0.1 Millman's Theorem

Statement:

In a network if it contains a several voltage sources $E_1,\ E_2,\ E_3...$ ***with an internal impedances*** $Z_1,\ Z_2,\ Z_3...$ ***, which are connected in parallel may be replaced by a single voltage source E with internal impedance Z, where E and Z are***

$$E = \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3}$$

and

$$Z = \frac{1}{Y} = \frac{1}{Y_1 + Y_2 + Y_3}$$

Consider a network containing source $E_1,\ E_2,\ E_3...$ with an internal impedances $Z_1,\ Z_2,\ Z_3...$, which are connected in parallel is as shown in Figure 1.

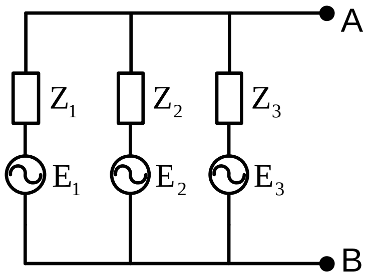


Figure 1

Replace the each voltage source by current source with its internal resistance connected in parallel, which is as shown in Figure 2

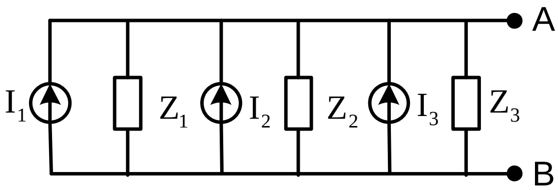


Figure 2

All the current source are added to form a single current source I where I

$$I = I_1 + I_2 + I_3$$

Replace the impedances by a single impedance Z where Z is

$$Y = \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

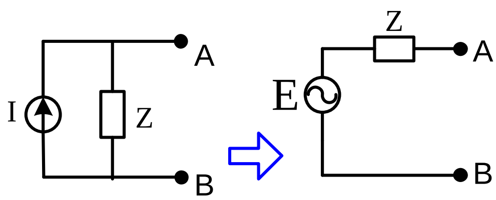


Figure 3

Next the current source by voltage source E where E is

$$E = IZ = \frac{I_1 + I_2 + I_3}{\frac{1}{Z}} = \frac{\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}$$

$$E = \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3}$$

and

$$Y = \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$
$$Y = Y_1 + Y_2 + Y_3$$

2019-JULLY State and explain Millman's theorem.

2019-JAN For the circuit shown in Figure 4 find the current through $(6 + j8)\Omega$, impedance using Millman's theorem.

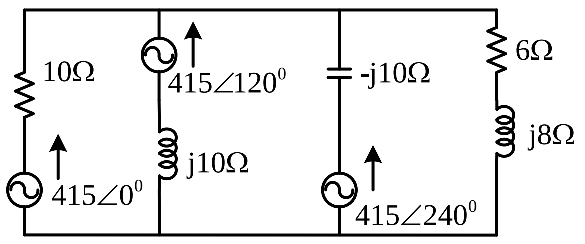


Figure 4

Solution:

$$
\begin{aligned}
Y_1 &= \frac{1}{Z_1} = \frac{1}{10} = 0.1 \\
Y_2 &= \frac{1}{Z_2} = \frac{1}{j10} = -j0.1 \\
Y_3 &= \frac{1}{Z_3} = \frac{1}{-j10} = j0.1
\end{aligned}
$$

$$
\begin{aligned}
Y &= Y_1 + Y_2 + Y_3 = 0.1 - j0.1 + j0.1 \\
&= 0.1 \\
Z &= \frac{1}{Y} = \frac{1}{0.1} = 10\Omega
\end{aligned}
$$

$$
\begin{aligned}
E_1Y_1 &= 415 \times 0.1 = 41.5 \\
E_2Y_2 &= 415\angle 120 \times -j0.1 = 415\angle 120 \times 0.1\angle -90 \\
&= 41.5\angle 30 = 36 + j20.75 \\
E_3Y_3 &= 415\angle 240 \times j0.1 = 41.5\angle 330 \\
&= 36 - j20.75
\end{aligned}
$$

$$
\begin{aligned}
E &= \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3} \\
&= \frac{41.5 + 36 + j20.75 + 36 - j20.75}{0.1} \\
&= \frac{113.5}{0.1} \\
&= 1135
\end{aligned}
$$

Millman's equivalent circuit is is as shown in Figure 5.

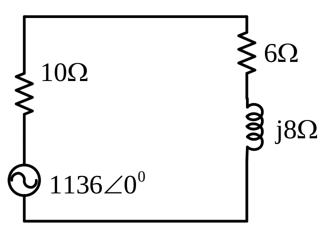


Figure 5

The current through $(6 + j8)\Omega$, impedance is

$$
\begin{aligned}
I &= \frac{E}{Z + Z_L} = \frac{1135}{10 + 6 + j8} \\
&= \frac{1135}{16 + j8} = \frac{1135}{17.88\angle 26.56} \\
&= 63.47\angle -26.56
\end{aligned}
$$

2018-JULY Using Millman's theorem find current through R_L for the circuit shown in Figure 6.

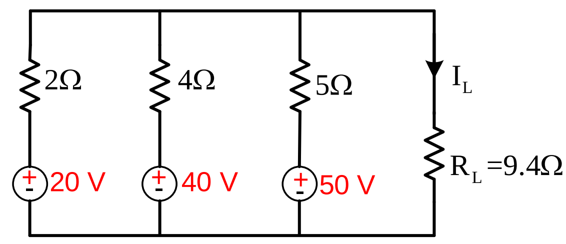


Figure 6

Solution:

$$
\begin{aligned}
Y_1 &= \frac{1}{Z_1} = \frac{1}{2} = 0.5 \\
Y_2 &= \frac{1}{Z_2} = \frac{1}{4} = 0.25 \\
Y_3 &= \frac{1}{Z_3} = \frac{1}{5} = 0.2
\end{aligned}
$$

$$
\begin{aligned}
Y &= Y_1 + Y_2 + Y_3 = 0.5 + 0.25 + 0.2 \\
&= 0.95
\end{aligned}
$$

$$
Z = \frac{1}{Y} = \frac{1}{0.95} = 1.052\Omega
$$

$$
\begin{aligned}
E &= \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3} \\
&= \frac{20 \times 0.5 + 40 \times 0.25 + 50 \times 0.2}{0.95} \\
&= \frac{30}{0.95} \\
&= 31.57
\end{aligned}
$$

Millman's equivalent circuit is is as shown in Figure 7.

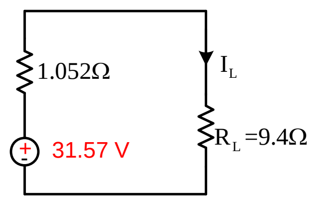


Figure 7

The current I_L through R_L is

$$\begin{aligned} I_L &= \frac{31.57}{Z+Z_L} = \frac{31.57}{1.052+9.4} \\ &= 3.02A \end{aligned}$$

2017-JAN Apply Millman's theorem to find V_O and I_O for the circuit shown in Figure 8.

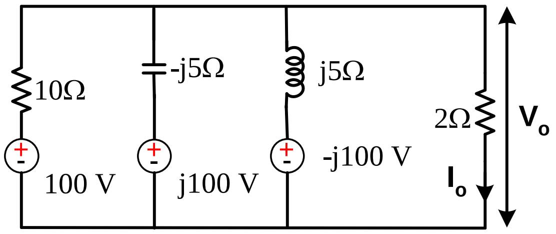


Figure 8

Solution:

$$\begin{aligned} Y_1 &= \frac{1}{Z_1} = \frac{1}{10} = 0.1 \\ Y_2 &= \frac{1}{Z_2} = \frac{1}{-j5} = j0.2 \\ Y_3 &= \frac{1}{Z_3} = \frac{1}{j5} = -j0.2 \end{aligned}$$

$$\begin{aligned} Y &= Y_1+Y_2+Y_3 = 0.1+j0.2-j0.2 \\ &= 0.1 \end{aligned}$$

$$Z = \frac{1}{Y} = \frac{1}{0.1} = 10\Omega$$

$$\begin{aligned} E &= \frac{E_1Y_1+E_2Y_2+E_3Y_3}{Y_1+Y_2+Y_3} \\ &= \frac{100\times 0.1 + j100\times j0.2 + j100\times -j0.2}{0.1} \\ &= \frac{-30}{0.1} \\ &= -300 \end{aligned}$$

Millman's equivalent circuit is is as shown in Figure 7.

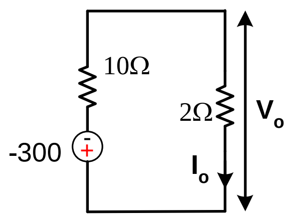


Figure 9

The current I_L through R_L is

$$\begin{aligned} I_L &= \frac{-300}{Z+Z_L} = \frac{-300}{10+2} \\ &= 25A \\ V_O &= 25\times 2 = 50V \end{aligned}$$

2014-JULLY Using Millman's theorem find the current I_L through R_L for the network shown in Figure 10.

