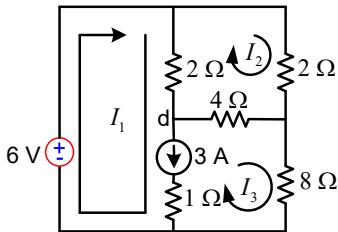


Figure 29

Solution:

For the given circuit there is current source, to apply KVL current source has to be converted into voltage source, the modified circuit is as shown in Figure 30

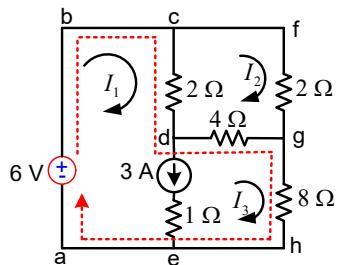


Figure 30

KVL for I_2 is

$$\begin{aligned} 2I_2 + 4(I_2 - I_3) + 2(I_2 - I_1) &= 0 \\ -2I_1 + 8I_2 - 4I_3 &= 0 \end{aligned}$$

KVL for supermesh I_2 is

$$\begin{aligned} 2(I_1 - I_2) + 4(I_3 - I_2) + 8I_3 - 6 &= 0 \\ 2I_1 - 6I_2 + 12I_3 &= 6 \end{aligned}$$

$$I_1 - I_3 = 3$$

$$I_3 = I_1 - 3$$

$$-2I_1 + 8I_2 - 4I_3 = 0$$

$$2I_1 - 6I_2 + 12I_3 = 6$$

$$I_1 + 0I_2 - I_3 = 3$$

Solving the above equations

$$I_1 = 3.4736A, I_2 = 1.105A, I_3 = 0.4736A$$

Q 7) In the circuit shown in Figure 31 determine the current I_x

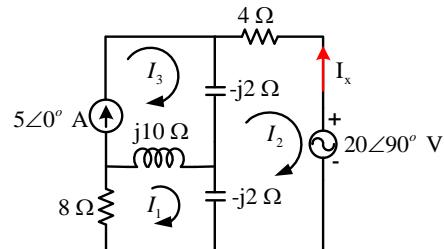


Figure 31

Solution: Applying KVL to mesh I_1

$$\begin{aligned} 8I_1 + j10(I_1 - I_3) - j2(I_1 - I_2) &= 0 \\ (8 + j8)I_1 + j2I_2 - j10I_3 &= 0 \end{aligned}$$

Applying KVL to mesh I_2

$$\begin{aligned} -j2(I_2 - I_1) + 4I_2 - j2(I_2 - I_3) + 20∠90 &= 0 \\ j2I_1 + (4 - j4)I_2 + j2I_3 &= -20∠90 \end{aligned}$$

For mesh I_3 , $I_3 = 5A$

$$\begin{aligned} (8 + j8)I_1 + j2I_2 - j10I_3 &= 0 \\ (8 + j8)I_1 + j2I_2 - j50 &= 0 \\ (8 + j8)I_1 + j2I_2 &= j50 \end{aligned}$$

$$\begin{aligned} j2I_1 + (4 - j4)I_2 + j2I_3 &= -20∠90 \\ j2I_1 + (4 - j4)I_2 + j10 &= -j20 \\ j2I_1 + (4 - j4)I_2 &= -j30 \end{aligned}$$

$$\begin{aligned} (8 + j8)I_1 + j2I_2 &= j50 \\ j2I_1 + (4 - j4)I_2 &= -j30 \end{aligned}$$

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32 - j32 + j32 + 32 + 4 = 68$$

$$i_1 = \frac{\begin{vmatrix} j50 & j2 \\ -j30 & 4 - j4 \end{vmatrix}}{\Delta} = \frac{j200 + 200 - 60}{68} = \frac{140 + j200}{68} = \frac{214.13∠55}{68} = 3.15∠55$$

$$I_2 = \frac{\begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix}}{\Delta} = \frac{-j240 + 240 + 100}{68} = \frac{340 - j240}{68}$$

$$= \frac{416.17∠-35.2}{68} = 6.12∠-35.21 = 6.12∠144.78$$

$$I_x = -I_2 = 6.12∠144.78$$

Q 8) Use loop analysis to find V_x in the circuit shown in Figure 32 such that the current through $2 + j3 \Omega$ is zero.

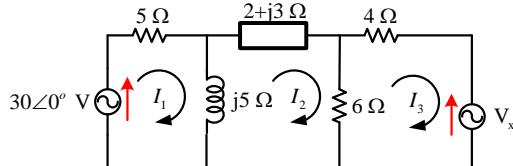


Figure 32

Solution: Applying KVL to mesh I_1

$$5I_1 + j5(I_1 - I_2) - 30\angle 0^\circ = 0 \\ (5 + j5)I_1 - j5I_2 = 30\angle 0^\circ$$

Applying KVL to mesh I_2

$$(2 + j2)I_2 + 6(I_2 - I_3) + j5(I_2 - I_1) = 0 \\ -j5I_1 + (8 + j8)I_2 - 6I_3 = 0$$

Applying KVL to mesh I_3

$$4I_3 + 6(I_3 - I_2) + V_x = 0 \\ 0I_1 - 6I_2 + 10I_3 = -V_x$$

$$\Delta = \begin{vmatrix} 5 + j5 & -j5 & 0 \\ -j5 & 8 + j8 & -6 \\ 0 & -6 & 10 \end{vmatrix}$$

$$= (5 + j5)(80 + j80 - 36) + j5(-j50)$$

$$(5 + j5)(44 + j80) + 250 = 220 + j400 + j220 + 400 + 250$$

$$870 + j620 = 1068\angle 35.47$$

$$I_2 = \frac{\begin{vmatrix} 5 + j5 & 30 & 0 \\ -j5 & 0 & -6 \\ 0 & -V_x & 10 \end{vmatrix}}{\Delta} = (5 + j5)(-6V_x) - 30(-j50)$$

Since I_2

$$0 = (5 + j5)(-6V_x) + j1500$$

$$6V_x = \frac{j1500}{(5 + j5)} = \frac{1500\angle 90^\circ}{7.07\angle 45^\circ}$$

$$V_x = 35.36\angle 45^\circ$$

Q 9) Use loop analysis to find V_x in the circuit shown in Figure 60 such that the current through $2 + j3 \Omega$ is zero.

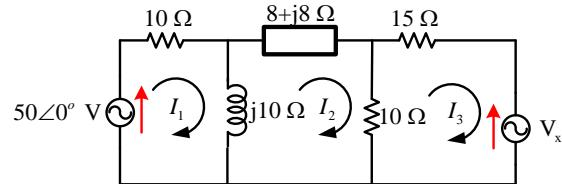


Figure 33

Solution: Applying KVL to mesh I_1

$$105I_1 + j10(I_1 - I_2) - 50\angle 0^\circ = 0 \\ (10 + j10)I_1 - j10I_2 = 50\angle 0^\circ$$

Applying KVL to mesh I_2

$$(8 + j8)I_2 + 10(I_2 - I_3) + j10(I_2 - I_1) = 0 \\ -j10I_1 + (18 + j18)I_2 - 10I_3 = 0$$

Applying KVL to mesh I_3

$$15I_3 + 10(I_3 - I_2) + V_x = 0 \\ 0I_1 - 10I_2 + 25I_3 = -V_x$$

$$\Delta = \begin{vmatrix} 10 + j10 & -j10 & 0 \\ -j10 & 18 + j18 & -10 \\ 0 & -10 & 25 \end{vmatrix} =$$

$$(10 + j10)(450 + j450 - 100) + j10(-j250)$$

$$(10 + j10)(350 + j450) + 2500$$

$$3500 + j4500 + j3500 - 4500 + 2500 = 1500 + j8000$$

$$1500 + j8000 = 8139.4\angle 79.38^\circ$$

$$I_2 = \frac{\begin{vmatrix} 10 + j10 & 50 & 0 \\ -j10 & 0 & -10 \\ 0 & -V_x & 25 \end{vmatrix}}{\Delta}$$

Since I_2

$$0 = (10 + j10)(-10V_x) - 50(-j250)$$

$$(10 + j10)(-10V_x) + j12500 =$$

$$10V_x = \frac{j12500}{(10 + j10)} = \frac{12500\angle 90^\circ}{14.14\angle 45^\circ}$$

$$V_x = 88.4\angle 45^\circ$$

0.3 Question Papers

Q 2019-DEC 1 b) Use mesh analysis determine the current I_1, I_2, I_3 as shown in Figure 34

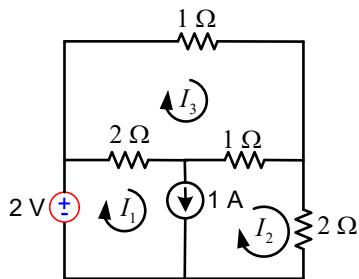


Figure 34: 2019-DEC-1b(2018-scheme)

Solution:

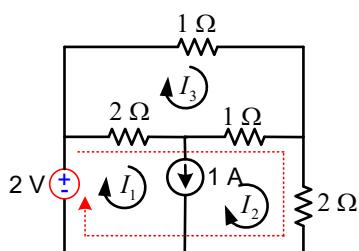


Figure 35

Q 2019-JAN) Using mesh analysis determine the current I_1, I_2, I_3 shown in Figure 36

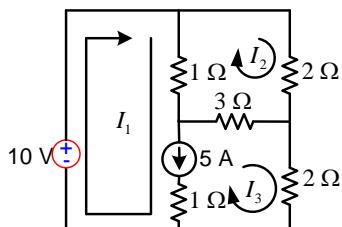


Figure 36

Solution:

For the given circuit there is current source between mesh1 and mesh2. Convert the modified circuit is as shown in Figure 37

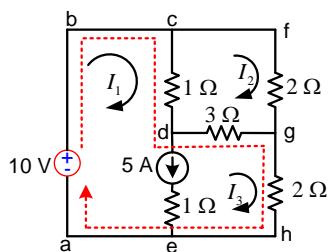


Figure 37

KVL for I_2 is

$$\begin{aligned} 2I_2 + 3(I_2 - I_3) + 1(I_2 - I_1) &= 0 \\ -1I_1 + 6I_2 - 3I_3 &= 0 \end{aligned}$$

KVL for supermesh is

$$\begin{aligned} 1(I_1 - I_2) + 3(I_3 - I_2) + 2I_3 - 10 &= 0 \\ 1I_1 - 4I_2 + 5I_3 &= 10 \end{aligned}$$

$$I_1 - I_3 = 5$$

$$I_1 + 6I_2 - 3I_3 = 0$$

$$I_1 - 4I_2 + 5I_3 = 10$$

$$I_1 + 0I_2 - I_3 = 5$$

Solving the above equations

$$I_1 = 5.357A, I_2 = -0.714A, I_3 = 0.357A$$

JAN-2018-CBCS 2-a Determine the loop currents I_1, I_2, I_3 and I_4 for the network shown in Figure 38.

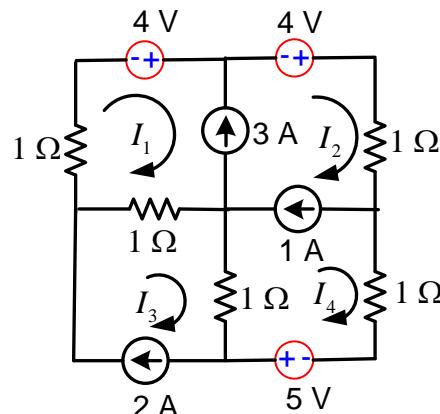


Figure 38: 2018-CBCS-Question Paper

Solution:

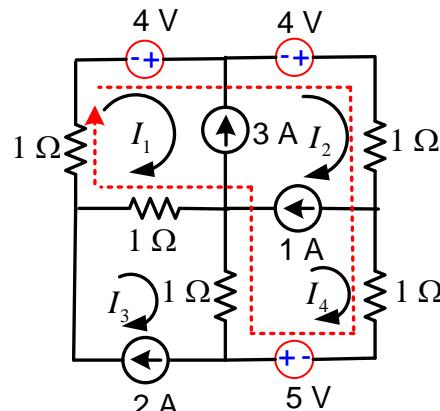


Figure 39

$$I_3 = 2$$

$$I_2 - I_1 = 3$$

$$-I_1 + I_2 = 3$$

$$I_2 - I_4 = 1$$



Applying the KVL for the Supermesh

$$\begin{aligned} -4 - 4 + 1I_2 + 1I_4 - 5 + 1(I_4 - I_3) &= 0 \\ 2I_1 + I_2 - 2I_3 + 2I_4 &= 13 \\ 2I_1 + I_2 - 2 \times 2 + 2I_4 &= 13 \\ 2I_1 + I_2 + 2I_4 &= 17 \end{aligned}$$

Simultaneous equations are

$$\begin{aligned} -I_1 + I_2 + 0I_4 &= 3 \\ 0I_1 + I_2 - I_4 &= 1 \\ 2I_1 + I_2 + 2I_4 &= 17 \end{aligned}$$

Solving the above equations

$$I_1 = 2A, I_2 = 5A, I_3 = 2A, I_4 = 4A$$

JAN-2017-CBCS Find the voltage across 20Ω resistor in the network shown in Figure ?? by mesh analysis.

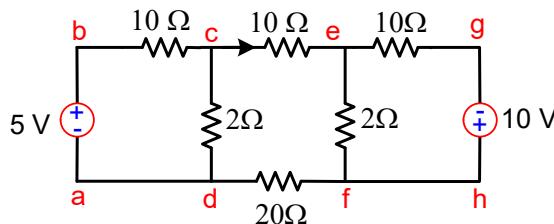


Figure 40

Solution:

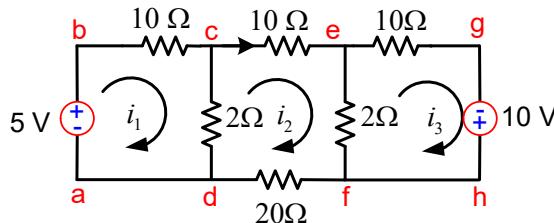


Figure 41

Applying the KVL for the loop abcda

$$\begin{aligned} 10i_1 + 2(i_1 - i_2) - 5 &= 0 \\ 12i_1 - 2i_2 &= 5 \end{aligned}$$

For the loop cefdc

$$\begin{aligned} 10i_2 + 2(i_2 - i_3) + 20i_2 + 2(i_2 - i_1) &= 0 \\ -2i_1 + 34i_2 - 2i_3 &= 0 \end{aligned}$$

For the loop eghfe

$$\begin{aligned} 10i_3 + 2(i_3 - i_2) - 10 &= 0 \\ 0i_1 - 2i_2 + 12i_3 &= 10 \end{aligned}$$

The three mesh equations are,

$$\begin{aligned} 12i_1 - 2i_2 + 0i_3 &= 5 \\ -2i_1 + 34i_2 - 2i_3 &= 0 \\ 0i_1 - 2i_2 + 12i_3 &= 10 \end{aligned}$$

Solving the above equations

$$I_1 = 0.429A, I_2 = 0.075A, I_3 = 0.8453A$$

Voltage across 20Ω is

$$i_2 \times 20 = 0.075 \times 20 = 1.56V$$

JULY 2017 CBCS) Use mesh analysis to determine the three mesh currents i_1, i_2, i_3 as shown in Figure 42

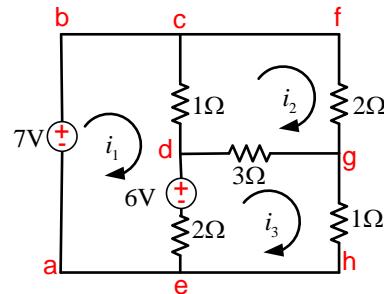


Figure 42

Solution:

Applying the KVL for the loop abcdea

$$\begin{aligned} 1(i_1 - i_2) + 2(i_1 - i_3) + 6 - 7 &= 0 \\ 3i_1 - i_2 - 2i_3 &= 1 \end{aligned}$$

For the loop cfgdc

$$\begin{aligned} 2i_2 + 3(i_2 - i_3) + 1(i_2 - i_1) &= 0 \\ -i_1 + 6i_2 - 3i_3 &= 0 \end{aligned}$$

For the loop dghed

$$\begin{aligned} 3(i_3 - i_2) + 2(i_3 - i_1) + i_3 - 6 &= 0 \\ -2i_1 - 3i_2 + 6i_3 &= 6 \end{aligned}$$

The three mesh equations are,

$$\begin{aligned} 3i_1 - i_2 - 2i_3 &= 1 \\ -i_1 + 6i_2 - 3i_3 &= 0 \\ -2i_1 - 3i_2 + 6i_3 &= 6 \end{aligned}$$

Solving these equations

$$i_1 = 3A, i_2 = 2A, i_3 = 3A$$

July-2016 1-b Using mesh current method find current through 10Ω resistor in the network shown in Figure 43.



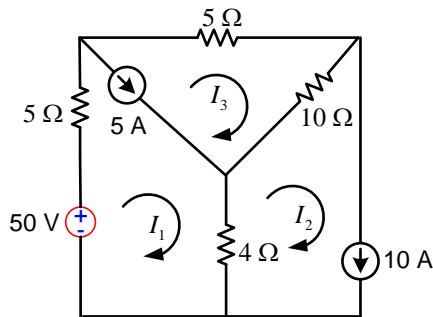


Figure 43: July-2016-Question Paper

Solution: Applying KVL to mesh I_1

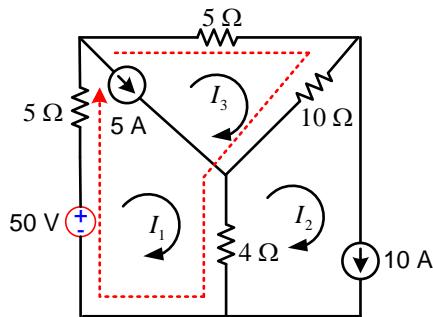


Figure 44: July-2016-Question Paper

$$I_1 - I_3 = 5$$

$$I_2 = 10$$

Applying the KVL for the Supermesh

$$\begin{aligned} 5I_3 + 10(I_3 - I_2) + 4(I_1 - I_2) - 50 + 5I_1 &= 0 \\ 9I_1 - 14I_2 + 15I_3 &= 50 \\ 9I_1 - 14 \times 10 + 15I_3 &= 50 \\ 9I_1 + 15I_3 &= 190 \end{aligned}$$

Simultaneous equations are

$$\begin{aligned} I_1 - I_3 &= 5 \\ 9I_1 + 15I_3 &= 190 \end{aligned}$$

Solving above equations

$$I_1 = 11.04A, I_2 = 10A, I_3 = 6.04A$$

JAN-2015) For the circuit shown in Figure 60 find the power supplied 10 V source using mesh analysis.

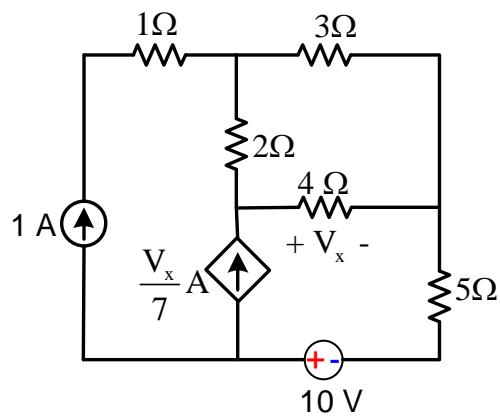


Figure 45

Solution:

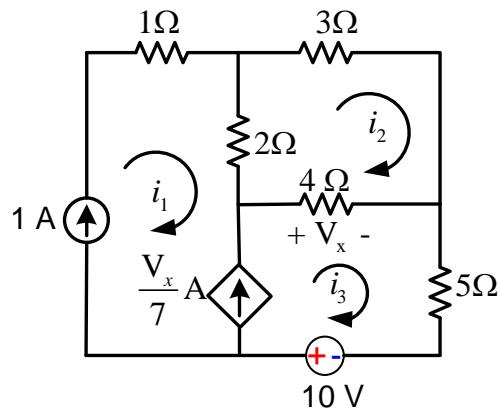


Figure 46

$$V_x = 4(i_3 - i_2)$$

$$i_1 = 1A$$

$$i_3 - i_1 = \frac{V_x}{7} = 0.142V_x$$

$$i_3 - 1 = 0.142 \times 4(i_3 - i_2)$$

$$i_3 - 1 = 0.568i_3 - 0.568i_2$$

$$0.568i_2 + 0.432i_3 = 1$$

Applying KVL to mesh i_2

$$\begin{aligned} 3i_2 + 4(i_2 - i_3) + 2(i_2 - i_1) &= 0 \\ -2i_1 + 9i_2 - 4i_3 &= 0 \\ -2 \times 1 + 9i_2 - 4i_3 &= 0 \\ 9i_2 - 4i_3 &= 2 \end{aligned}$$

The simultaneous equations are

$$0.568i_2 + 0.432i_3 = 1$$

$$9i_2 - 4i_3 = 2$$

$$\Delta = \begin{vmatrix} 0.568 & 0.432 \\ 9 & -4 \end{vmatrix} = 2.272 - 3.888 = -1.616$$

$$i_3 = \frac{\begin{vmatrix} 0.568 & 1 \\ 9 & 2 \end{vmatrix}}{\Delta} = \frac{1.136 - 9}{-1.616} = \frac{-7.864}{-1.616} = 4.866A$$

The power dissipated by 10V source is

$$10 \times i_3 = 10 \times 4.866A = 48.66W$$

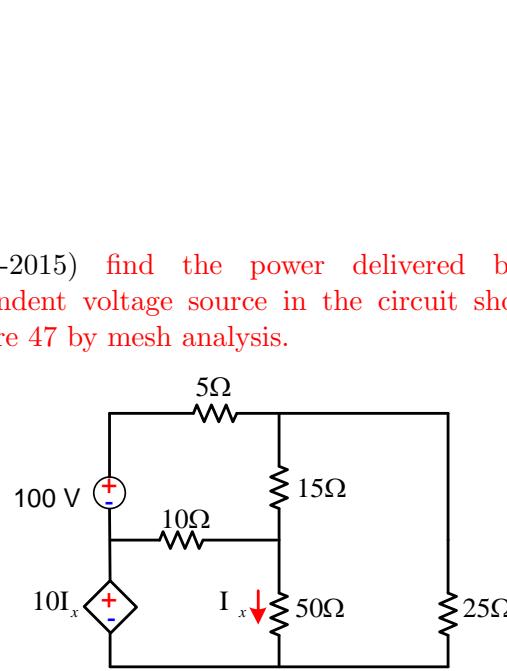


Figure 47

Solution:

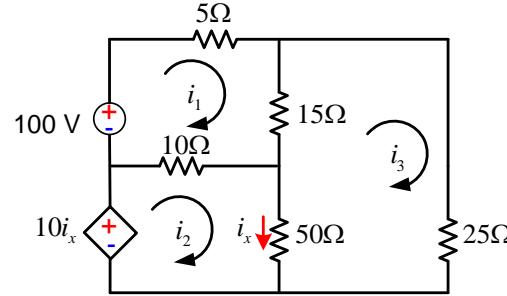


Figure 48

KVL for loop i_1

$$\begin{aligned} 5i_1 + 15(i_1 - i_3) + 10(i_1 - i_2) - 100 &= 0 \\ 30i_1 - 10i_2 - 15i_3 &= 100 \\ i_x = 50(i_2 - i_3) &= 50i_2 - 50i_3 \end{aligned}$$

KVL for loop i_2

$$\begin{aligned} 10(i_2 - i_1) + 50(i_2 - i_3) - 10i_x &= 0 \\ -10i_1 + 60i_2 - 50i_3 - 10i_x &= 0 \\ -10i_1 + 60i_2 - 50i_3 - 10(50i_2 - 50i_3) &= 0 \\ -10i_1 - 440i_2 + 450i_3 &= 0 \end{aligned}$$

KVL for loop i_3

$$\begin{aligned} 25i_3 + 50(i_3 - i_2) + 15(i_3 - i_1) &= 0 \\ -15i_1 - 50i_2 + 90i_3 &= 0 \end{aligned}$$

Simultaneous equations are

$$\begin{aligned} 30i_1 - 10i_2 - 15i_3 &= 100 \\ -10i_1 - 440i_2 + 450i_3 &= 0 \\ -15i_1 - 50i_2 + 90i_3 &= 0 \end{aligned}$$

$$\Delta = \begin{vmatrix} 30 & -10 & -15 \\ -10 & -440 & 450 \\ -15 & -50 & 90 \end{vmatrix}$$

$$30[-39600+22500]+10[-900+6750]-15[500-6600]$$

$$\begin{aligned} &= 30[-17100] + 10[5850] - 15[-6100] \\ &= -513000 + 58500 + 91500 = -363000 \end{aligned}$$

DEC-2015) find the power delivered by the dependent voltage source in the circuit shown in Figure 47 by mesh analysis.

Bridge Network) Use mesh analysis to determine the three mesh currents i_1, i_2, i_3 also determine the voltage drop across 2Ω for the circuit shown in Figure 49

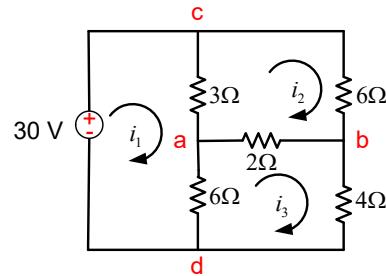


Figure 49

Solution:

Applying the KVL for the loop cadc

$$\begin{aligned} 3(i_1 - i_2) + 6(i_1 - i_3) - 30 &= 0 \\ 9i_1 - 3i_2 - 6i_3 &= 30 \end{aligned}$$

For the loop cabc

$$\begin{aligned} 6i_2 + 2(i_2 - i_3) + 3(i_2 - i_1) &= 0 \\ -3i_1 + 11i_2 - 2i_3 &= 0 \end{aligned}$$

For the loop abda

$$\begin{aligned} 2(i_3 - i_2) + 6(i_3 - i_1) + 4i_3 &= 0 \\ -6i_1 - 2i_2 + 12i_3 &= 0 \end{aligned}$$

The three mesh equations are,

$$\begin{aligned} 9i_1 - 3i_2 - 6i_3 &= 30 \\ -3i_1 + 11i_2 - 2i_3 &= 0 \\ -6i_1 - 2i_2 + 12i_3 &= 0 \end{aligned}$$

$i_1 = 6.6667A$ $i_2 = 2.5A$ $i_3 = 3.75A$ The voltage drop across 2Ω resistor is

$$\begin{aligned} V_3 &= (i_3 - i_2)2 \\ &= (3.75 - 2.5)2 \\ &= 2.5V \end{aligned}$$



Bridge Network) Use mesh analysis to determine the mesh currents i_1, i_2, i_3, i_4 for the circuit shown in Figure 50

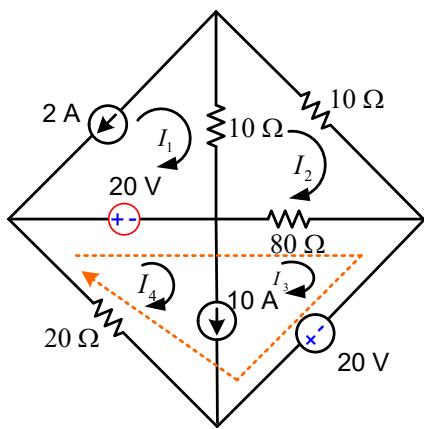


Figure 50

Applying the KVL for the Mesh2

$$\begin{aligned} 10i_2 + 10(i_2 - i_1) + 80(i_2 - i_3) - 30 &= 0 \\ -10i_1 + 100i_2 - 80i_3 &= 0 \\ 100i_2 - 80i_3 &= -20 \end{aligned}$$

$$i_4 - i_3 = 10A$$

Applying supermesh

$$\begin{aligned} 80(i_3 - i_2) + 20i_4 + 20 - 20 &= 0 \\ -80i_2 + 80i_3 + 20i_4 &= 0 \end{aligned}$$

Solving simultaneous equations

$$\begin{aligned} 100i_2 - 80i_3 + 0i_4 &= -20 \\ 0i_2 - i_3 + i_4 &= 10A \\ -80i_2 + 80i_3 + 20i_4 &= 0 \end{aligned}$$

$$i_1 = -2A$$

$$i_2 = -5A \quad i_3 = -6A \quad i_4 = 4A$$

Important: All the diagrams are redrawn and solutions are prepared. While preparing this study material most of the concepts are taken from some text books or it may be Internet. This material is just for class room teaching to make better understanding of the concepts on Network analysis: Not for any commercial purpose



0.4 Node Analysis

Steps to find a current flowing in a circuit using Node Analysis

1. Identify nodes in a circuit and Label all the nodes.
2. Select one of node as reference node.
3. Apply KCL to each node.
4. Solve the resulting simultaneous linear equations for the unknown node voltages using Cramer's Rule.
5. Branch currents can be calculated using node voltages

Q 1) In the circuit shown in Figure 51 determine all branch currents by node analysis.

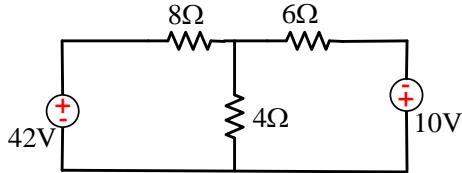


Figure 51

Solution: The circuit is labeled by nodes which is as shown in Figure 52

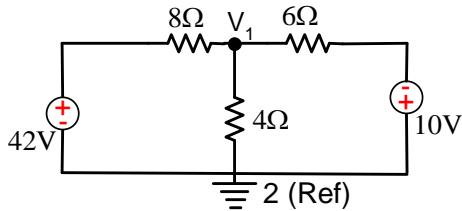


Figure 52

Apply KCL to node V_1

$$\begin{aligned} \left[\frac{V_1 - 42}{8} + \frac{V_1}{4} + \frac{V_1 + 10}{6} \right] &= 0 \\ [0.125 + 0.25 + 0.167] V_1 - 5.25 + 1.67 &= 0 \\ [0.5416] V_1 &= 3.58 \\ V_1 &= 6.61 \end{aligned}$$

Current through 8Ω resistor is

$$I_8 = \frac{42 - 6.61}{8} = 4.42A$$

Current through 6Ω resistor is

$$I_6 = \frac{10 + 6.61}{6} = 2.77A$$

Current through 4Ω resistor is

$$I_4 = \frac{6.61}{4} = 1.65A$$

Q 2) In the circuit shown in Figure 53 determine all branch currents and the voltage across the 5Ω resistor by node analysis.