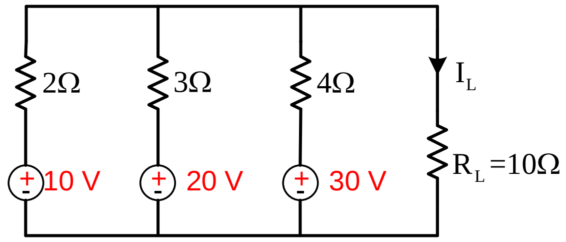


Figure 10

Solution:

$$\begin{aligned} Y_1 &= \frac{1}{Z_1} = \frac{1}{2} = 0.5 \\ Y_2 &= \frac{1}{Z_2} = \frac{1}{3} = 0.33 \\ Y_3 &= \frac{1}{Z_3} = \frac{1}{4} = 0.25 \end{aligned}$$

$$\begin{aligned} Y &= Y_1+Y_2+Y_3 = 0.5+0.33+0.25 \\ &= 1.0833 \end{aligned}$$

$$Z = \frac{1}{Y} = \frac{1}{1.0833} = 0.923\Omega$$

$$\begin{aligned} E &= \frac{E_1Y_1+E_2Y_2+E_3Y_3}{Y_1+Y_2+Y_3} \\ &= \frac{10\times 0.5+20\times 0.33+30\times 0.25}{1.0833} \\ &= \frac{19.16}{1.0833} \\ &= 17.689 \end{aligned}$$

Millman's equivalent circuit is is as shown in Figure 11.

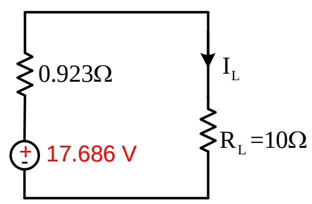


Figure 11

The current I_L through R_L is

$$\begin{aligned} I_L &= \frac{E}{Z+Z_L} = \frac{17.689}{17.689+10} \\ &= 1.6191A \end{aligned}$$

2013-JAN Using Millman's theorem find the current I_L through R_L for the network shown in Figure 12.

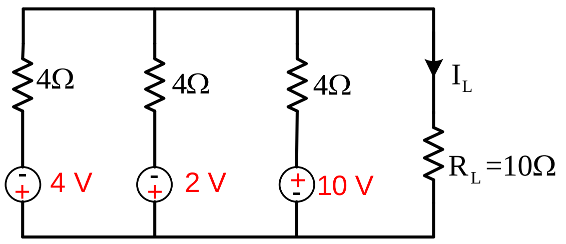


Figure 12

Solution:

$$Y_1 = \frac{1}{Z_1}$$

$$\begin{aligned} Y_1 &= \frac{1}{Z_1} = \frac{1}{4} = 0.25 \\ Y_2 &= \frac{1}{Z_2} = \frac{1}{4} = 0.25 \\ Y_3 &= \frac{1}{Z_3} = \frac{1}{4} = 0.25 \end{aligned}$$

$$\begin{aligned} Y &= Y_1 + Y_2 + Y_3 = 0.25 + 0.25 + 0.25 \\ &= 0.75 \end{aligned}$$

$$Z = \frac{1}{Y} = \frac{1}{0.75} = 1.333\Omega$$

$$\begin{aligned} E &= \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3} \\ &= \frac{-4 \times 0.25 - 2 \times 0.25 + 10 \times 0.25}{0.75} \\ &= \frac{1}{0.75} \\ &= 1.333 \end{aligned}$$

Millman's equivalent circuit is is as shown in Figure 13.

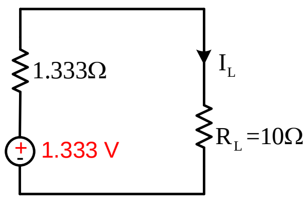


Figure 13

The current I_L through R_L is

$$\begin{aligned} I_L &= \frac{E}{Z + Z_L} = \frac{1.333}{1.333 + 10} \\ &= 0.1176A \end{aligned}$$

2013-JULY Using Millman's theorem find the current I_L through R_L for the network shown in Figure 14.

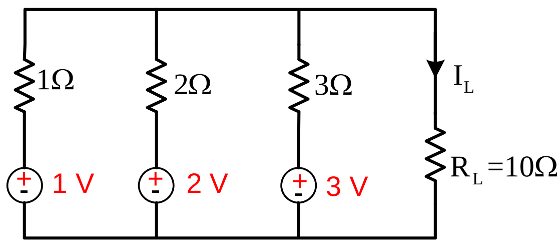


Figure 14

Solution:

$$Y_1 = \frac{1}{Z_1}$$

$$\begin{aligned} Y_1 &= \frac{1}{Z_1} = \frac{1}{1} = 1 \\ Y_2 &= \frac{1}{Z_2} = \frac{1}{2} = 0.5 \\ Y_3 &= \frac{1}{Z_3} = \frac{1}{3} = 0.333 \end{aligned}$$

$$\begin{aligned} Y &= Y_1 + Y_2 + Y_3 = 1 + 0.5 + 0.333 \\ &= 1.833 \end{aligned}$$

$$Z = \frac{1}{Y} = \frac{1}{1.833} = 0.5454\Omega$$

$$\begin{aligned} E &= \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3} \\ &= \frac{1 \times 1 + 2 \times 0.5 + 3 \times 0.333}{0.333} \\ &= \frac{1}{1.833} \\ &= 1.636V \end{aligned}$$

Millman's equivalent circuit is is as shown in Figure 15.

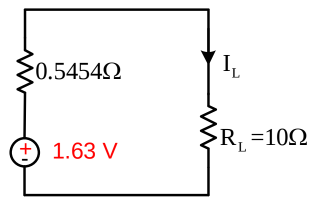


Figure 15

The current I_L through R_L is

$$\begin{aligned} I_L &= \frac{E}{Z + Z_L} = \frac{1.636}{0.5454 + 10} \\ &= 0.1552A \end{aligned}$$

2012-JULY Using Millman's theorem find the current I_L through R_L for the network shown in Figure 16.

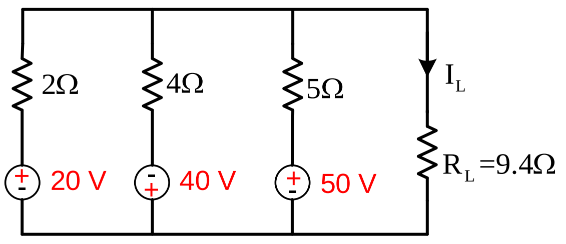


Figure 16

Solution:

$$\begin{aligned} Y_1 &= \frac{1}{Z_1} = \frac{1}{2} = 0.5 \\ Y_2 &= \frac{1}{Z_2} = \frac{1}{4} = 0.25 \\ Y_3 &= \frac{1}{Z_3} = \frac{1}{5} = 0.2 \end{aligned}$$

$$\begin{aligned} Y &= Y_1 + Y_2 + Y_3 = 0.5 + 0.25 + 0.2 \\ &= 0.95 \end{aligned}$$

$$Z = \frac{1}{Y} = \frac{1}{0.95} = 1.052\Omega$$

$$\begin{aligned} E &= \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3} \\ &= \frac{20 \times 0.5 + 40 \times 0.25 + 50 \times 0.2}{0.95} \\ &= \frac{30}{0.95} \\ &= 31.57V \end{aligned}$$

Millman's equivalent circuit is is as shown in Figure 17.

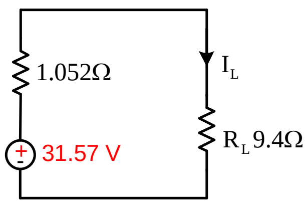


Figure 17

The current I_L through R_L is

$$\begin{aligned} I_L &= \frac{E}{Z + Z_L} = \frac{31.57}{1.052 + 9.4} \\ &= 3.02A \end{aligned}$$

2012-JAN Using Millman's theorem determine voltage V_S of the network shown in Figure 18 given that $E_R = 230\angle 0\ V$, $E_Y = 230\angle -120\ V$ $E_B = 230\angle 120\ V$.

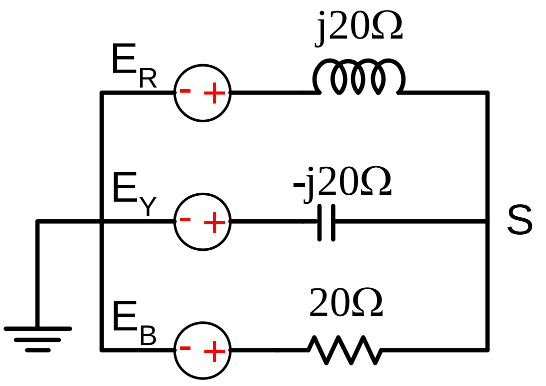


Figure 18

Solution:

$$\begin{aligned} Y_1 &= \frac{1}{Z_1} = \frac{1}{j20} = -j0.05 \\ Y_2 &= \frac{1}{Z_2} = \frac{1}{-j20} = j0.05 \\ Y_3 &= \frac{1}{Z_3} = \frac{1}{20} = 0.05 \end{aligned}$$

$$\begin{aligned} Y &= Y_1 + Y_2 + Y_3 = -j0.05 + j0.05 + 0.05 \\ &= 0.05 \end{aligned}$$

$$Z = \frac{1}{Y} = \frac{1}{0.05} = 20\Omega$$

$$E_1Y_1 = 230 \times -j0.05 = -j11.5$$

$$E_2Y_2 = 230\angle -120 \times j0.05$$

$$E_2Y_2 = 230\angle -120 \times 0.05\angle 90 = 11.5\angle -30$$

$$E_3Y_3 = 230\angle 120 \times 0.05 = -j11.5$$

$$E_3Y_3 = 230\angle 120 \times 0.05 = 11.5\angle 120$$

$$\begin{aligned} E &= \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3} \\ &= \frac{-j11.5 + 11.5\angle -30 + 11.5\angle 120}{0.05} \\ &= \frac{-j11.5 + 9.95 - j5.75 - 5.75 + j9.95}{0.05} \\ &= \frac{4.2 - j7.3}{0.05} = \frac{8.42\angle -60}{0.05} \\ &= 168.4\angle -60V \end{aligned}$$

Millman's equivalent circuit is is as shown in Figure 19.

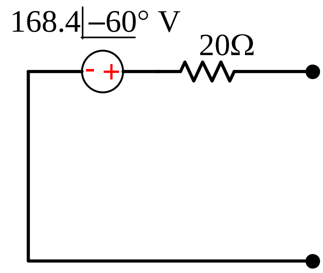


Figure 19

Using Millman's theorem determine current flowing through $(4 + j3)$ Ω of the network shown in Figure 20

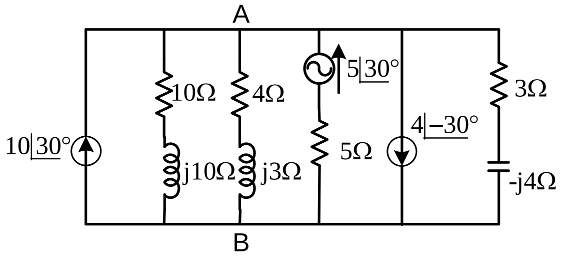


Figure 20

Solution:

Replace the current source and parallel resistance by a voltage source

$$Z_1 = 10 + j10 = 14.14\angle 45°\Omega$$

$$E_1 = 10\angle 30 \times 14.14\angle 45° = 141.4\angle 75°$$

Similarly

$$Z_3 = 3 - j14 = 5\angle - 53.13°\Omega$$

$$E_3 = 4\angle - 30 \times 5\angle - 53.13° = 20\angle - 83.13°$$

The modified network as shown in Figure 21

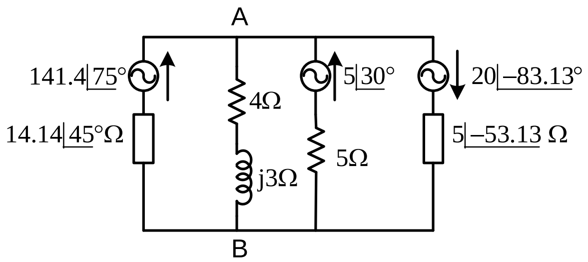


Figure 21

$$\begin{aligned}
Y_1 &= \frac{1}{Z_1} = \frac{1}{14.14\angle 45°} = 0.0707\angle - 45° \\
Y_1 &= 0.05 - j0.05 \\
Y_2 &= \frac{1}{Z_2} = \frac{1}{5} = 0.2 \\
Y_3 &= \frac{1}{Z_3} = \frac{1}{5\angle - 53.13°} = 0.2\angle 53.13° \\
&= 0.12 + j0.16
\end{aligned}$$

$$\begin{aligned}
Y &= Y_1 + Y_2 + Y_3 0.12 + j0.16 \\
&= 0.37 + j0.11 \\
&= 0.386\angle 16.56
\end{aligned}$$

$$Z = \frac{1}{Y} = \frac{1}{0.386\angle 16.56} = 2.6\angle - 16.56\Omega$$

$$\begin{aligned}
E_1Y_1 &= 141.14\angle 75 \times 0.0707\angle - 45° = \\
&= 9.978\angle 30 = 8.641 + j4.989 \\
E_2Y_2 &= 5\angle 30 \times 0.2 = 1\angle 30 = 0.8666 + j0.5 \\
E_3Y_3 &= 20\angle - 83.13 \times 0.2\angle 53.13 \\
&= 4\angle - 30 = 3.464 - j2
\end{aligned}$$

$$\begin{aligned}
E &= \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3} \\
&= \frac{8.641 + j4.989 + 0.8666 + j0.5 + 3.464 - j2}{0.386\angle 16.56} \\
&= \frac{12.971 + j3.489}{0.386\angle 16.56} = \frac{13.432\angle 15}{0.386\angle 16.56} \\
&= 168.4\angle - 60V
\end{aligned}$$

Millman's equivalent circuit is is as shown in Figure 19.

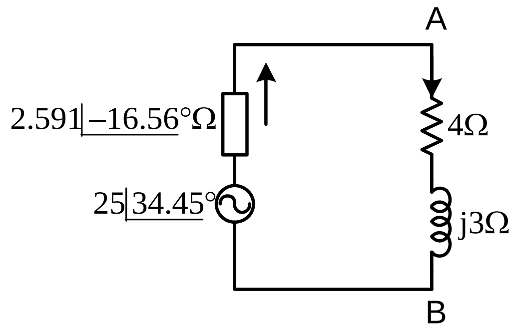


Figure 22

0.2 Thevenin's and Norton's Theorems

0.2.1 Thevenin's Theorem

Statement:

Any linear, bilateral network with two terminals can be replaced by a single voltage source E_{TH} in series with an impedance Z_{TH}, where the E_{TH} is an open circuit voltage at the terminals and an impedance Z_{TH} is the equivalent impedance as viewed from the terminals into the network.

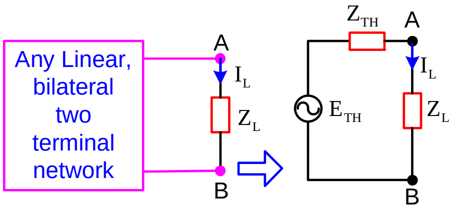


Figure 23

Proof:

Consider a network as shown in Figure 24. The current in the terminal AB is

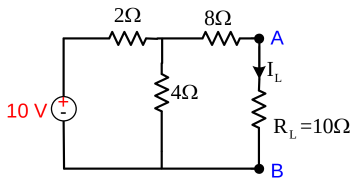


Figure 24

$$
\begin{aligned}
\frac{V-10}{2}+\frac{V}{4}+\frac{V}{18} &= 0\\
V[0.5+0.25+0.0556] &= 5\\
V &= \frac{5}{0.8056}=6.026\\
I_L &= \frac{6.026}{18}=0.3448A
\end{aligned}
$$

- **Step 1: Determine the Open Circuit Voltage**

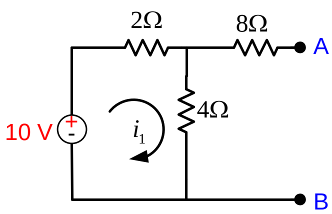


Figure 25

$$
\begin{aligned}
6i_1-10 &= 0\\
i_1 &= 1.666A\\
E_{TH} &= 1.666A\times 4=6.667V
\end{aligned}
$$

- **Step 2: Determine the Tehevenin's Impedance**

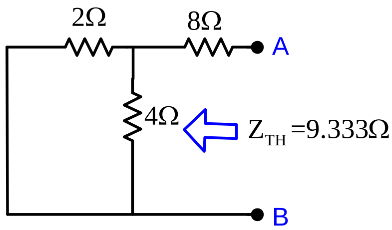


Figure 26

$$Z_{TH}=8+\frac{2\times 4}{2+4}=9.333\Omega$$

- **Step 3: Tehevenin's Equivalent Circuit**

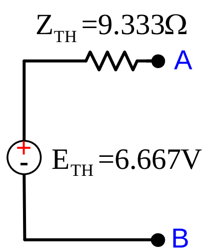


Figure 27

- **Step 4: Current through I_L is**

$$I_L=\frac{E_{TH}}{Z_{TH}+Z_L}=\frac{6.667V}{9.333+10}=0.344\ A$$

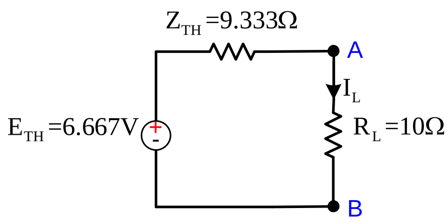


Figure 28

0.2.2 Norton's Theorem

Statement:

Any linear, bilateral network with two terminals can be replaced by a single current source I_N in parallel with an impedance Z_N, where the I_N is an short circuit current through the terminals and an impedance Z_N is the equivalent impedance as viewed from the terminals into the network.