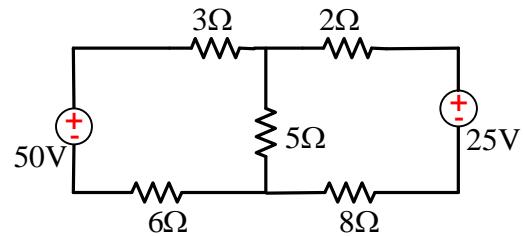


Figure 53

Solution:

The circuit is labeled by nodes which is as shown in Figure 54

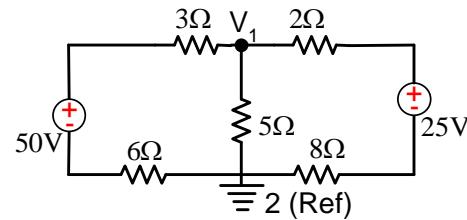


Figure 54

Apply KCL to node V_1

$$\begin{aligned} \left[\frac{V_1 - 50}{3 + 6} + \frac{V_1 - 25}{2 + 8} + \frac{V_1}{5} \right] &= 0 \\ [0.111 + 0.1 + 0.2] V_1 - 5.556 - 2.5 &= 0 \\ [0.4111] V_1 &= 8.056 \\ V_1 &= 19.59 \end{aligned}$$

Current through 3Ω and 6Ω resistor is

$$I_3 = \frac{V_1 - 50}{3 + 6} = \frac{19.59 - 50}{3 + 6} = -3.37A$$

Current through 2Ω and 8Ω resistor is

$$I_5 = \frac{V_1 - 25}{2 + 8} = \frac{19.59 - 25}{2 + 8} = -0.541A$$

Current through 5Ω resistor is

$$I_5 = \frac{8.056}{5} = 3.918A$$

Voltage across 5Ω resistor is

$$V = I_5 \times 5 = 3.918 \times 5 = 19.59V$$

Q 3) In the circuit shown in Figure 56 determine all branch currents and the voltage across the 5Ω resistor by node analysis.

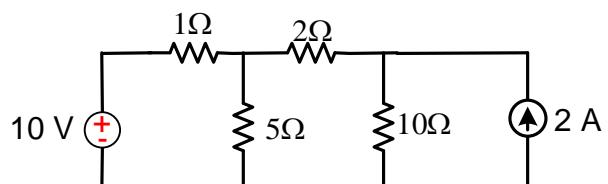


Figure 55

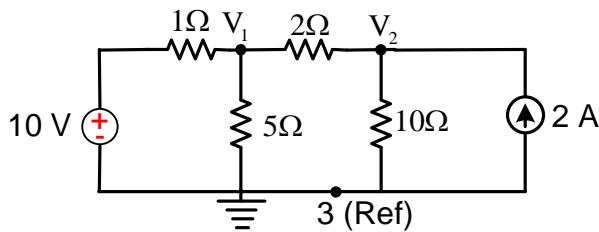
Solution:

Figure 56

Apply KCL at node V_1

$$\begin{aligned} \left[\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} \right] &= 0 \\ V_1[1 + 0.2 + 0.5] - 10 - 0.5V_2 &= 0 \\ V_1[1 + 0.2 + 0.5] - 0.5V_2 &= 10 \\ 1.7V_1 - 0.5V_2 &= 10 \end{aligned}$$

Apply KCL at node V_2

$$\begin{aligned} \left[\frac{V_2}{10} + \frac{V_2 - V_1}{2} - 2 \right] &= 0 \\ -0.5V_1 + V_2[0.1 + 0.5] &= 2 \\ -0.5V_1 + 0.6V_2 &= 2 \end{aligned}$$

The simultaneous equations are

$$\begin{aligned} 1.7V_1 - 0.5V_2 &= 10 \\ -0.5V_1 + 0.6V_2 &= 2 \end{aligned}$$

$$\Delta = \begin{vmatrix} 1.7 & -0.5 \\ -0.5 & 0.6 \end{vmatrix} = 1.02 - .25 = 0.25 = 0.77$$

$$V_1 = \frac{\begin{vmatrix} 10 & -0.5 \\ 2 & 0.6 \end{vmatrix}}{\Delta} = \frac{6 + 1}{0.77} = 9.09$$

$$V_2 = \frac{\begin{vmatrix} 1.7 & 10 \\ -0.5 & 2 \end{vmatrix}}{\Delta} = \frac{3.4 + 5}{0.77} = 10.91$$

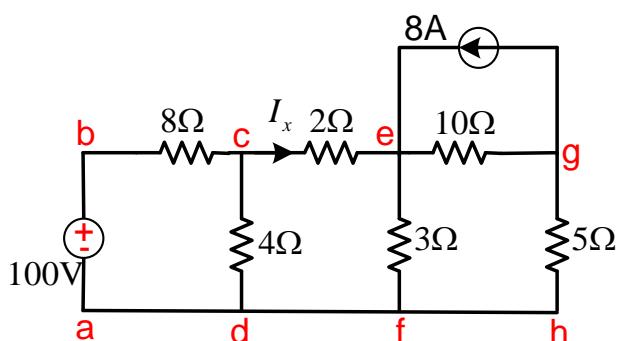
 V_1 is the voltage across 5Ω resistor which is 9.09 VQ 5) In the circuit shown in Figure 57 determine the current I_x 

Figure 57

Solution:

For the given circuit there is current source, to apply KCL current source has to be converted into voltage source, the modified circuit is as shown in Figure 58

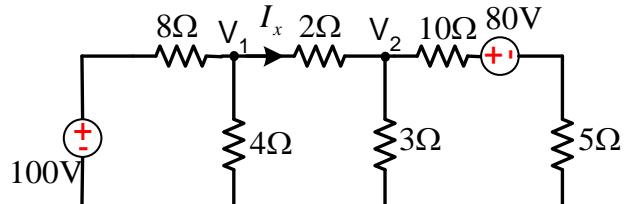


Figure 58

$$\begin{aligned} \left[\frac{V_1 - 100}{8} + \frac{V_1}{4} + \frac{V_1 - V_2}{2} \right] &= 0 \\ [0.125 + 0.25 + 0.5] V_1 - 12.5 - 0.5V_2 &= 0 \\ 0.875V_1 - 0.5V_2 &= 12.5 \\ \left[\frac{V_2}{3} + \frac{V_2 - 80}{15} + \frac{V_2 - V_1}{2} \right] &= 0 \\ -0.5V_1 + [0.33 + 0.067 + 0.5] V_2 - 5.33 &= 0 \\ -0.5V_1 + 0.897V_2 &= 5.33 \end{aligned}$$

The simultaneous equations are

$$\begin{aligned} 0.875V_1 - 0.5V_2 &= 12.5 \\ -0.5V_1 + 0.897V_2 &= 5.33 \\ \Delta = \begin{vmatrix} 0.875 & -0.5 \\ -0.5 & 0.897 \end{vmatrix} &= 0.7848 - .25 = 0.5348 \end{aligned}$$

$$V_1 = \frac{\begin{vmatrix} 12.5 & -0.5 \\ 5.33 & 0.897 \end{vmatrix}}{\Delta} = \frac{11.212 + 2.665}{0.5348} = \frac{13.877}{0.5348} = 25.94V$$

$$V_2 = \frac{\begin{vmatrix} 0.875 & 12.5 \\ -0.5 & 5.33 \end{vmatrix}}{\Delta} = \frac{4.66 + 6.25}{0.5348} = \frac{10.91}{0.5348} = 20.4V$$

$$I_x = \frac{V_1 - V_2}{2} = \frac{25.94 - 20.4}{2} = 2.77A$$

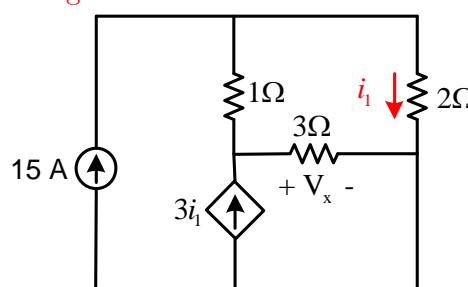
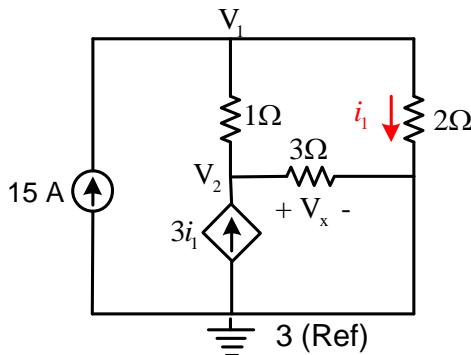
Q 6) Find the loop currents i_1, i_2, i_3 in the circuit shown in Figure 59

Figure 59





Solution:

Figure 60

Applying KCL to mesh V_1

$$\begin{aligned}\frac{V_1 - V_2}{1} + \frac{V_1}{2} - 15 &= 0 \\ (1 + 0.5)V_1 - V_2 &= 15 \\ 1.5V_1 - V_2 &= 15\end{aligned}$$

$$i_1 = \frac{V_1}{2}$$

Applying KCL to mesh V_2

$$\begin{aligned}\frac{V_2 - V_1}{1} + \frac{V_2}{3} - 3i_1 &= 0 \\ -V_1 + [1 + 0.333]V_2 - 3i_1 &= 0 \\ -V_1 + 1.333V_2 - 3i_1 &= 0 \\ -V_1 + 1.333V_2 - 3(0.5V_1) &= 0 \\ -2.5V_1 + 1.333V_2 &= 0 \\ 1.5V_1 - V_2 &= 15 \\ -2.5V_1 + 1.333V_2 &= 0\end{aligned}$$

$$\Delta = \begin{vmatrix} 1.5 & -1 \\ -2.5 & 1.333 \end{vmatrix} = 2 - 2.5 = -0.5$$

$$V_1 = \frac{\begin{vmatrix} 15 & 1 \\ 0 & 1.333 \end{vmatrix}}{\Delta} = \frac{20}{-0.5} = -40V$$

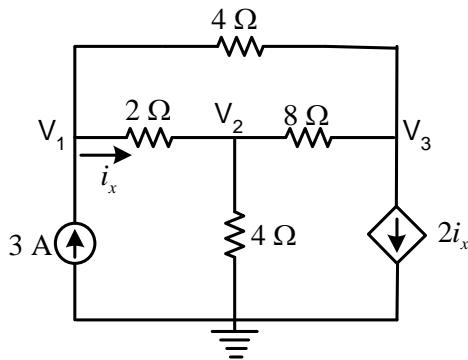
$$V_2 = \frac{\begin{vmatrix} 1.5 & 15 \\ -2.5 & 0 \end{vmatrix}}{\Delta} = \frac{37.5}{-0.5} = -75V$$

Power delivered by dependent current source is

$$i_1 = \frac{V_1}{2} = \frac{-40}{2} = -20$$

$$V_2 \times 3i_1 = -75 \times 3 \times -20 = 4.5kW$$

Q 7) In the network shown in Figure 61 find the node voltages using nodal analysis

Figure 61
Solution: Applying KCL to node V_1

$$\begin{aligned}\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} - 3 &= 0 \\ [0.5 + 0.25]V_1 - 0.5V_2 - 0.25V_3 &= 3 \\ 0.75V_1 - 0.5V_2 - 0.25V_3 &= 3 \\ 3V_1 - 2V_2 - V_3 &= 12\end{aligned}$$

Applying KCL to node V_2

$$\begin{aligned}\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{8} + \frac{V_2}{4} &= 0 \\ -0.5V_1 + 0.875V_2 - 0.125V_3 &= 0 \\ -4V_1 + 7V_2 - V_3 &= 0 \\ i_x &= \frac{V_1 - V_2}{2}\end{aligned}$$

Applying KCL to node V_3

$$\begin{aligned}\frac{V_3 - V_1}{4} + \frac{V_3 - V_2}{8} + 2i_x &= 0 \\ \frac{V_3 - V_1}{4} + \frac{V_3 - V_2}{8} + 2\frac{V_1 - V_2}{2} &= 0 \\ 0.75V_1 - 1.125V_2 + 0.375V_3 &= 0 \\ 2V_1 - 3V_2 + V_3 &= 0\end{aligned}$$

Linear equations are

$$\begin{aligned}3V_1 - 2V_2 - V_3 &= 12 \\ -4V_1 + 7V_2 - V_3 &= 0 \\ 2V_1 - 3V_2 + V_3 &= 0\end{aligned}$$

$$\begin{aligned}\Delta &= \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{vmatrix} = 3(7-3) + 2(-4+2) - 1(12-14) \\ &= 12 - 4 + 2 = 10\end{aligned}$$

$$V_1 = \frac{\begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \end{vmatrix}}{\Delta} = \frac{12(7-3)}{10} = 4.8V$$

$$V_2 = \frac{\begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \end{vmatrix}}{\Delta} = \frac{-12(-4+2)}{10} = 2.4V$$



$$V_3 = \frac{\begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{vmatrix}}{\Delta} = \frac{12(12-14)}{10} = -2.4V$$

Q 8) In the network shown in Figure 62 find the node voltages using nodal analysis

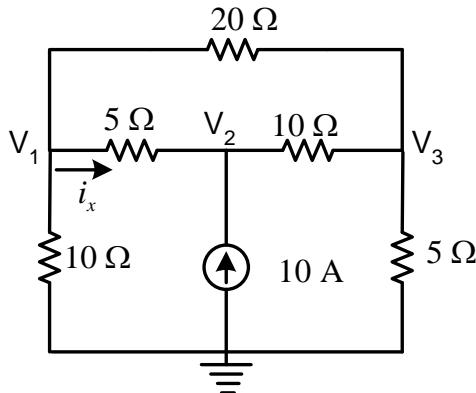


Figure 62

Solution: Applying KCL to node V_1

$$\begin{aligned} \frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{20} + \frac{V_1}{10} &= 0 \\ [0.1 + 0.2 + 0.05] V_1 - 0.2V_2 - 0.05V_3 &= 0 \\ 0.35V_1 - 0.2V_2 - 0.05V_3 &= 0 \\ 7V_1 - 4V_2 - V_3 &= 0 \end{aligned}$$

Applying KCL to node V_2

$$\begin{aligned} \frac{V_2 - V_1}{5} + \frac{V_2 - V_3}{10} - 10 &= 0 \\ -0.2V_1 + [0.2 + 0.1] V_2 - 0.1V_3 &= 10 \\ -0.2V_1 + 0.3V_2 - 0.1V_3 &= 10 \\ -2V_1 + 3V_2 - V_3 &= 100 \end{aligned}$$

Applying KCL to node V_3

$$\begin{aligned} \frac{V_3 - V_1}{20} + \frac{V_3 - V_2}{10} + \frac{V_3}{5} &= 0 \\ -0.05V_1 - 0.1V_2 + [0.2 + 0.1 + 0.05] V_3 &= 0 \\ -0.05V_1 - 0.1V_2 + 0.35V_3 &= 0 \\ -V_1 - 2V_2 + 7V_3 &= 0 \end{aligned}$$

$$\begin{aligned} 7V_1 - 4V_2 - V_3 &= 0 \\ -2V_1 + 3V_2 - V_3 &= 100 \\ -V_1 - 2V_2 + 7V_3 &= 0 \end{aligned}$$

$$\Delta = \begin{vmatrix} 7 & -4 & -1 \\ -2 & 3 & -1 \\ -1 & -2 & 7 \end{vmatrix} = 7(21-2) + 4(-14-1) - 1(4+3) = 133 - 60 - 7 = 66$$

$$V_1 = \frac{\begin{vmatrix} 0 & -4 & -1 \\ 100 & 3 & -1 \\ 0 & -2 & 7 \end{vmatrix}}{\Delta}$$

$$= \frac{4(700) - 1(-200)}{66} = \frac{2800 + 200}{66} = 45.45V$$

$$V_2 = \frac{\begin{vmatrix} 7 & 0 & -1 \\ -2 & 100 & -1 \\ -1 & 0 & 7 \end{vmatrix}}{\Delta}$$

$$= \frac{7(700) - 1(100)}{66} = \frac{4900 - 100}{66} = 72.73V$$

$$V_3 = \frac{\begin{vmatrix} 7 & -4 & 0 \\ -2 & 3 & 100 \\ -1 & -2 & 0 \end{vmatrix}}{\Delta}$$

$$= \frac{7(200) + 4(100)}{66} = \frac{1400 + 400}{66} = 27.27V$$

Q 9) In the network shown in Figure ?? find the current I by node voltage method.