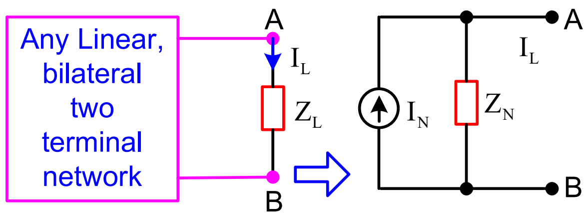


Figure 29

0.2.3 Maximum Power Transfer Theorem

Statement:

In Any linear, bilateral network maximum power is delivered to the load R_L by the source when the load resistance R_L is equal to the Thevenin's resistance R_{TH}.

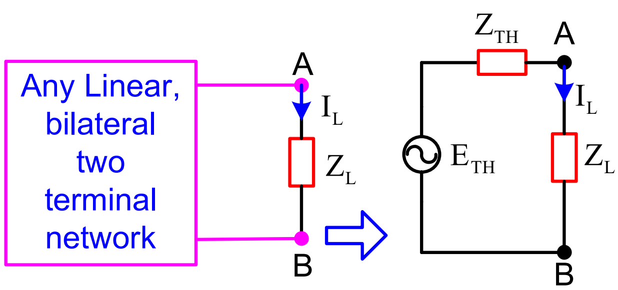


Figure 30

Q 1) Find the Thevenin and Norton equivalent for the circuit shown in Figure 31 with respect terminals A-B

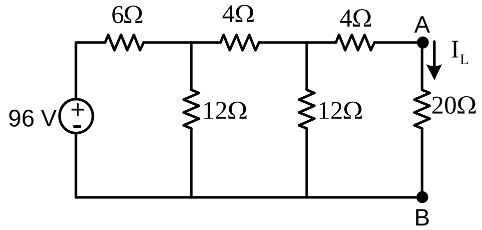


Figure 31

Solution:

Determine the Thevenin voltage V_{TH}. Apply KVL for the circuit shown in Figure 32.

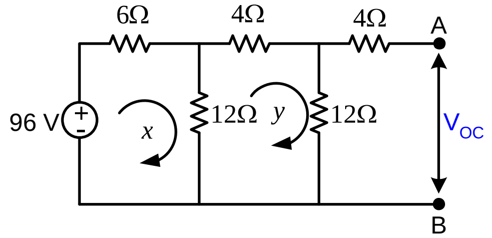


Figure 32

$$\begin{aligned} 18x - 12y &= 96 \\ -12x + 28y &= 0 \end{aligned}$$

By Solving

$$x = 7.467A \quad y = 3.2A$$

$$V_{OC} = 7.467 \times 12A = 38.4V$$

$$\begin{aligned} Z_{TH} &= [(6||12) + 4]||12 + 4 \\ &= [4 + 4]||12 + 4 \\ &= 8||12 + 4 = 4.8 + 4 \\ &= 8.8\Omega \end{aligned}$$

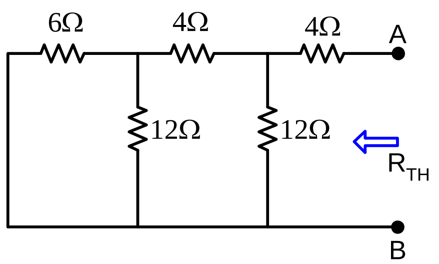


Figure 33

To determine the short circuit current Apply KVL for the circuit shown in Figure 34

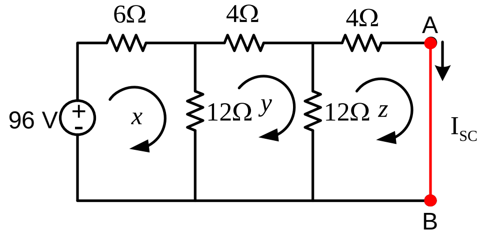


Figure 34

$$\begin{aligned} 18x - 12y + 0z &= 96 \\ -12x + 28y - 12z &= 0 \\ 0x - 12y + 16z &= 0 \end{aligned}$$

By Solving

$$x = 9.212A \quad y = 5.818A \quad z = 4.36A$$

$$I_{SC} = I_N = z = 4.36A$$

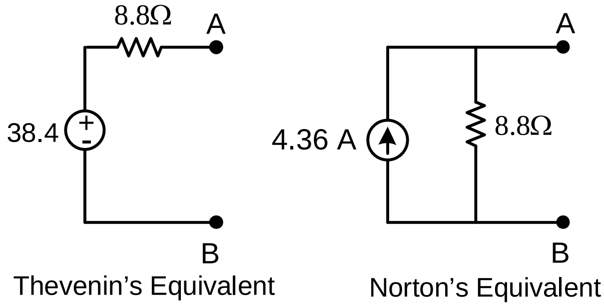


Figure 35

Current through R_L is

$$I_L = \frac{V_{OC}}{R_L} = \frac{38.4}{20 + 8.8} = 1.33A$$

Power through is

$$P_L = I_L^2 R_L = (1.33)^2 \times 20 = 35.56W$$

Maximum power is R_L is

$$I_L = \frac{V_{OC}}{R_N + R_L} = \frac{38.4}{8.8 + 8.8} = 2.18A$$

$$P_L = I_L^2 R_L = (2.18)^2 \times 8.8 = 41.856W$$

Q 2) Find the Thevenin and Norton equivalent for the circuit shown in Figure 36 with respect terminals A-B

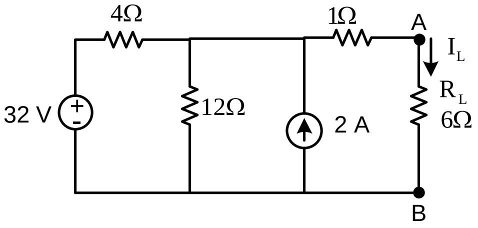


Figure 36

Solution:

Determine the Thevenin voltage V_{TH}. Apply KVL for the for the circuit shown in Figure 37.

$$\begin{aligned} y &= -2A \\ 16x - 12y &= 32 \\ 16x - 24 &= 32 \\ x &= \frac{32-24}{16} = 0.5A \end{aligned}$$

$$V_{OC} = 12[0.5A - (-2)].5A \times 3 = 30V$$

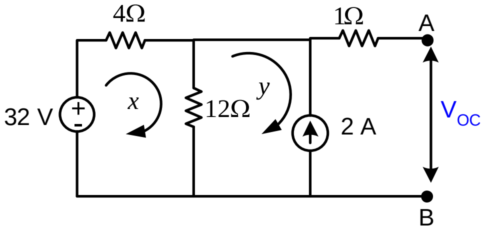


Figure 37

$$\begin{aligned} Z_{TH} &= (4||12) + 1 \\ &= 3 + 1 = 4\Omega \end{aligned}$$

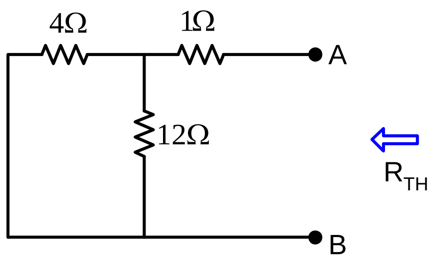
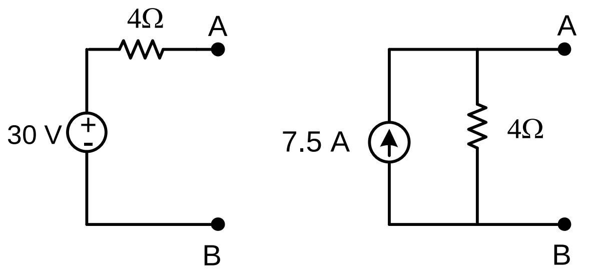


Figure 38



Thevenin's Equivalent   Norton's Equivalent

Figure 39

Q 3) Find the Thevenin and Norton equivalent for the circuit shown in Figure 40 with respect terminals A-B

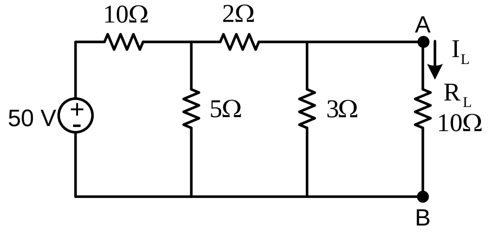


Figure 40

Solution:

Determine the Thevenin voltage V_{TH}. Apply the KVL for the circuit shown in Figure 41.

$$\begin{aligned} 15x - 5y+ &= 50 \\ -5x + 10y &= 0 \end{aligned}$$

By Solving

$$x = 4A \quad y = 2A$$

$$V_{OC} = 2A \times 3 = 6V$$

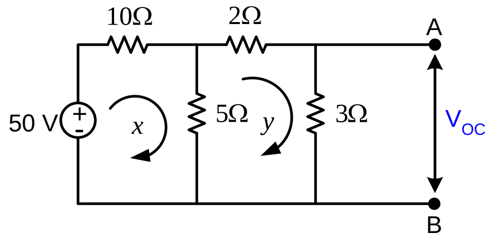


Figure 41

$$\begin{aligned} Z_{TH} &= [(10||5) + 2]||3 \\ &= [3.33 + 2]||3 \\ &= 5.33||3 = 1.92\Omega \end{aligned}$$

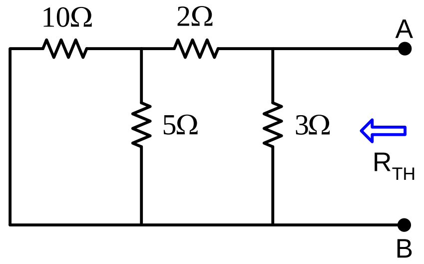


Figure 42

Determine the short circuit current by Applying KVL for the circuit 43

$$\begin{aligned} 15x - 5y &= 50 \\ -5x + 7y &= 0 \end{aligned}$$

By Solving

$$x = 4.375A \quad y = 3.125A$$

$$I_{SC} = I_N = y = 3.125A$$

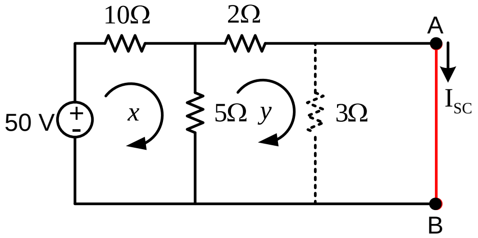


Figure 43

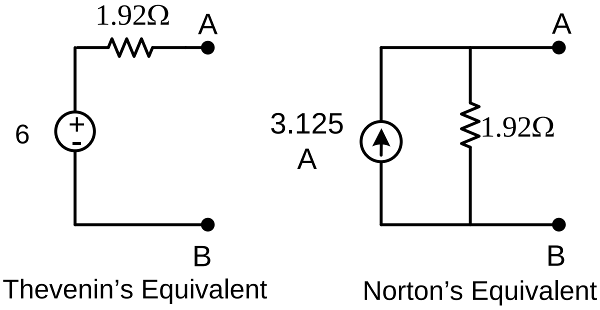


Figure 44

Current through R_L is

$$I_L = \frac{V_{OC}}{R_L} = \frac{6}{1.92 + 10} = 0.503A$$

Power through is

$$P_L = I_L^2 R_L = (0.503)^2 \times 10 = 2.53W$$

Maximum power is R_L is

$$I_L = \frac{V_{OC}}{R_N + R_L} = \frac{6}{1.92 + 1.92} = 1.5625A$$

$$P_L = I_L^2 R_L = (1.5625)^2 \times 1.92 = 4.68W$$

Q 4) Find the Thevenin and Norton equivalent for the circuit shown in Figure **??** with respect terminals a-b

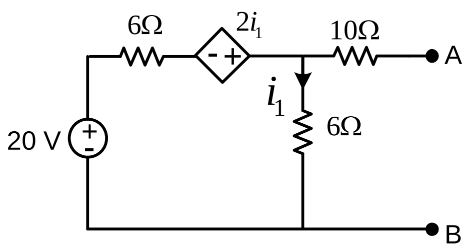


Figure 45

Solution:

Determine the Thevenin voltage V_{TH}. Apply KVL for the circuit shown in Figure 46.

By KVL around the loop

$$\begin{aligned} 6i - 2i + 6i - 20 &= 0 \\ 10i &= 20 \\ i &= 2A \end{aligned}$$

Voltage across AB $V_{OC} = V_{TH}$ is

$$V_{OC} = 6i = 6 \times 2 = 12V$$

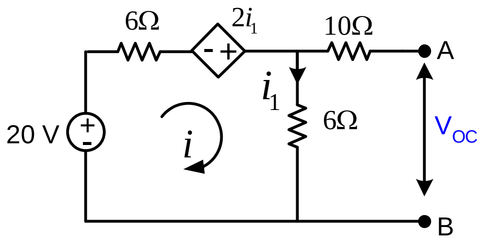


Figure 46

When dependant voltage sources are present then Thevenin Resistance R_{TH} is calculated by determining the short circuit current at terminals AB:

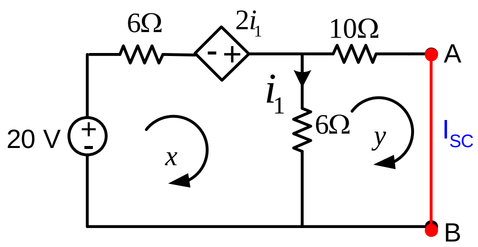


Figure 47

$$x - y = i_1$$

KVL for loop x

$$\begin{aligned} 12x - 2i_1 - 6y - 20 &= 0 \\ 12x - 2(x - y) - 6y &= 20 \\ 10x - 4y &= 20 \end{aligned}$$

KVL for loop y

$$\begin{aligned} -6x + 16y &= 0 \\ 6x - 16y &= 0 \end{aligned}$$

Solving the following simultaneous equations

$$\begin{aligned} 10x - 4y &= 20 \\ 6x - 16y &= 0 \end{aligned}$$

$$x = 2.353 \quad y = 0.882$$

$$I_{SC} = y = 0.882A$$

Thevenin's resistance is

$$R_{TH} = \frac{V_{TH}}{I_{SC}} = \frac{12}{0.882} = 13.6\Omega$$

Thevenin and Norton equivalent circuits as shown in Figure 48

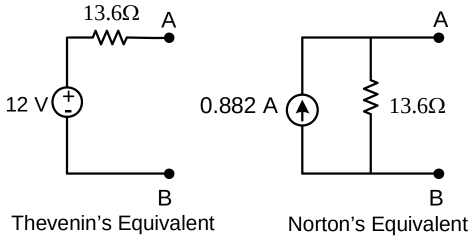


Figure 48

Q 5) Find the Thevenin and Norton equivalent for the circuit shown in Figure 49 with respect terminals a-b

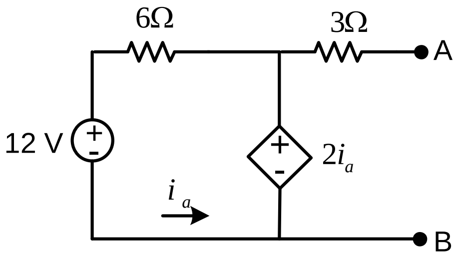


Figure 49

Solution:

Determine the Thevenin voltage V_{TH}. Apply the KVL for the circuit shown in Figure 50. From the figure it is observed that

$$i_a = -x$$

By KVL for the loop x

$$\begin{aligned} 6x + 2i_a - 12 &= 0 \\ 6x - 2x &= 12 \\ x &= \frac{12}{4} = 3A \end{aligned}$$

$$i_a = -x = -3A$$

Open Circuit voltage V_{OC} is

$$V_{OC} = 2i_a = 2 \times (-3) = -6V$$

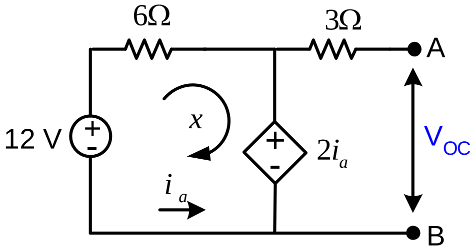


Figure 50

When dependant voltage sources are present then Thevenin Resistance R_{TH} is calculated by determining the short circuit current at terminals AB. Apply KVL for loop x

$$\begin{aligned} 6x + 2i_a - 12 &= 0 \\ 6x - 2x &= 12 \\ x &= \frac{12}{4} = 3A \end{aligned}$$

$$i_a = -x = -3A$$

Apply KVL for loop y

$$\begin{aligned} 3y - 2i_a &= 0 \\ 3y - 2(-3) &= 0 \\ y &= \frac{-6}{3} = -2A \end{aligned}$$

Short circuit current I_{SC}

$$I_{SC} = y = -2A$$

Thevenin resistance is R_{TH}

$$R_{TH} = \frac{-6}{-2} = 3\Omega$$

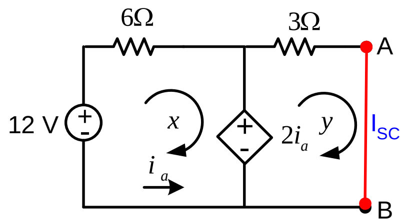


Figure 51

Thevenin's and Norton's Circuits are as shown in Figure 52

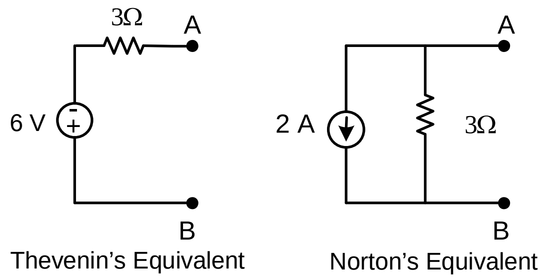


Figure 52

Q 6) Find the Thevenin and Norton equivalent for the circuit shown in Figure 53 with respect terminals a-b

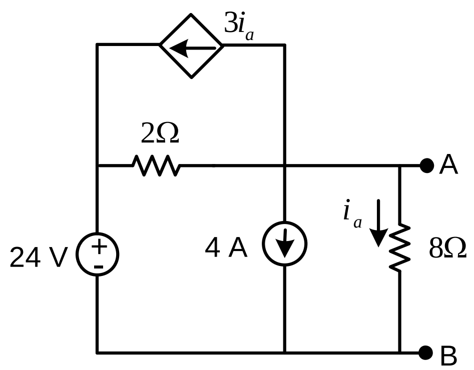


Figure 53

Solution:

Determine the Thevenin voltage V_{TH} for circuit shown in Figure 50. It is observed that

$$V_2 = 24$$
$$i_a = \frac{V_1}{8}$$

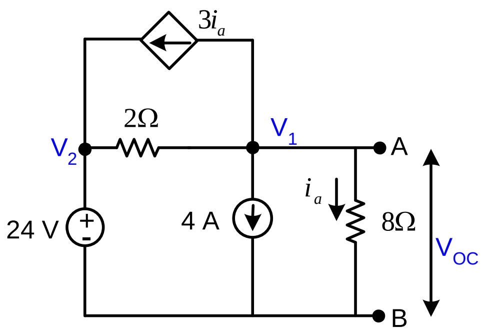


Figure 54

Apply (KCL) node voltage for the circuit shown in Figure 54 for node 1

$$\begin{aligned}
\frac{V_1 - V_2}{2} + 4 + \frac{V_1}{8} + 3i_a &= 0 \\
\frac{V_1 - 24}{2} + 4 + \frac{V_1}{8} + 3\frac{V_1}{8} &= 0 \\
\frac{V_1}{2} + \frac{V_1}{8} + 3\frac{V_1}{8} &= 8 \\
V_1 &= 8
\end{aligned}$$

To find the short current I_{sc} short the output terminals a and b.

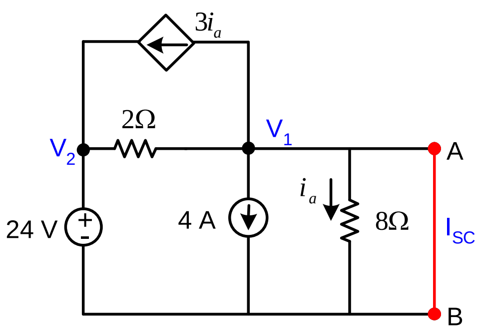


Figure 55

When it is short circuited the current $I_a = 0$ then dependent source becomes zero. The modified circuit is as shown in Figure 56.

$$\begin{aligned}
\frac{V_1 - 24}{2} + 4 + I_{SC} &= 0 \\
\frac{0 - 24}{2} + 4 + I_{SC} &= 0 \\
I_{SC} &= -4 + 12 = 8A
\end{aligned}$$

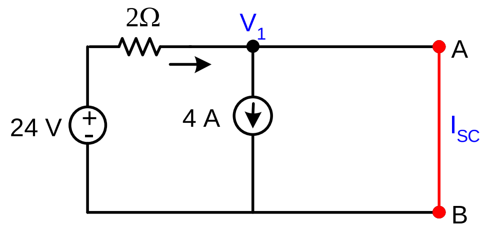


Figure 56

$$R_{TH} = \frac{8}{8} = 1\Omega$$

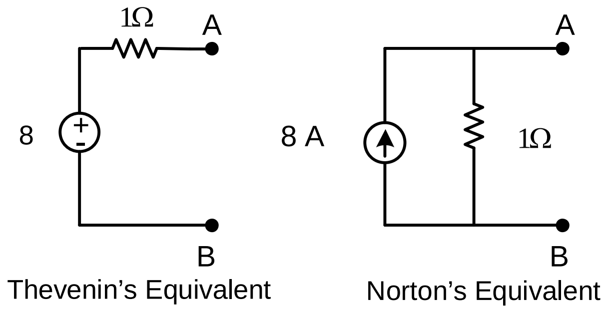


Figure 57

Q 7) Find the Thevenin and Norton equivalent for the circuit shown in Figure 58 with respect terminals a-b

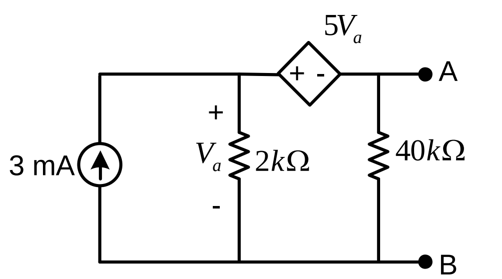


Figure 58

Solution:

Determine the Thevenin voltage V_{TH} by apply node voltage method for the circuit shown in Figure 59

$$\begin{aligned}
\frac{V_1}{2k\Omega} + \frac{V_2}{40k\Omega} - 3 \times 10^{-3} &= 0 \\
0.5 \times 10^{-3} V_1 + 0.025 \times 10^{-3} V_2 &= 3 \times 10^{-3} \\
0.5V_1 + 0.025V_2 &= 3
\end{aligned}$$

It is observed that

$$V_a = V_1$$

$$\begin{aligned}
V_1 - V_2 &= 5V_a \\
V_1 - V_2 - 5V_a &= 0 \\
V_1 - V_2 - 5V_1 &= 0 \\
4V_1 + V_2 &= 0
\end{aligned}$$

Solving following equations

$$
\begin{aligned}
0.5V_1 + 0.025V_2 &= 3 \\
4V_1 + V_2 &= 0
\end{aligned}
$$

$$V_1 = 7.5V,\ V_2 = -30V$$

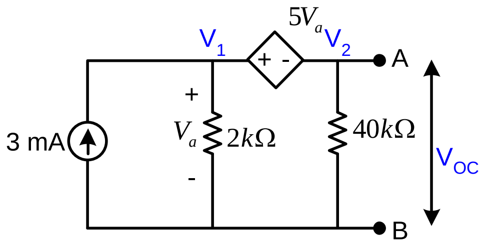


Figure 59

Determine the short circuit current I_{SC} for the circuit shown in Figure 60. It is observed that 40 kΩ is also shorted hence entire current is flowing through shortened terminals AB.

$$
\begin{aligned}
x &= 3mA \\
y &= -x = -3mA
\end{aligned}
$$

$$
\begin{aligned}
V_{OC} &= -30V \\
Z_{TH} &= \frac{V_{OC}}{I_{SC}} = \frac{-30V}{-3mA} = 10k\Omega
\end{aligned}
$$

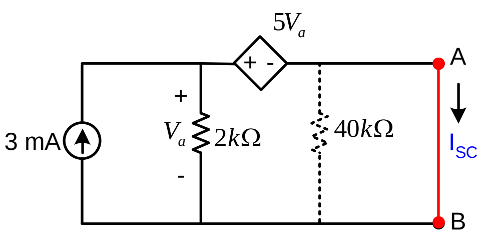


Figure 60

Thevenin and Norton circuits are as shown in Figure 61

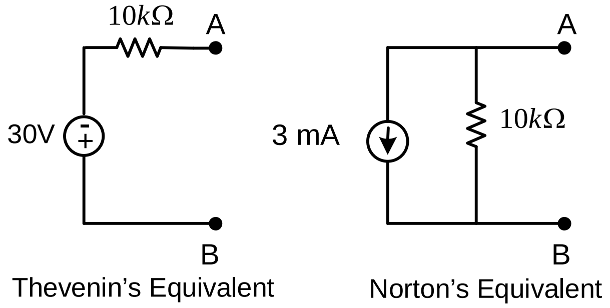


Figure 61

Q 8) Find the Thevenin and Norton equivalent for the circuit shown in Figure refThevenin-dependent8-1 with respect terminals a-b

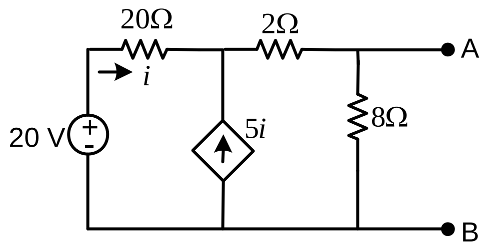


Figure 62

Solution:

Determine the Thevenin voltage V_{TH} by applying KVL for the circuit shown in Figure 63. It is observed that there is a current source between two loops, hence apply supermesh analysis.

$$
\begin{aligned}
y - x &= 5i = 5x \\
6x - y &= 0 \\
20x + 10y &= 20
\end{aligned}
$$

Solving the above equations

$$x = 0.25A \quad y = 1.5A$$

$$V_{OC} = V_{TH} = 1.5A \times 8\Omega = 6V$$

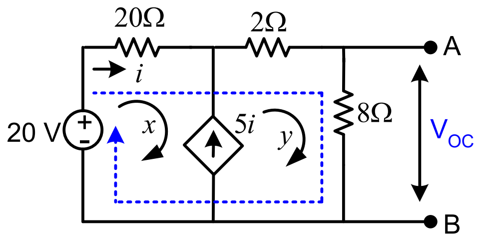


Figure 63

Determine the I_{SC} for the circuit shown in Figure 64.

$$
\begin{aligned}
y - x &= 5i = 5x \\
6x - y &= 0 \\
20x + 2y &= 20
\end{aligned}
$$

Solving the above equations

$$x = 0.625A \quad y = 3.75A$$

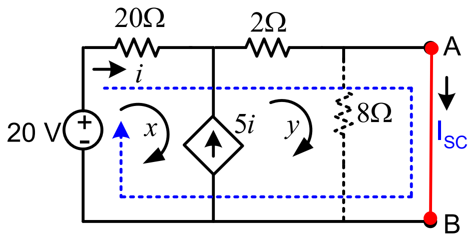


Figure 64

$$Z_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{6}{3.75} = 1.6\Omega$$

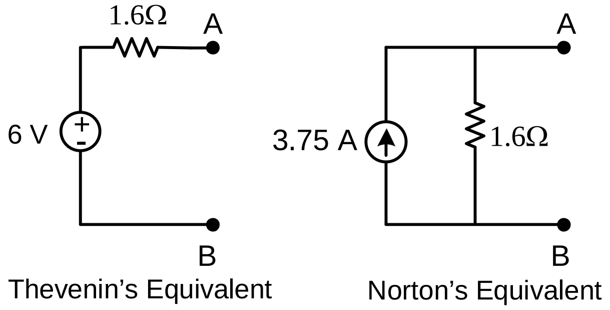


Figure 65

Q 9) Find the Thevenin and Norton equivalent for the circuit shown in Figure 66 with respect terminals a-b

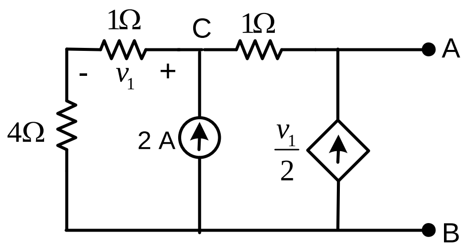


Figure 66

Solution:

Determine the Thevenin voltage V_{TH} for circuit shown in Figure 67. It is observed that

$$v_1 = \frac{V_C}{5} \times 1 = 0.2V_C$$

Apply KCL

$$\begin{aligned}
\frac{V_C}{5} - 2 - \frac{v_1}{2} &= 0 \\
\frac{V_C}{5} - 2 - 0.2 \times \frac{V_C}{2} &= 0 \\
0.2V_C - 0.1V_C &= 2 \\
V_C &= \frac{2}{0.1} = 20
\end{aligned}$$

$$\begin{aligned}
V_{OC} = V_{TH} &= V_C + 1 \times \frac{v_1}{2} \\
&= 20 + 1 \times \frac{0.2 \times 20}{2} \\
&= 22
\end{aligned}$$

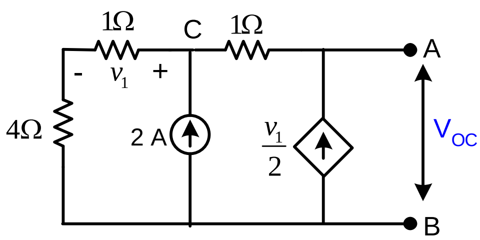


Figure 67

Applying KCL for the circuit shown in Figure 68.

$$V_A = 0V$$

$$\begin{aligned}
\frac{V_C}{5} - 2 + \frac{V_C - V_A}{1} &= 0 \\
0.2V_C + 1V_C &= 2 \\
V_C &= \frac{2}{1.2} = 1.67V
\end{aligned}$$

$$\begin{aligned}
\frac{V_A - V_C}{1} - \frac{v_1}{2} + I_{SC} &= 0 \\
\frac{-V_C}{1} - \frac{0.2V_C}{2} &= -I_{SC} \\
\frac{1.67}{1} + \frac{0.2 \times 1.67}{2} &= I_{SC} \\
I_{SC} = 1.67 + 0.167 &= 1.837A
\end{aligned}$$

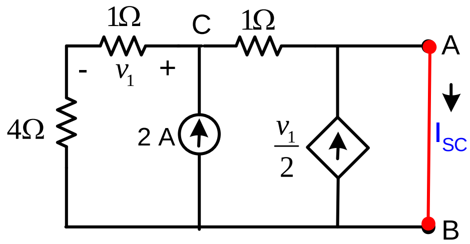


Figure 68

$$Z_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{22}{1.837A} = 12\Omega$$

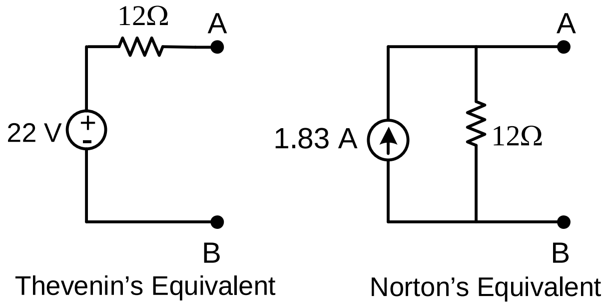


Figure 69

By Test voltage method. Apply a test voltage of 1 V at the output terminals AB and determine the applied source current. The modified circuit is as shown in Figure 70.

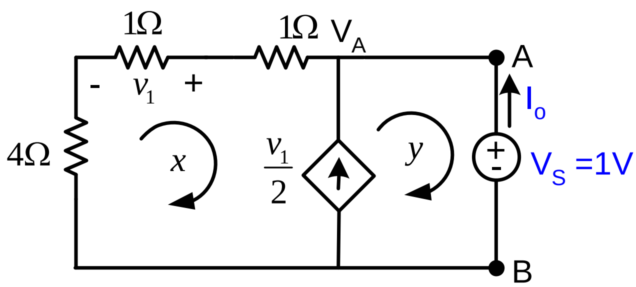


Figure 70

It is observed that

$$\begin{aligned} v_1 &= -1x = -x \\ y - x &= \frac{v_1}{2} = \frac{-x}{2} \\ 0.5x - y &= 0 \end{aligned}$$

By supermesh analysis method

$$\begin{aligned} 6x + 1 &= 0 \\ x &= -\frac{1}{6} = -0167A \\ y &= 0.5x = 0.5 \times (-0167A) = -0.0833A \end{aligned}$$

The circuit impedance is

$$Z_{TH} = \frac{1V}{0.0833A} = 12\Omega$$

Q 9-1) Find the Thevenin and Norton equivalent for the circuit shown in Figure 71 with respect terminals a-b

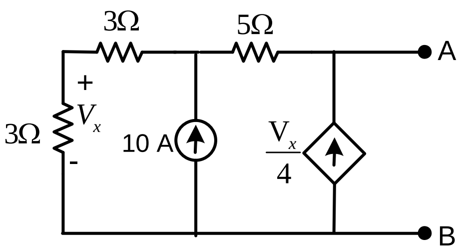


Figure 71

Solution:

Determine the Thevenin voltage V_{TH} for circuit shown in Figure 72. It is observed that

$$v_x = \frac{V_C}{6} \times 3 = 0.5V_C$$

Apply KCL

$$\begin{aligned} \frac{V_C}{6} - 10 - \frac{V_x}{4} &= 0 \\ \frac{V_C}{6} - 10 - \frac{0.5V_C}{4} &= 0 \\ V_C(0.166 - 0.125) &= 10 \\ 0.0416V_C &= 10 \\ V_C &= 240.38 \end{aligned}$$

$$\begin{aligned} V_{OC} = V_{TH} &= V_C + 5 \times \frac{V_x}{4} \\ &= 240.38 + 5 \times \frac{0.5V_C}{4} \\ &= 240.38 + 5 \times \frac{0.5 \times 240.38}{4} \\ &= 240.38 + 150 = 390.38 \end{aligned}$$