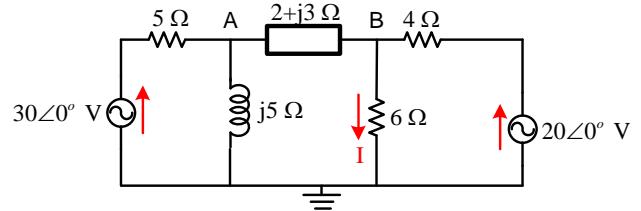


Figure 63

Solution: Applying KCL to node V_A

$$\begin{aligned} \frac{V_A - 30\angle 0}{5} + \frac{V_A - V_B}{2+j3} + \frac{V_A}{j5} &= 0 \\ V_A \left[\frac{1}{5} + \frac{1}{j5} + \frac{1}{2+j3} \right] - V_B \left[\frac{1}{2+j3} \right] - \frac{30}{5} &= 0 \\ \left[.2 - j0.2 + \frac{1}{3.6\angle 56.3} \right] V_A - \frac{V_B}{3.6\angle 56.3} - \frac{30\angle 0}{5} &= 0 \\ [.2 - j.2 + .277\angle -56.3] V_A - .277\angle -56.3 V_B &= 6 \\ [.2 - j.2 + .153 - j.23] V_A - .277\angle -56.3 V_B &= 6 \\ [.353 - j.43] V_A - .277\angle -56.3 V_B &= 6 \\ [.556\angle -50.6] V_A - .277\angle -56.3 V_B &= 6 \end{aligned}$$

Applying KCL to node V_B

$$\begin{aligned} \frac{V_B - 20\angle 0}{4} + \frac{V_B - V_A}{2+j3} + \frac{V_B}{6} &= 0 \\ V_B \left[\frac{1}{4} + \frac{1}{2+j3} + \frac{1}{6} \right] - V_A \left[\frac{1}{2+j3} \right] - \frac{20\angle 0}{4} &= 0 \\ \left[.16 + .25 + \frac{1}{3.6\angle 56.3} \right] V_A - \frac{V_A}{3.6\angle 56.3} - \frac{20\angle 0}{4} &= 0 \\ [.16 + .25 + .277\angle -56.3] V_B - .277\angle -56.3 V_A &= 5 \\ -.277\angle -56.3 V_A + [.16 + .25 + .153 - j.23] V_B &= 6 \\ -.277\angle -56.3 V_A + [0.563 - j0.23] V_B &= 5 \\ -.277\angle -56.3 V_A + 0.6\angle -22.3 V_B &= 5 \end{aligned}$$

$$\begin{aligned} 0.556\angle -50.6 V_A - 0.277\angle -56.3 V_2 &= 6 \\ -0.277\angle -56.3 V_A + 0.6\angle -22.3 V_B &= 5 \end{aligned}$$

Applying KCL to node V_B

$$\Delta = \begin{vmatrix} 0.556\angle - 50.6 & -0.277\angle - 56.3 \\ -0.277\angle - 56.3 & 0.6\angle - 22.3 \end{vmatrix}$$

$$= 0.336\angle - 72.9 - 0.076\angle - 112.6$$

$$0.1 - j0.321 + 0.029 + j0.07 = 0.129 - j0.251 = 0.282\angle - 62.79$$

$$V_B = \frac{\begin{vmatrix} 0.556\angle - 50.6 & 6\angle 0 \\ -0.277\angle - 56.3 & 5\angle 0 \end{vmatrix}}{\Delta} =$$

$$\begin{aligned} &= \frac{2.78\angle - 50.6 + 1.662\angle - 56.3}{0.282\angle - 62.79} \\ &= \frac{1.764 - j2.148 + 0.922 - j1.38}{0.282\angle - 62.79} \\ &= \frac{2.686 - j3.528}{0.282\angle - 62.79} = \frac{4.43\angle - 52.7}{0.282\angle - 62.79} \\ &= 15.87\angle 10.01 \end{aligned}$$

$$I = \frac{V_B}{6} = \frac{15.87\angle 10.01}{6} = 2.64\angle 10.01A$$

Q 10) In the network shown in Figure 64 find the value of E_2 such that current through the $8+j8\Omega$ is zero.

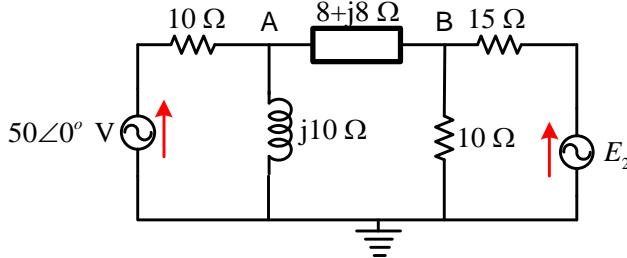


Figure 64

Solution:

It is given that current through the $8+j8\Omega$ is zero
this is possible only when $V_A = V_B$ i., $V_A - V_B = 0$

Applying KCL to node V_A

$$\begin{aligned} \frac{V_A - 50}{10} + \frac{V_A}{j10} &= 0 \\ \left[\frac{1}{10} + \frac{1}{j10} \right] V_A - \frac{50}{10} &= 0 \\ [0.1 - j0.1] V_A &= 5 \\ 0.141\angle - 45V_A &= 5 \\ V_A &= 35.46\angle 45 \end{aligned}$$

$$\begin{aligned} \frac{V_B - E_2}{15} + \frac{V_B}{10} &= 0 \\ \left[\frac{1}{10} + \frac{1}{15} \right] V_B - \frac{E_2}{15} &= 0 \\ [0.1 + 0.066] V_B - \frac{E_2}{15} &= 0 \\ \frac{E_2}{15} &= 0.166V_B \\ E_2 &= 2.5V_B \end{aligned}$$

$$V_A = V_B$$

$$\begin{aligned} E_2 &= 2.5 \times 35.46\angle 45 \\ E_2 &= 88.65\angle 45 \end{aligned}$$

Q 11) In the network shown in Figure 65 find the value of E_2 such that current through the $2+j3\Omega$ is zero.

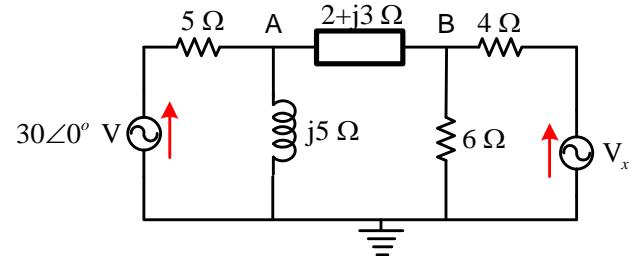


Figure 65

Solution:

It is given that current through the $2+j3\Omega$ is zero
this is possible only when $V_A = V_B$ i., $V_A - V_B = 0$

Applying KCL to node V_A

$$\begin{aligned} \frac{V_A - 30}{5} + \frac{V_A}{j5} &= 0 \\ \left[\frac{1}{5} + \frac{1}{j5} \right] V_A - \frac{30}{5} &= 0 \\ [0.2 - j0.2] V_A &= 6 \\ 0.282\angle - 45V_A &= 6 \\ V_A &= 21.27\angle 45 \end{aligned}$$



Applying KCL to node V_B

$$\begin{aligned}\frac{V_B - V_x}{4} + \frac{V_B}{6} &= 0 \\ \left[\frac{1}{6} + \frac{1}{4}\right] V_B - \frac{V_x}{4} &= 0 \\ [0.166 + 0.25] V_B - \frac{V_x}{4} &= 0 \\ \frac{V_x}{4} &= 0.416 V_B \\ V_x &= 1.664 V_B\end{aligned}$$

$$\begin{aligned}V_A &= V_B \\ V_x &= 1.664 \times 21.27 \angle 45 \\ V_x &= 35.39 \angle 45\end{aligned}$$

simultaneous equations are

$$\begin{aligned}0.355\angle - 22V_1 + 0.133\angle 90V_2 &= -5\angle 90 \\ 0.133\angle 90V_1 + 0.212\angle - 38.7V_2 &= 10 \\ \Delta &= \begin{vmatrix} 0.355\angle - 22 & 0.133\angle 90 \\ 0.133\angle 90 & 0.212\angle - 38.7 \end{vmatrix} \\ &= 0.0752\angle - 60.7 - 0.0177\angle 180 \\ &= 0.0368 - j0.0655 + 0.0177 = 0.0545 - j0.0655 \\ &= 0.0852\angle - 50.2\end{aligned}$$

$$\begin{aligned}V_2 &= \frac{\begin{vmatrix} 0.355\angle - 22 & -5\angle 90 \\ 0.133\angle 90 & 10 \end{vmatrix}}{\Delta} \\ &= \frac{3.55\angle - 22 + 0.665\angle 180}{\Delta} \\ &= \frac{3.3 - j1.33 - 0.665}{\Delta} = 2.635 - j1.3 \\ &= \frac{2.93\angle - 26.25}{0.0852\angle - 50.2} \\ &= 34.38\angle 24\end{aligned}$$

Q 12) In the network shown in Figure 66 find the value of V_2 using nodal analysis

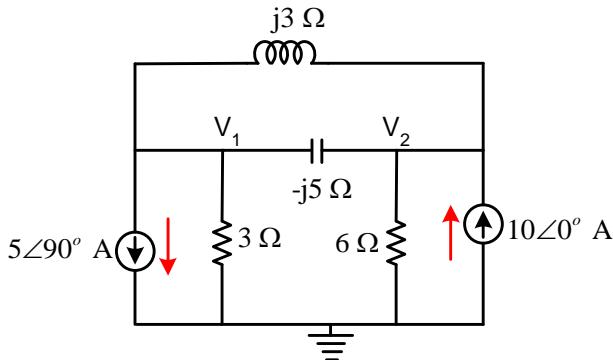


Figure 66

Solution: Applying KCL to node V_1

$$\begin{aligned}\frac{V_1}{3} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j3} + 5\angle 90 &= 0 \\ [0.33 + j0.2 - j0.33] V_1 - [j0.2 - j0.33] V_2 &= -5\angle 90 \\ [0.33 - j0.133] V_1 + j0.133 V_2 &= -5\angle 90 \\ 0.355\angle - 22V_1 + 0.133\angle 90V_2 &= -5\angle 90\end{aligned}$$

Applying KCL to node V_1

$$\begin{aligned}\frac{V_2}{6} + \frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j3} - 10 &= 0 \\ -[j0.2 - j0.33] V_1 + [0.166 + j0.2 - j0.33] V_2 &= 10 \\ j0.133 V_1 + [0.166 - j0.133] V_2 &= 10 \\ 0.133\angle 90 V_1 + 0.212\angle - 38.7 V_2 &= 10\end{aligned}$$

0.5 Supernode

- When an voltage source appears between two nonreference nodes and any elements connected in parallel with it then combining these nodes is called a supernode.

Q 1) In the circuit shown in Figure 67 determine the node voltages

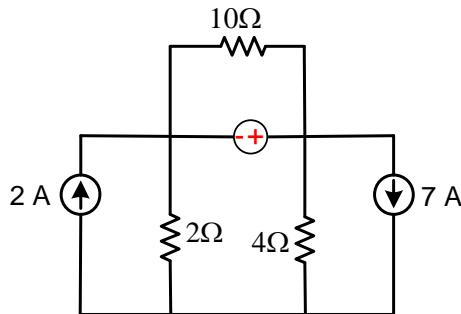


Figure 67

Solution:

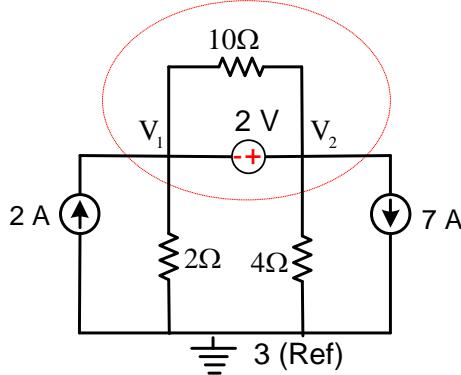


Figure 68

There is voltage source between V_1 and V_2 and by supernode equation is

$$\begin{aligned} \frac{V_1}{2} + \frac{V_2}{4} - 2 + 7 &= 0 \\ 0.5V_1 + 0.25V_2 &= -5 \end{aligned}$$

$$V_2 - V_1 = 2$$

$$V_1 - V_2 = -2$$

Simultaneous equations are

$$0.5V_1 + 0.25V_2 = -5$$

$$V_1 - V_2 = -2$$

where Δ is

$$\Delta = \begin{vmatrix} 0.5 & 0.25 \\ 1 & -1 \end{vmatrix} = -0.5 - 0.25 = -0.75$$

$$V_1 = \frac{\begin{vmatrix} -5 & 0.25 \\ -2 & -1 \end{vmatrix}}{\Delta} = \frac{5 + 0.5}{-0.75} = -7.33$$

$$V_2 = \frac{\begin{vmatrix} 0.5 & -5 \\ 1 & -2 \end{vmatrix}}{\Delta} = \frac{-1 + 5}{-0.75} = -5.33$$

Q 2) In the circuit shown in Figure 69 determine the node voltages

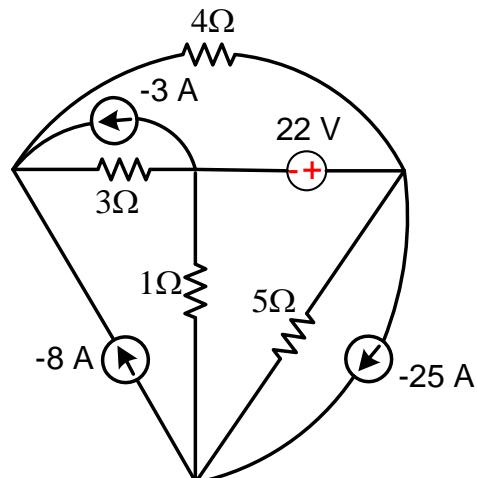


Figure 69

Solution:

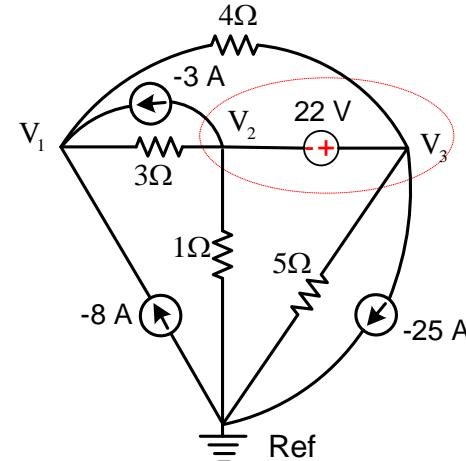


Figure 70

There is voltage source between V_3 and V_2 i.e.,

$$V_3 - V_2 = 22$$

$$V_3 = V_2 + 22$$

Apply KCL for node V_1

$$\frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4} - (-8) - (-3) = 0$$

$$0.5833V_1 - 0.333V_2 - 0.25V_3 = -11$$

$$0.5833V_1 - 0.333V_2 - 0.25(V_2 + 22) = -11$$

$$0.5833V_1 - 0.583V_2 = -5.5 \quad (1)$$



There is voltage source between V_2 V_3 and by applying supernode equation

$$\frac{V_2 - V_1}{3} + \frac{V_2}{1} + \frac{V_3 - V_1}{4} + \frac{V_3}{5} + (-3) + (-25) = 0$$

$$\begin{aligned} -0.5833V_1 + 1.333V_2 + 0.45V_3 &= 28 \\ -0.5833V_1 + 1.333V_2 + 0.45(V_2 + 22) &= 28 \\ -0.5833V_1 + 1.783V_2 &= 18.1 \quad (2) \end{aligned}$$

Simultaneous equations are

$$\begin{aligned} 0.5833V_1 - 0.583V_2 &= -5.5 \\ -0.5833V_1 + 1.783V_2 &= 18.1 \end{aligned}$$

where Δ is

$$\Delta = \begin{vmatrix} 0.5833 & -0.583 \\ -0.583 & 1.783 \end{vmatrix} = 1.04 - 0.339 = 0.7$$

$$V_1 = \frac{\begin{vmatrix} -5.5 & -0.583 \\ 18.1 & 1.783 \end{vmatrix}}{\Delta} = \frac{-9.8 + 10.5}{0.7} = 1V$$

$$V_1 = \frac{\begin{vmatrix} 0.5833 & -5.5 \\ -0.5833 & 18.1 \end{vmatrix}}{\Delta} = \frac{10.55 - 3.2}{0.7} = 10.48V$$

$$V_3 = V_2 + 22 = 10.48 + 22 = 32.48$$

Q 3) In the circuit shown in Figure 71 determine the node voltages

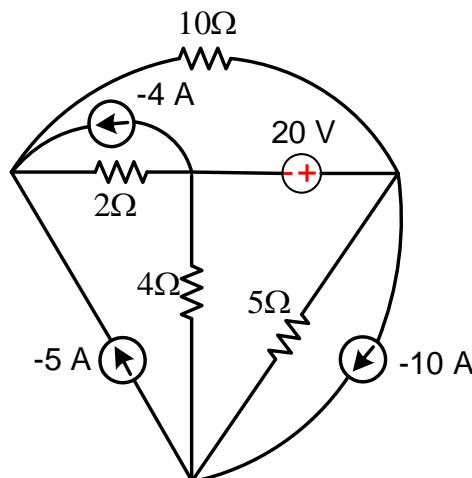


Figure 71

Solution:

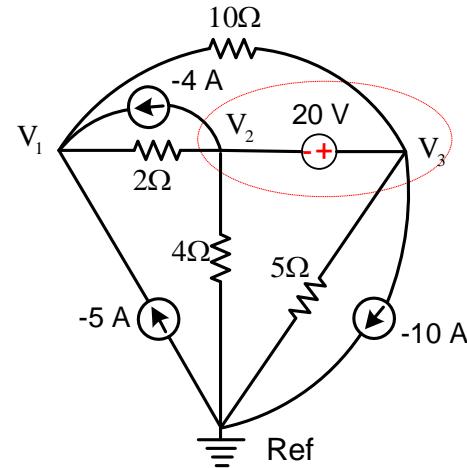


Figure 72

There is voltage source between V_3 and V_2 i.e.,

$$\begin{aligned} V_3 - V_2 &= 20 \\ V_3 &= V_2 + 20 \end{aligned}$$

Apply KCL for node V_1

$$\begin{aligned} \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{10} - (-5) - (-4) &= 0 \\ 0.6V_1 - 0.5V_2 - 0.1V_3 &= -9 \\ 0.6V_1 - 0.5V_2 - 0.1(V_2 + 20) &= -9 \\ 0.6V_1 - 0.6V_2 &= -7 \quad (1) \end{aligned}$$

There is voltage source between V_2 V_3 and by applying supernode equation

$$\begin{aligned} \frac{V_2 - V_1}{2} + \frac{V_2}{4} + \frac{V_3 - V_1}{10} + \frac{V_3}{5} + (-4) + (-10) &= 0 \\ -0.6V_1 + 0.75V_2 + 0.3V_3 &= 14 \\ -0.6V_1 + 0.75V_2 + 0.3(V_2 + 20) &= 14 \\ -0.6V_1 + 1.05V_2 &= 8 \quad (2) \end{aligned}$$

Simultaneous equations are

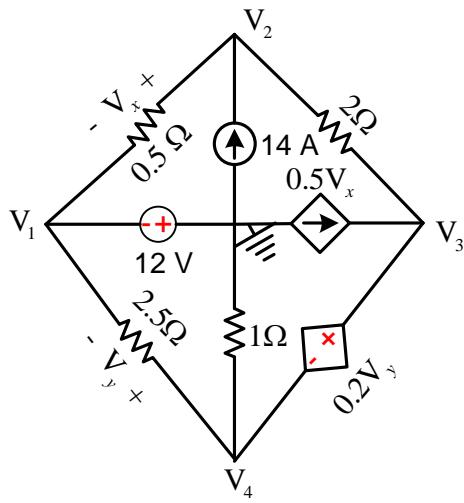
$$\begin{aligned} 0.6V_1 - 0.6V_2 &= -7 \\ -0.6V_1 + 1.05V_2 &= 8 \end{aligned}$$

where Δ is

$$\Delta = \begin{vmatrix} 0.6 & -0.6 \\ -0.6 & 1.05 \end{vmatrix} = 0.63 - 0.36 = 0.27$$

$$\begin{aligned} V_1 &= \frac{\begin{vmatrix} -7 & -0.6 \\ 8 & 1.05 \end{vmatrix}}{\Delta} = \frac{-7.35 + 4.8}{0.27} = -9.44V \\ V_2 &= \frac{\begin{vmatrix} 0.6 & -7 \\ -0.6 & 8 \end{vmatrix}}{\Delta} = \frac{4.8 - 4.2}{0.27} = 2.22V \\ V_3 &= V_2 + 20 = 2.22 + 20 = 22.22V \end{aligned}$$

Q 4) In the circuit shown in Figure ?? determine the node voltages V_1, V_2, V_3, V_4



Solution:

$$\begin{aligned}
 V_3 - V_4 &= 0.2V_y \\
 V_3 - V_4 &= 0.2(V_4 - V_1) \\
 V_3 - 1.2V_4 &= -0.2V_1 = -0.2 \times (-12) \\
 V_3 - 1.2V_4 &= 2.4 \\
 -1.2V_4 &= 2.4 - V_3 \\
 V_4 &= 0.833V_3 - 2
 \end{aligned}$$

Apply KCL for super nodes V_3 and V_4

$$\begin{aligned}
 \frac{V_3 - V_2}{2} - 0.5V_x + \frac{V_4 - V_1}{2.5} + \frac{V_4}{1} &= 0 \\
 -.4V_1 - .5V_2 + .5V_3 + 1.4V_4 - .5V_x &= 0 \\
 -.4V_1 - .5V_2 + .5V_3 + 1.4V_4 - .5(V_2 - V_1) &= 0 \\
 0.1V_1 - V_2 + 0.5V_3 + 1.4V_4 &= 0 \\
 -V_2 + 0.5V_3 + 1.4V_4 &= 1.2 \\
 -V_2 + 0.5V_3 + 1.4(0.833V_3 - 2) &= 1.2 \\
 -V_2 + 1.66V_3 &= 4 \quad (2)
 \end{aligned}$$

Simultaneous equations are

$$\begin{aligned}
 2.5V_2 - 0.5V_3 &= -10 \\
 -V_2 + 1.66V_3 &= 4
 \end{aligned}$$

where Δ is

$$\Delta = \begin{vmatrix} 2.5 & -0.5 \\ -1 & 1.66 \end{vmatrix} = 4.15 - 0.5 = 3.65$$

$$V_2 = \frac{\begin{vmatrix} -10 & -0.5 \\ 4 & 1.66 \end{vmatrix}}{\Delta} = \frac{-16.6 + 2}{3.65} = -4V$$

$$V_3 = \frac{\begin{vmatrix} 2.5 & -10 \\ -1 & 4 \end{vmatrix}}{\Delta} = \frac{10 - 10}{3.65} = 0V$$

$$V_4 = 0.833V_3 - 2 = 0.833 \times 0 - 2 = -2$$

Figure 73

From the figure 73 it is observed that

$$V_1 = -12V$$

Apply KCL for node V_2

$$V_1 = -12, V_2 = -4, V_3 = 0, V_4 = -2$$

$$\begin{aligned}
 \frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} - 14 &= 0 \\
 -2V_1 + [2 + 0.5]V_2 - 0.5V_3 &= 14 \\
 -2(-12) + 2.5V_2 - 0.5V_3 &= 14 \\
 2.5V_2 - 0.5V_3 &= -10 \quad (1)
 \end{aligned}$$

There is voltage source between V_3 and V_4 i.e.,

$$V_3 - V_4 = 0.2V_y$$

$$V_y = V_4 - V_1$$

$$V_x = V_2 - V_1$$

Q 5) In the circuit shown in Figure 74 determine the node voltages

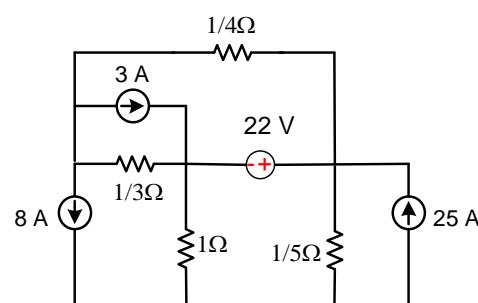


Figure 74

Solution:



The circuit is redrawn with node labeling is as shown in Figure 75

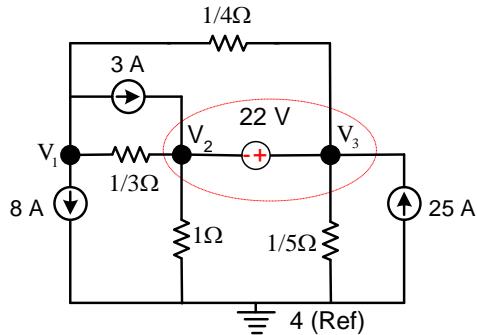


Figure 75

From the figure it is observed that there is voltage source between node V_2 and V_3

$$V_3 - V_2 = 22$$

$$V_3 = V_2 + 22$$

Apply KCL for node V_1

$$\begin{aligned} \frac{V_1 - V_2}{1/3} + \frac{V_1 - V_3}{1/4} + 8 + 3 &= 0 \\ 7V_1 - 3V_2 - 4V_3 &= -11 \\ 7V_1 - 3V_2 - 4(V_2 + 22) &= -11 \\ 7V_1 - 7V_2 &= 77 \quad (1) \end{aligned}$$

There is voltage source between V_2 and V_3 i.e.,

$$\begin{aligned} \frac{V_2 - V_1}{1/3} + \frac{V_2}{1} + \frac{V_3 - V_1}{1/4} + \frac{V_3}{1/5} - 25 - 3 &= 0 \\ -7V_1 + 4V_2 + 9V_3 &= 28 \\ -7V_1 + V_2 + 9(V_2 + 22) &= 28 \\ -7V_1 + 13V_2 &= -170 \end{aligned}$$

Simultaneous equations are

$$\begin{aligned} 7V_1 - 7V_2 &= 77 \\ -7V_1 + 13V_2 &= -170 \end{aligned}$$

$$\Delta = \begin{vmatrix} 7 & -7 \\ -7 & 13 \end{vmatrix} = 91 - 49 = 42$$

$$V_1 = \frac{\begin{vmatrix} 77 & -7 \\ -170 & 13 \end{vmatrix}}{\Delta} = \frac{1001 - 1190}{42} = -4.5V$$

$$V_2 = \frac{\begin{vmatrix} 7 & 77 \\ -7 & -170 \end{vmatrix}}{\Delta} = \frac{-1190 + 539}{42} = -15.5V$$

$$V_3 = V_2 + 22 = -15.5 + 22 = 6.5V$$

Q 6) In the circuit shown in Figure 76 determine the node voltages and also current I_o using nodal analysis

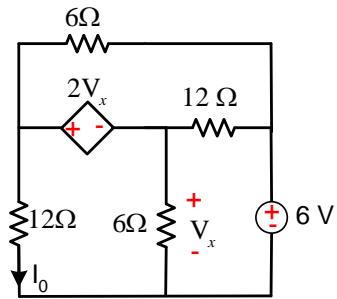


Figure 76

Solution:

The circuit is redrawn with node labeling is as shown in Figure 77

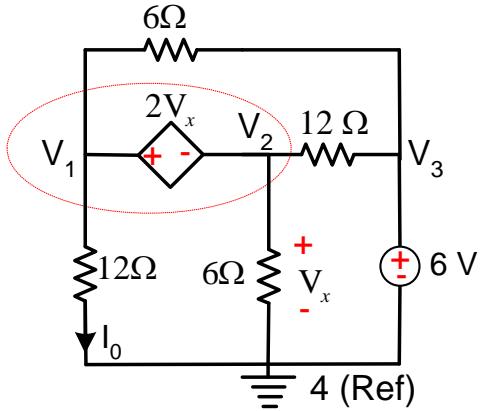


Figure 77

From the figure it is observed that there is dependent voltage source between node V_1 and V_2

$$\begin{aligned} V_3 &= 6V \\ V_2 &= V_x \\ V_1 - V_2 &= 2V_x \\ V_1 &= V_2 + 2V_x \\ V_1 &= V_x + 2V_x = 3V_x \end{aligned}$$

There is voltage source between V_1 and V_2 Apply KCL for supernode V_1 and V_2

$$\begin{aligned} \frac{V_1 - V_3}{6} + \frac{V_1}{12} + \frac{V_2}{6} + \frac{V_2 - V_3}{12} &= 0 \\ \frac{2V_1 - 2V_3 + V_1 + 2V_2 + V_2 - V_3}{12} &= 0 \\ 6V_x - 2V_3 + 3V_x + 2V_x + V_x - V_3 &= 0 \\ 12V_x &= 3V_3 \\ V_x &= \frac{3 \times 6}{12} = 1.5V \\ V_1 &= 3V_x = 3 \times 1.5 = 4.5 \\ I_o &= \frac{V_1}{12} = \frac{4.5}{12} = 0.375A \end{aligned}$$

Q 7) In the circuit shown in Figure 78 determine all the node voltages using nodal analysis



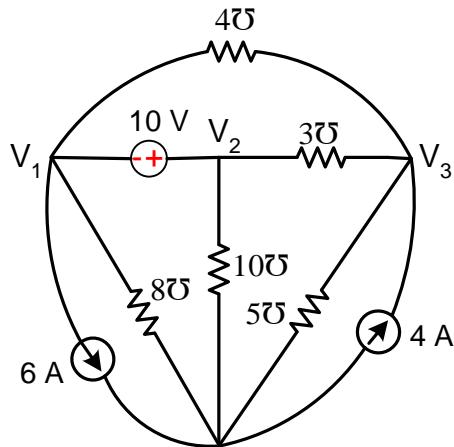


Figure 78

Solution:

From the figure it is observed that there is a voltage source between node V_1 and V_2

$$V_2 - V_1 = 10$$

$$-V_1 + V_2 + 0V_3 = 10$$

There is voltage source between V_1 and V_2 . Apply KCL for supernode V_1 and V_2

$$8V_1 + 4V_1 - 4V_3 + 6 + 10V_2 + 3V_2 - 3V_3 = 0$$

$$12V_1 + 13V_2 - 7V_3 = -6$$

Apply KCL for node V_3

$$-4V_1 - 3V_2 + 12V_3 - 4 = 0$$

$$-4V_1 - 3V_2 + 12V_3 = 4$$

$$\begin{aligned} -V_1 + V_2 + 0V_3 &= 10 \\ 12V_1 + 13V_2 - 7V_3 &= -6 \\ -4V_1 - 3V_2 + 12V_3 &= 4 \end{aligned}$$

Solving above simultaneous equations

$$V_1 = 5.5V \quad V_2 = 4.46V \quad V_3 = -0.406V$$

Q 8) In the circuit shown in Figure 79 determine all the node voltages using nodal analysis

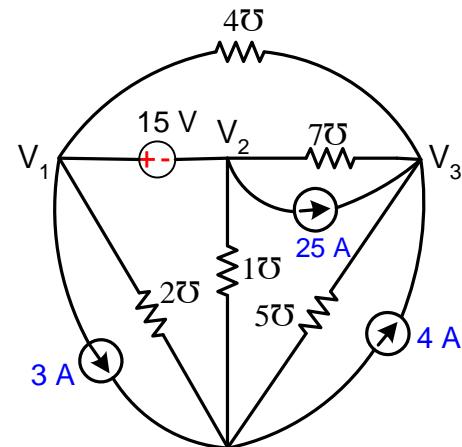


Figure 79

Solution:

From the figure it is observed that there is a voltage source between node V_1 and V_2 .

$$\begin{aligned} V_2 - V_1 &= -15 \\ -V_1 + V_2 + 0V_3 &= -15 \end{aligned}$$

There is voltage source between V_1 and V_2 . Apply KCL for supernode V_1 and V_2

$$\begin{aligned} 2V_1 + 4V_1 + 3 - 4V_3 + 1V_2 + 7V_2 - 7V_3 + 25 &= 0 \\ 6V_1 + 8V_2 - 11V_3 &= -28 \end{aligned}$$

Apply KCL for node V_3

$$\begin{aligned} -4V_1 - 7V_2 + 16V_3 - 25 - 4 &= 0 \\ -4V_1 - 7V_2 + 16V_3 &= 29 \end{aligned}$$

$$-V_1 + V_2 + 0V_3 = -15$$

$$6V_1 + 8V_2 - 11V_3 = -28$$

$$-4V_1 - 7V_2 + 16V_3 = 29$$

Solving above simultaneous equations

$$V_1 = 6.17V \quad V_2 = -8.82V \quad V_3 = -0.504V$$

Q 9) In the circuit shown in Figure 80 determine all the node voltages using nodal analysis

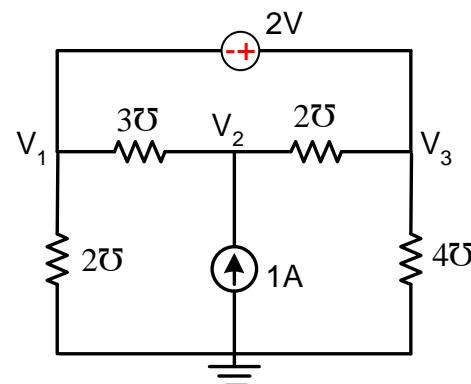


Figure 80

Solution:

From the figure it is observed that there is a voltage source between node V_1 and V_3

$$\begin{aligned} V_3 - V_1 &= 2 \\ -V_1 + 0V_2 + V_3 &= 2 \end{aligned}$$

There is voltage source between V_1 and V_3 . Apply KCL for supernode V_1 and V_3

$$\begin{aligned} 2V_1 + 3V_1 - 4V_2 + 4V_3 + 2V_3 - 2V_2 &= 0 \\ 5V_1 - 5V_2 + 6V_3 &= 0 \end{aligned}$$

Apply KCL for node V_2

$$\begin{aligned} -3V_1 + 5V_2 - 2V_3 - 1 &= 0 \\ -3V_1 + 5V_2 - 2V_3 &= 1 \end{aligned}$$

$$\begin{aligned} -V_1 + 0V_2 + V_3 &= 2 \\ 5V_1 - 5V_2 + 6V_3 &= 0 \\ -3V_1 + 5V_2 - 2V_3 &= 1 \end{aligned}$$

Solving above simultaneous equations

$$V_1 = -1.667V \quad V_2 = -0.1667V \quad V_3 = 0.833V$$

The node equations of a network are

$$\left[\frac{1}{5} + \frac{1}{j5} + \frac{1}{-j5} \right] V_1 - \frac{1}{j5} V_2 = \frac{50\angle 0}{5} \quad (1)$$

$$-\frac{1}{j5} V_1 + \left[\frac{1}{j5} + \frac{1}{2+j5} + \frac{1}{5} \right] V_2 = -\frac{50\angle 90}{5} \quad (2)$$

Derive the network

Solution:

From equation (5) it is observed that

- From node 1 to common node there is 5Ω resistor in series with $50\angle 0$ voltage source.
- The inductance of 5Ω is connected between node 1 and node 2
- The capacitance of 5Ω is connected from node 1 to common node

Similarly From equation (6) it is observed that

- From node 2 to common node there is 5Ω resistor in series with $50\angle 90$ voltage source.
- The inductance of 5Ω is connected between node 1 and node 2

- The resistor of 2Ω in series with inductance of 5Ω is connected from node 1 to common node.

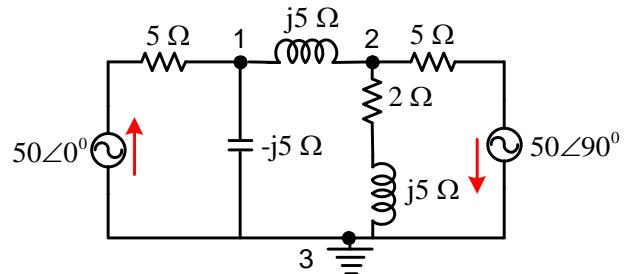


Figure 81

The node equations of a network are

$$\left[\frac{1}{6} + \frac{1}{j5} + \frac{1}{-j10} \right] V_1 - \frac{1}{j5} V_2 = \frac{10\angle 30}{6} \quad (3)$$

$$-\frac{1}{j5} V_1 + \left[\frac{1}{j5} + \frac{1}{3+j8} + \frac{1}{4} \right] V_2 = -\frac{50\angle -30}{4} \quad (4)$$

Derive the network

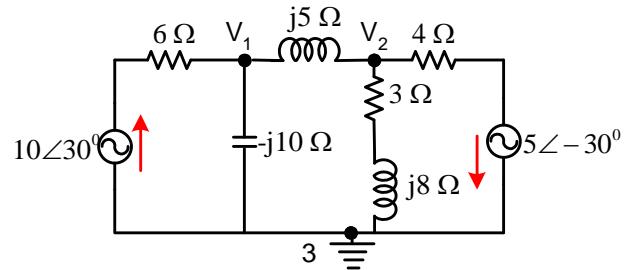
Solution:

Figure 82

The node equations of a network are

$$\left[\frac{1}{5} + \frac{1}{j2} + \frac{1}{-j2} \right] V_1 - \frac{1}{4} V_2 = \frac{50\angle 0}{5} \quad (5)$$

$$-\frac{1}{4} V_1 + \left[\frac{1}{4} + \frac{1}{-j2} + \frac{1}{2} \right] V_2 = \frac{50\angle 90}{2} \quad (6)$$

Derive the network

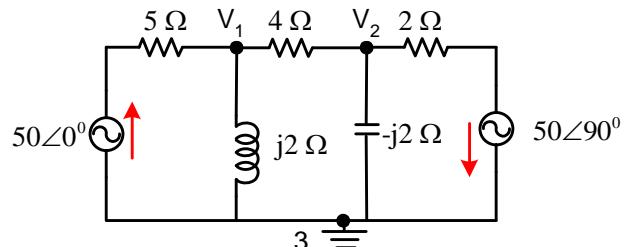
Solution:

Figure 83



Derive the network for the following mesh equations.

$$\begin{bmatrix} 5+j5 & -j5 & 0 \\ -j5 & 8+j8 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50\angle 30^\circ \\ -5\angle 0^\circ \\ -20\angle 0^\circ \end{bmatrix} \quad (7)$$

Solution:

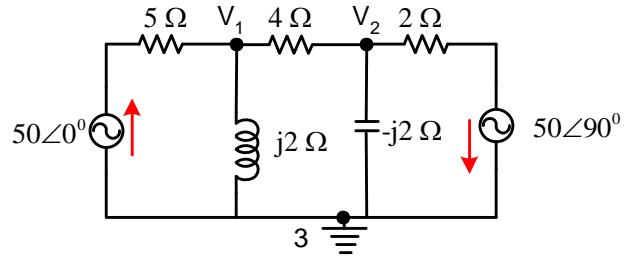


Figure 84

0.6 Question Papers

2020-Aug-CBCS 2-b) Find I_1 in the circuit shown in Figure 85 using nodal analysis.

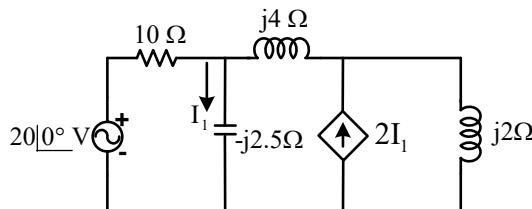


Figure 85: 2020-Aug

Applying KCL to node 1

$$\begin{aligned} \frac{V_1 - 30}{5 + j15} + \frac{V_1}{-j10} + \frac{V_1 + 10}{8 - j5} &= 0 \\ \left[\frac{1}{5 + j15} - \frac{1}{j10} + \frac{1}{8 - j5} \right] V_1 &= \frac{30}{5 + j15} - \frac{10}{8 - j5} \\ 0.146\angle 41.194 V_1 &= 2.38\angle -97 \\ V_1 &= \frac{2.38\angle -97}{0.146\angle 41.194} = 16.3\angle -138.19 \end{aligned}$$

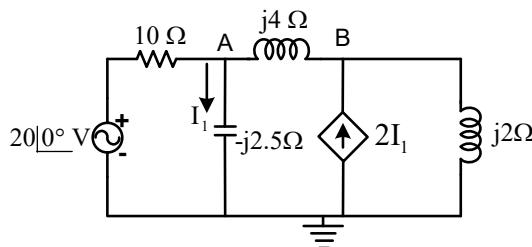


Figure 86: 2020-Aug

Applying KCL to node A

$$\begin{aligned} \frac{V_A - 20\angle 0}{10} + \frac{V_A}{-j2.5} + \frac{V_A - V_B}{j4} &= 0 \\ \left[\frac{1}{2} + \frac{1}{-j2} \right] V_B - \frac{50\angle 90}{2} &= 0 \\ [0.5 + j0.5] V_B &= 25\angle 90 \\ 0.707\angle 45 V_B &= 25\angle 90 \\ V_B &= 35.36\angle 45 \end{aligned}$$

2019-July-CBCS 2-b) Find the voltage across the capacitor 10Ω reactance of the network shown in Figure 88 using nodal analysis.

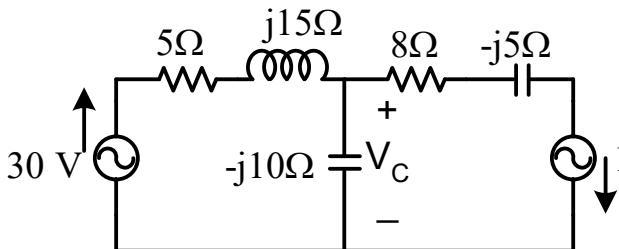


Figure 87: 2019-July

Solution:

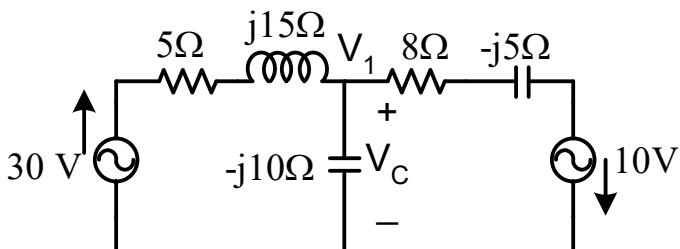


Figure 88: 2019-July

JAN-2018-CBCS 2-b JAN-2015 1d) For the network shown in Figure 89 find the value of voltage source V such that current through the 4Ω is zero. Use node voltage analysis.

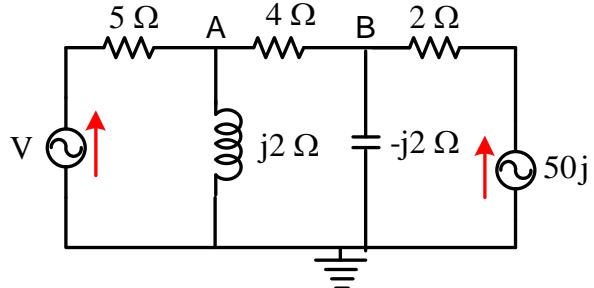


Figure 89: 2018-CBCS-Question Paper

Solution:

It is given that current through the 4Ω is zero this is possible only when $V_A = V_B$ i.e., $V_A - V_B = 0$

Applying KCL to node B

$$\begin{aligned} \frac{V_B - 50\angle 90}{2} + \frac{V_B}{-j2} &= 0 \\ \left[\frac{1}{2} + \frac{1}{-j2} \right] V_B - \frac{50\angle 90}{2} &= 0 \\ [0.5 + j0.5] V_B &= 25\angle 90 \\ 0.707\angle 45 V_B &= 25\angle 90 \\ V_B &= 35.36\angle 45 \end{aligned}$$

Applying KCL to node A

$$\begin{aligned} \frac{V_A - V}{5} + \frac{V_A}{j2} &= 0 \\ \left[\frac{1}{5} + \frac{1}{j2} \right] V_A - \frac{V}{5} &= 0 \\ [0.2 - j0.5] V_A - \frac{V}{5} &= 0 \\ 0.538\angle -68.2 V_A - 0.2V &= 0 \\ 0.2V &= 0.538\angle -68.2 V_A \\ V &= 2.69\angle -68.2 \times 35.36\angle 45 \\ V &= 95.11\angle -23.2 \end{aligned}$$



JAN-2017-CBCS Determine all the node voltages of the network shown in Figure 90 using nodal analysis.

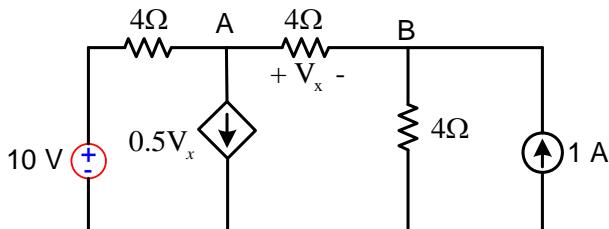


Figure 90

Solution:

$$V_x = V_A - V_B$$

Applying KCL to node A

$$\begin{aligned} \frac{V_A - 10}{4} + \frac{V_A - V_B}{4} + 0.5V_x &= 0 \\ \left[\frac{1}{4} + \frac{1}{4} \right] V_A - \frac{V_B}{4} + 0.5V_x - 2.5 &= 0 \\ 0.5V_A - 0.25V_B + 0.5(V_A - V_B) &= 2.5 \\ 0.5V_A - 0.25V_B + 0.5V_A - 0.5V_B &= 2.5 \\ V_A - 0.75V_B &= 2.5 \end{aligned}$$

Applying KCL to node B

$$\begin{aligned} \frac{V_B}{4} + \frac{V_B - V_A}{4} - 1 &= 0 \\ -0.25V_A + 0.5V_B &= 1 \end{aligned}$$

$$V_A - 0.75V_B = 2.5$$

$$-0.25V_A + 0.5V_B = 1$$

Solving the above equations

$$V_A = 6.4V \quad V_B = 5.2V$$

JULY-2017-CBCS 2 b) For the circuit shown in Figure 91 determine all the node voltages.

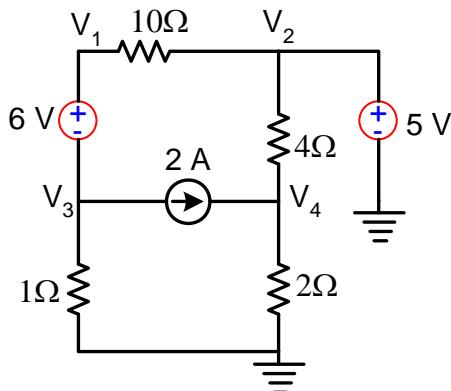


Figure 91

Solution: The circuit is redrawn which is as shown in Figure 92

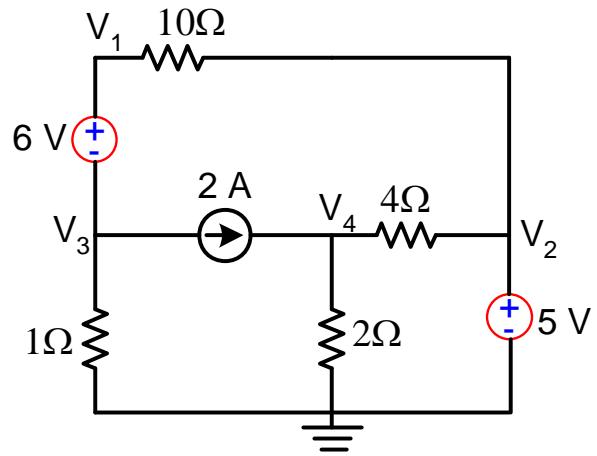


Figure 92

$$V_2 = 5$$

Applying KCL to node 3

$$\begin{aligned} \frac{V_3 + 6 - V_2}{10} + \frac{V_3}{1} + 2 &= 0 \\ \left[\frac{1}{10} + \frac{1}{1} \right] V_3 - \frac{V_2}{6} + \frac{6}{10} + 2 &= 0 \\ 1.1V_3 - 0.16V_2 &= -2.6 \\ 1.1V_3 - 0.16 \times 5 &= -2.6 \\ 1.1V_3 - 0.8 &= -2.6 \\ 1.1V_3 &= -1.8 \\ V_3 &= -1.63 \end{aligned}$$

Applying KCL to node 4

$$\begin{aligned} \frac{V_4}{2} + \frac{V_4 - V_2}{4} - 2 &= 0 \\ 0.75V_4 - 0.25V_2 - 2 &= 0 \\ 0.75V_4 - 0.25 \times 5 - 2 &= 0 \\ 0.75V_4 - 1.25 - 2 &= 0 \\ 0.75V_4 &= 3.25 \\ V_4 &= 4.33 \end{aligned}$$

$$V_1 = V_3 + 6 = -1.63 + 6 = 4.37$$

CBCS JAN 2017) Find i using nodal analysis for the network shown in Figure 93

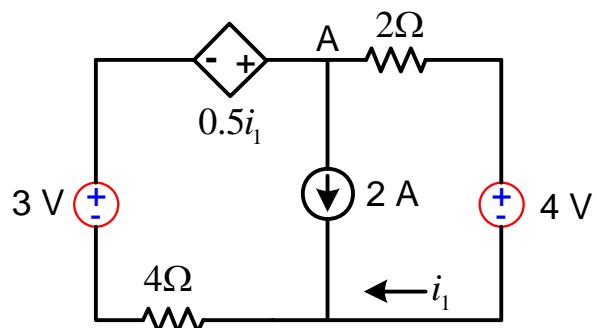


Figure 93

Solution:

$$i_1 = \frac{V_A - 4}{2} = 0.5V_A - 2$$

Applying KCL to node A

$$\begin{aligned} \frac{V_A - 3 - 0.5i_1}{4} + 2 + \frac{V_A - 4}{2} &= 0 \\ \left[\frac{1}{4} + \frac{1}{2} \right] V_A - \frac{0.5i_1}{4} + 2 - 0.75 - 2 &= 0 \\ 0.75V_A - 0.125i_1 &= 0.75 \\ 0.75V_A - 0.125(0.5V_A - 2) &= 0.75 \\ 0.75V_A - 0.0625V_A + 0.25 &= 0.75 \\ 0.6875V_A &= 0.5 \\ V_A &= 0.727v \end{aligned}$$

$$i_1 = 0.5V_A - 2 = 0.5 \times 0.727 - 2 = -1.636A$$

JULY 2016) In the circuit shown in Figure 94 determine all the node voltages using nodal analysis

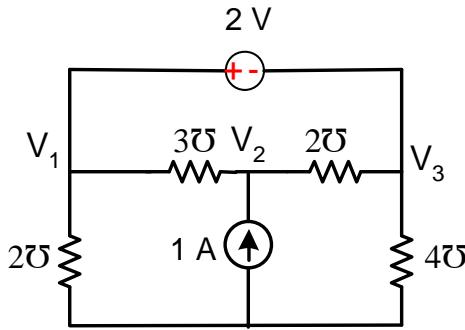


Figure 94

Solution:

From the figure it is observed that there is a voltage source between node V_1 and V_3

$$V_1 - V_3 = 2$$

$$V_1 + 0V_2 - V_3 = 2$$

There is voltage source between V_1 and V_3 . Apply KCL for supernode V_1 and V_3

$$2V_1 + 3V_1 - 3V_2 + 4V_3 + 2V_3 - 2V_2 = 0$$

$$5V_1 - 5V_2 + 6V_3 = 0$$

Apply KCL for node V_2

$$-3V_1 + 5V_2 - 2V_3 - 1 = 0$$

$$-3V_1 + 5V_2 - 2V_3 = 1$$

$$V_1 + 0V_2 - V_3 = 2$$

$$5V_1 - 5V_2 + 6V_3 = 0$$

$$-3V_1 + 5V_2 - 2V_3 = 1$$

Solving above simultaneous equations

$$V_1 = 1.5V \quad V_2 = 0.9V \quad V_3 = -0.5V$$

DEC-2015 1(c)) Find the current i In the circuit shown in Figure 95 using nodal analysis

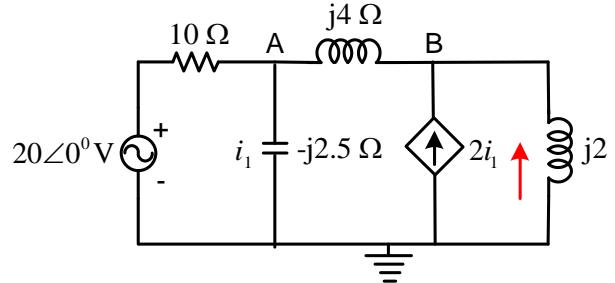


Figure 95

Solution:

$$i_1 = \frac{V_A}{-j2.5} = j0.4V_A$$

Applying KCL to node A

$$\begin{aligned} \frac{V_A - 20}{10} + \frac{V_A}{-j2.5} + \frac{V_A - V_B}{j4} &= 0 \\ \left[\frac{1}{10} + \frac{1}{j4} - \frac{1}{j2.5} \right] V_A - \frac{V_B}{j4} - 2 &= 0 \\ [0.1 - j0.25 + j0.4] V_A + j0.25V_B &= 2 \\ [0.1 + j0.15] V_A + j0.25V_B &= 2 \\ 0.18\angle 56.3V_A + 0.25\angle 90V_B &= 2 \end{aligned}$$

Applying KCL to node B

$$\begin{aligned} \frac{V_B - V_A}{j4} + \frac{V_B}{j2} - 2i_1 &= 0 \\ j0.25V_A + \left[\frac{1}{j4} + \frac{1}{j2} \right] V_B - 2(j0.4V_A) &= 0 \\ -j0.55V_A + [-j0.25 - j0.5] V_B &= 0 \\ -j0.55V_A - j0.75V_B &= 0 \\ 0.55\angle -90V_A + 0.75\angle -90V_B &= 0 \end{aligned}$$

Simultaneous equations are

$$0.18\angle 56.3V_A + 0.25\angle 90V_B = 2$$

$$0.55\angle -90V_A + 0.75\angle -90V_B = 0$$

$$\Delta = \begin{vmatrix} 0.18\angle 56.3 & 0.25\angle 90 \\ 0.55\angle -90 & 0.75\angle -90 \end{vmatrix}$$

$$0.135\angle -33.7 - 0.1375\angle 0 = 0.112 - j0.75 - 0.1375$$

$$-0.0255 - j0.75 = 0.75\angle -92$$



$$V_A = \frac{\begin{vmatrix} 2 & 0.25\angle 90 \\ 0 & 0.75\angle -90 \end{vmatrix}}{\Delta} = \frac{1.5\angle -90}{0.75\angle -92} = 2\angle 2V$$

$$V_B = \frac{\begin{vmatrix} 0.18\angle 56.3 & 2 \\ 0.55\angle -90 & 0 \end{vmatrix}}{\Delta} = \frac{-1.1\angle -90}{0.75\angle -92} = -1.46\angle 2V$$

$$\begin{aligned} \frac{V_2 - V_1}{2} + \frac{V_2}{2} + \frac{V_2 - V_3}{1} + \frac{V_4 - V_1}{2} + \frac{V_4 - V_3}{2} &= 0 \\ -V_1 + 2V_2 - 1.5V_3 + V_4 &= 0 \\ +12 + 2V_2 - 1.5 \times 24 + V_2 + 24 &= \\ 2V_2 &= 0 \\ V_2 &= 0 \end{aligned}$$

$$i_1 = \frac{V_A}{-j2.5} = j0.4V_A = 0.4\angle 90 \times 2\angle 2 = 0.8\angle 92$$

2014-JUNE 1(c)) For the network shown in the Figure ?? find the current in I_0 using nodal analysis.

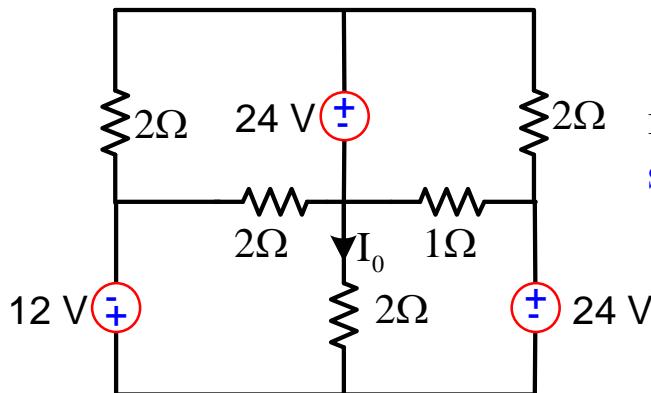


Figure 96

Solution:

From the figure it is observed that $V_1 = -12V$, $V_3 = 24V$. Also it is observed that there is a voltage source between V_2 and V_4 this forms the supernode between node 2 and node 4.

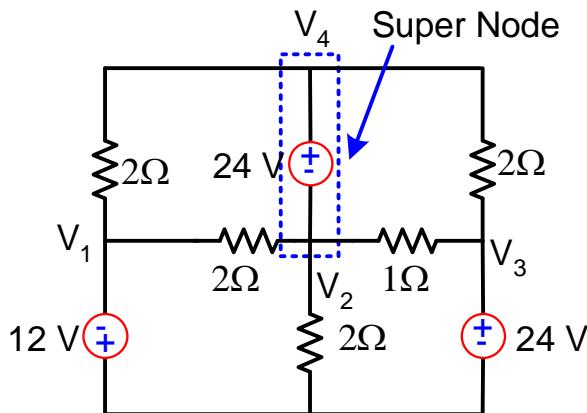


Figure 97

$$V_4 - V_2 = 24$$

$$V_4 = V_2 + 24$$

By KCL for the supernode

The current I_0 is also zero

JULY-2014 1(a)) The node equations of a network are

$$\left[\frac{1}{5} + \frac{1}{j2} + \frac{1}{4} \right] V_1 - \frac{1}{4} V_2 = \frac{50\angle 0}{5} \quad (1)$$

$$-\frac{1}{4} V_1 + \left[\frac{1}{4} - \frac{1}{j2} + \frac{1}{2} \right] V_2 = \frac{50\angle 90}{2} \quad (2)$$

Derive the network

Solution: From equation (1) it is observed that

- From node 1 to common node there is 5Ω resistor in series with $50\angle 0$ voltage source.
- 4Ω resistor is connected between node 1 and node 2
- The inductor of 2Ω is connected from node 1 to common node

Similarly From equation (2) it is observed that

- From node 2 to common node there is 2Ω resistor in series with $50\angle 90$ voltage source.
- 4Ω resistor is connected between node 1 and node 2
- The capacitor of 2Ω is connected from node 1 to common node

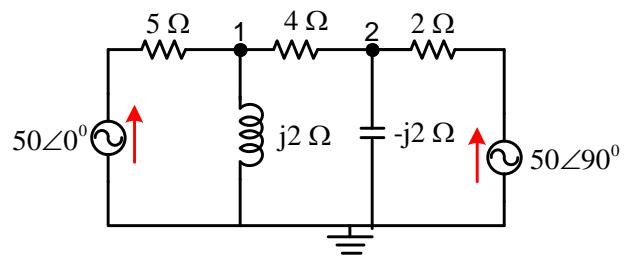


Figure 98

JULY-2013 1-b For the network shown in Figure 89 determine the node voltages by nodal analysis.

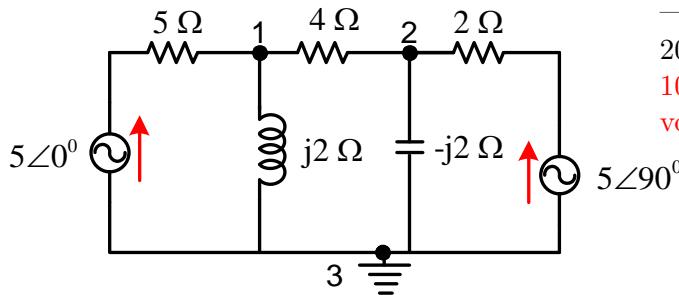


Figure 99: 2018-CBCS-Question Paper

Solution:

Applying KCL to node 1

$$\begin{aligned}\frac{V_1 - 5\angle 0}{5} + \frac{V_1}{j2} + \frac{V_1 - V_2}{4} &= 0 \\ \left[\frac{1}{5} + \frac{1}{j2} + \frac{1}{4} \right] V_1 - \frac{5\angle 0}{5} - \frac{V_2}{4} &= 0 \\ [0.45 - j0.5] V_1 - 0.25V_2 &= 1 \\ 0.672\angle - 48V_1 - 0.25V_2 &= 1\end{aligned}$$

Applying KCL to node 2

$$\begin{aligned}\frac{V_2 - 5\angle 90}{2} + \frac{V_2}{-j2} + \frac{V_2 - V_1}{4} &= 0 \\ -\frac{V_1}{4} + \left[\frac{1}{4} + \frac{1}{-j2} + \frac{1}{2} \right] V_2 - \frac{5\angle 90}{2} &= 0 \\ -0.25V_1 + [0.75 + j0.5] V_2 &= 2.5\angle 90 \\ -0.25V_1 + 0.9\angle 33.69V_2 &= 2.5\angle 90\end{aligned}$$

Simultaneous equations are

$$\begin{aligned}0.672\angle - 48V_1 - 0.25V_2 &= 1 \\ -0.25V_1 + 0.9\angle 33.69V_2 &= 2.5\angle 90\end{aligned}$$

$$\Delta = \begin{vmatrix} 0.672\angle - 48 & -0.25 \\ -0.25 & 0.9\angle 33.69 \end{vmatrix}$$

$$0.6\angle - 14.31 - 0.0625\angle 0 = 0.581 - j0.148 - 0.0625$$

$$0.518 - j0.148 = 0.538\angle - 16$$

$$V_1 = \frac{\begin{vmatrix} 1 & -0.25 \\ 2.5\angle 90 & 0.9\angle 33.69 \end{vmatrix}}{\Delta} = \frac{0.9\angle 33.69 + 0.625\angle 90}{0.538\angle - 16}$$

$$V_1 = \frac{0.75 + j0.5 + j0.625}{0.538\angle - 16} = \frac{1.35\angle 56.3}{0.538\angle - 16} = 2.5\angle 72.3$$

$$V_2 = \frac{\begin{vmatrix} 0.672\angle - 48 & 1 \\ -0.25 & 2.5\angle 90 \end{vmatrix}}{\Delta} = \frac{1.68\angle 42 + 0.25}{0.538\angle - 16}$$

$$V_2 = \frac{1.24 + j1.12 + 0.25}{0.538\angle - 16} = \frac{1.86\angle 36.9}{0.538\angle - 16} = 3.45\angle 52.9$$

2012-JUNE 1(c)) Find the power dissipated in the 10Ω resistor as shown in the Figure 100 by node voltage method.

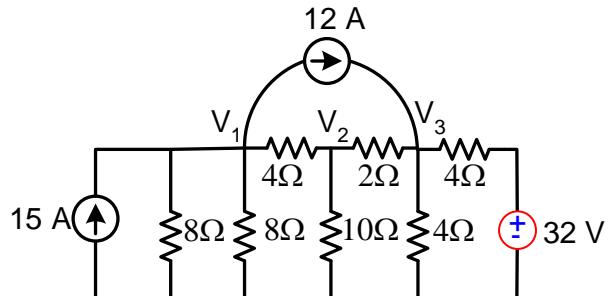


Figure 100

Solution:Applying the KCL for Node V_1

$$\begin{aligned}\frac{V_1}{8} + \frac{V_1}{8} + \frac{V_1 - V_2}{4} - 15 + 12 &= 0 \\ 0.5V_1 - 0.25V_2 - 0V_3 &= 3\end{aligned}$$

Applying the KCL for Node V_2

$$\begin{aligned}\frac{V_2}{10} + \frac{V_2 - V_1}{4} + \frac{V_2 - V_3}{2} &= 0 \\ -0.25V_1 + 0.85V_2 - 0.5V_3 &= 0\end{aligned}$$

Applying the KCL for Node V_3

$$\begin{aligned}\frac{V_3 - V_2}{2} + \frac{V_3}{4} + \frac{V_3 - 32}{4} - 12 &= 0 \\ 0V_1 - 0.5V_2 + 1V_3 &= 20\end{aligned}$$

Simultaneous equations are

$$\begin{aligned}0.5V_1 - 0.25V_2 - 0V_3 &= 3 \\ -0.25V_1 + 0.85V_2 - 0.5V_3 &= 0 \\ 0V_1 - 0.5V_2 + 1V_3 &= 20\end{aligned}$$

$$V_1 = 18.1 \quad V_2 = 24.21 \quad V_3 = 32.1$$

Power dissipated in the 10Ω resistor is

$$P_{10} = \frac{V_3^2}{10} = \frac{32.1^2}{10} = 58.6W$$

2011 DEC 1(b)) Find the currents in all the resistors in the circuit shown in Figure 101 by node voltage method.

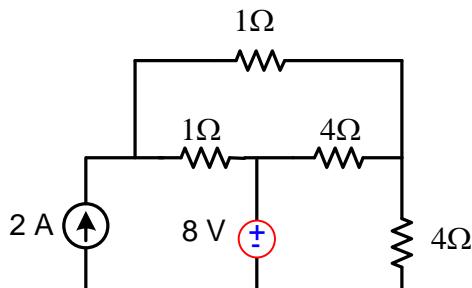


Figure 101

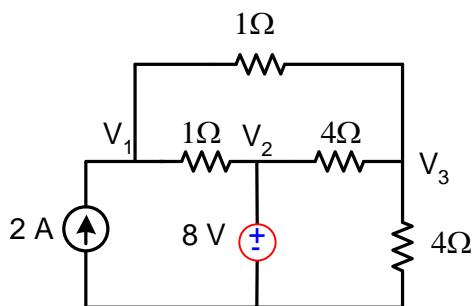
Solution:

Figure 102

From the figure it is observed that $V_2 = 8$
Applying the KCL for Node V_1

$$\begin{aligned}\frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{1} - 5 &= 0 \\ 2V_1 - V_2 - V_3 &= 2 \\ 2V_1 - 8 - V_3 &= 2 \\ 2V_1 - V_3 &= 10\end{aligned}$$

Applying the KCL for Node V_3

$$\begin{aligned}\frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{4} + \frac{V_3}{4} &= 0 \\ -V_1 - 0.25V_2 + 1.5V_3 &= 0 \\ -V_1 - 0.25 \times 8 + 1.5V_3 &= 0 \\ -V_1 + 1.5V_3 &= 2\end{aligned}$$

Simultaneous equations are

$$\begin{aligned}2V_1 - V_3 &= 10 \\ -V_1 + 1.5V_3 &= 2\end{aligned}$$

$$\Delta = \begin{vmatrix} 2 & -1 \\ -1 & 1.5 \end{vmatrix} = (3 - 1) = 2$$

$$\Delta = 2$$

$$V_1 = \frac{\begin{vmatrix} 10 & -1 \\ 2 & 1.5 \end{vmatrix}}{\Delta}$$

$$= \frac{(15 + 2)}{2} = 8.5V$$

$$V_3 = \frac{\begin{vmatrix} 2 & 10 \\ -1 & 2 \end{vmatrix}}{\Delta}$$

$$= \frac{(4 + 10)}{2} = 7V$$

$$i_{12} = \frac{V_1 - V_2}{1} = \frac{8.5V - 8}{1} = 0.5A$$

$$i_{13} = \frac{V_1 - V_3}{1} = \frac{8.5V - 7}{1} = 1.5A$$

$$i_{23} = \frac{V_2 - V_3}{1} = \frac{8V - 7}{4} = 0.25A$$

$$i_3 = \frac{V_3}{4} = \frac{7}{4} = 1.75A$$

JULY-2001 For the network shown in Figure 103 determine the current I by nodal analysis.

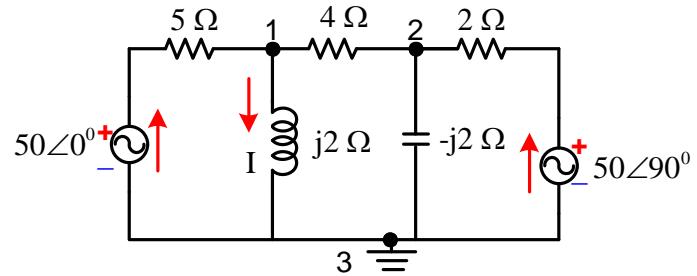


Figure 103: 2018-CBCS-Question Paper

Solution:

Applying KCL to node 1

$$\begin{aligned}\left[\frac{1}{5} + \frac{1}{j2} + \frac{1}{4} \right] V_1 - \frac{50\angle 0}{5} - \frac{V_2}{4} &= 0 \\ [0.45 - j0.5] V_1 - 0.25V_2 &= 10 \\ 0.672\angle -48V_1 - 0.25V_2 &= 10\end{aligned}$$

Applying KCL to node 2

$$\begin{aligned}\frac{V_2 - 50\angle 90}{2} + \frac{V_2}{-j2} + \frac{V_2 - V_1}{4} &= 0 \\ -\frac{V_1}{4} + \left[\frac{1}{4} + \frac{1}{-j2} + \frac{1}{2} \right] V_2 - \frac{5\angle 90}{2} &= 0 \\ -0.25V_1 + [0.75 + j0.5]V_2 &= 25\angle 90 \\ -0.25V_1 + 0.9\angle 33.69V_2 &= 25\angle 90\end{aligned}$$

Simultaneous equations are

$$\begin{aligned}0.672\angle -48V_1 - 0.25V_2 &= 1 \\ -0.25V_1 + 0.9\angle 33.69V_2 &= 2.5\angle 90\end{aligned}$$

$$\Delta = \begin{vmatrix} 0.672\angle -48 & -0.25 \\ -0.25 & 0.9\angle 33.69 \end{vmatrix}$$



$$0.6\angle - 14.31 - 0.0625\angle 0 = 0.581 - j0.148 - 0.0625$$

$$0.518 - j0.148 = 0.538\angle - 16$$

$$V_1 = \frac{\begin{vmatrix} 1 & -0.25 \\ 2.5\angle 90 & 0.9\angle 33.69 \end{vmatrix}}{\Delta} = \frac{0.9\angle 33.69 + 0.625\angle 90}{0.538\angle - 16}$$

$$V_1 = \frac{0.75 + j0.5 + j0.625}{0.538\angle - 16} = \frac{1.35\angle 56.3}{0.538\angle - 16} = 2.5\angle 72.3$$

$$V_2 = \frac{\begin{vmatrix} 0.672\angle - 48 & 1 \\ -0.25 & 2.5\angle 90 \end{vmatrix}}{\Delta} = \frac{1.68\angle 42 + 0.25}{0.538\angle - 16}$$

$$V_2 = \frac{1.24 + j1.12 + 0.25}{0.538\angle - 16} = \frac{1.86\angle 36.9}{0.538\angle - 16} = 3.45\angle 52.9$$

For the network shown in Figure ?? determine the current Node Voltages by nodal analysis.

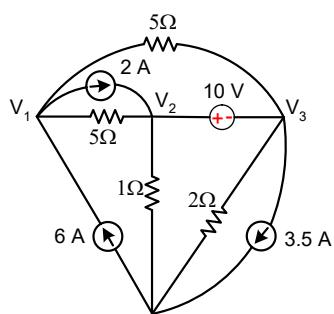


Figure 104: supernode10-1

$$V_2 - V_3 = 10$$

Apply supermesh for node 2 and 3

$$\begin{aligned} \frac{V_2 - V_1}{5} + \frac{V_2}{1} + \frac{V_3 - V_1}{5} + \frac{V_3}{2} - 2 + 3.5 &= 0 \\ [-0.2 - 0.2] V_1 + V_2[1 + 0.2] + V_3[0.2 + 0.5] &= -1.5 \\ -0.4V_1 + 1.2V_2 + 0.7V_3 &= -1.5 \end{aligned}$$

Simultaneous equations are

$$\begin{aligned} 0.4V_1 - 0.2V_2 - 0.2V_3 &= 4 \\ 0V_1 + V_2 - V_3 &= 10 \\ -0.4V_1 + 1.2V_2 + 0.7V_3 &= -1.5 \end{aligned}$$

$$V_1 = 10V \quad V_2 = 5V \quad V_3 = -5V$$

Solution:

Applying KCL to node 1

$$\begin{aligned} \frac{V_1 - V_3}{5} + \frac{V_1 - V_2}{5} + 2 - 6 &= 0 \\ [0.2 + 0.2] V_1 - 0.2V_2 - 0.2V_3 &= 4 \\ 0.4V_1 - 0.2V_2 - 0.2V_3 &= 4 \end{aligned}$$

Important: All the diagrams are redrawn and solutions are prepared. While preparing this study material most of the concepts are taken from some text books or it may be Internet. This material is just for class room teaching to make better understanding of the concepts on Network analysis: Not for any commercial purpose