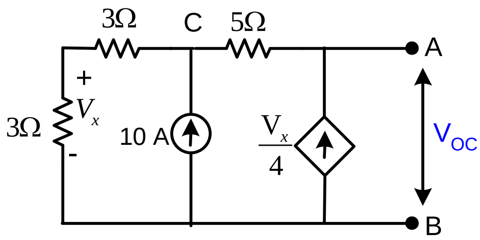


Figure 72

Applying KCL for the circuit shown in Figure 73.

$$V_A = 0V$$

$$\begin{aligned} \frac{V_C}{6} - 10 + \frac{V_C - V_A}{5} &= 0 \\ 0.166V_C + 0.2V_C &= 10 \\ 0.366V_C &= 10 \\ V_C &= \frac{10}{0.366} = 27.27V \end{aligned}$$

$$\begin{aligned} \frac{V_A - V_C}{5} - \frac{V_x}{4} + I_{SC} &= 0 \\ \frac{-V_C}{5} - \frac{0.5V_C}{4} &= -I_{SC} \\ \frac{27.27}{5} + \frac{0.5 \times 27.27}{4} &= I_{SC} \\ I_{SC} = 5.454 + 3.408 &= 8.862A \end{aligned}$$

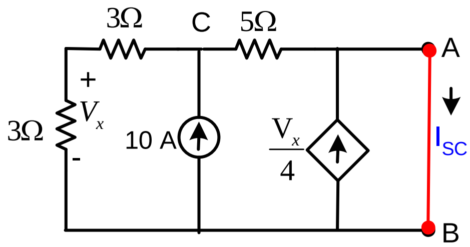


Figure 73

$$Z_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{390.38}{8.862A} = 44\Omega$$

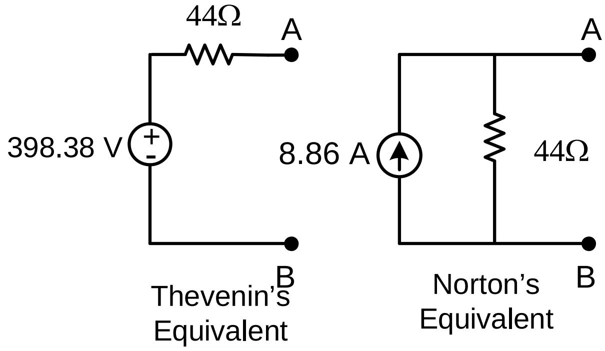


Figure 74

Q 10) Find the Thevenin equivalent for the circuit shown in Figure 75 with respect terminals a-b

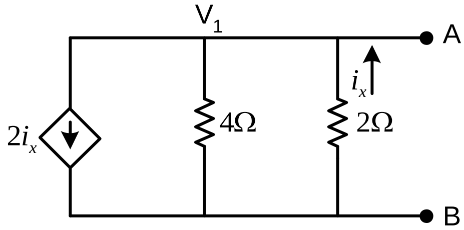


Figure 75

Solution:

For this circuit it does not have any independent sources. Apply a test voltage of 1 V at the output terminals AB and determine the applied source current. The modified circuit is as shown in Figure 76.

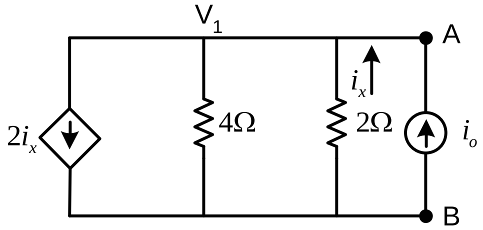


Figure 76

It is observed that

$$\begin{aligned} i_x &= -\frac{V_1}{2} \\ &= -0.5V_1 \end{aligned}$$

Apply KCL for the node V_1

$$\begin{aligned} +\frac{V_1}{4} + \frac{V_1}{2} + 2i_x - i_o &= 0 \\ V_1[0.25 + .5] + 2(-0.5V_1) - i_o &= 0 \\ V_1[0.25 + .5 - 1] - i_o &= 0 \\ -0.25V_1 - i_o &= 0 \\ -i_o &= 0.25V_1 \end{aligned}$$

$$R_{TH} = \frac{V_1}{i_o} = \frac{V_1}{0.25V_1} = 4\Omega$$

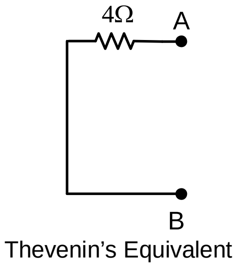


Figure 77

Q 11) Find the Thevenin equivalent for the circuit shown in Figure 78 with respect terminals a-b

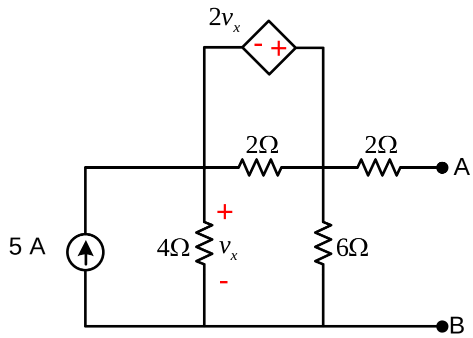


Figure 78

Solution:

Determine the Thevenin voltage V_{TH} for circuit shown in Figure 79.

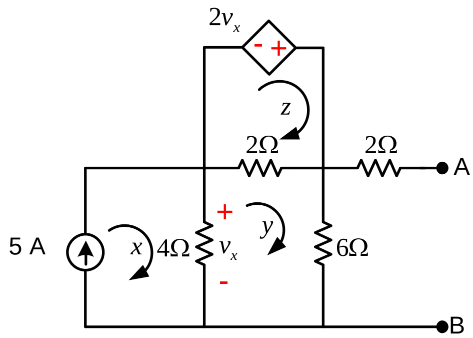


Figure 79

$$x = 5$$

It is observed that

$$\begin{aligned} v_x &= 4(x - y) \\ v_x &= 20 - 4y \end{aligned}$$

KVL for the mesh z

$$\begin{aligned} -2v_x + 2(z - y) &= 0 \\ -2y + 2z - 2(20 - 4y) &= \\ 6y + 2z &= 40 \end{aligned}$$

KVL for the mesh y

$$\begin{aligned} 4(y - x) + 2(y - z) + 6y &= 0 \\ -4x + 12y - 2z &= 0 \\ -4 \times 5 + 12y - 2z &= 0 \\ -20 + 12y - 2z &= 0 \\ 12y - 2z &= 20 \end{aligned}$$

Solving the following linear equations

$$\begin{aligned} 12y - 2z &= 20 \\ 6y + 2z &= 40 \end{aligned}$$

$$y = 3.33A \; x = 10A$$

$$V_{OC} = 6y = 6 \times 3.33A = 20V$$

Short circuit the output terminals AB and determine the short circuit current I_{SC}

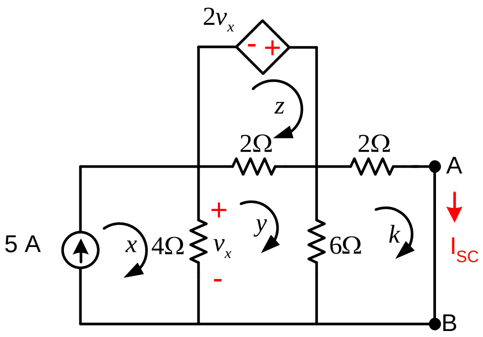


Figure 80

$$x = 5$$

It is observed that

$$\begin{aligned} v_x &= 4(x-y) \\ v_x &= 20-4y \end{aligned}$$

KVL for the mesh y

$$\begin{aligned} 4(y-x)+2(y-z)+6y-6k &= 0 \\ -4x+12y-2z-6k &= 0 \\ -4\times 5+12y-2z-6k &= 0 \\ -20+12y-2z-6k = & \ 0 \\ 12y-2z-6k &= 20 \end{aligned}$$

KVL for the mesh z

$$\begin{aligned} 2z-2y-2v_x &= 0 \\ -2y+2z-2(20-4y) &= 0 \\ -2y+2z-40+8y &= 0 \\ -2y+2z-2(20-4y) &= 0 \\ 6y+2z+0k &= 40 \end{aligned}$$

KVL for the mesh k

$$-6y+0z+8k = 0$$

Solving the following equations

$$\begin{aligned} 12y-2z-6k &= 20 \\ 6y+2z+0k &= 40 \\ -6y+0z+8k &= 0 \end{aligned}$$

$$y = 4.44, z = 6.667, k = 3.333$$

$$I_{SC} = k = 3.333$$

$$Z_{TH} = \frac{V_{TH}}{I_{SC}} = \frac{20}{3.333} = 6\Omega$$

Alternative Method: To determine the short circuit current I_{SC}, apply a test voltage of 1 V at the output terminals AB and determine the applied source current i_o. The modified circuit is as shown in Figure 81.

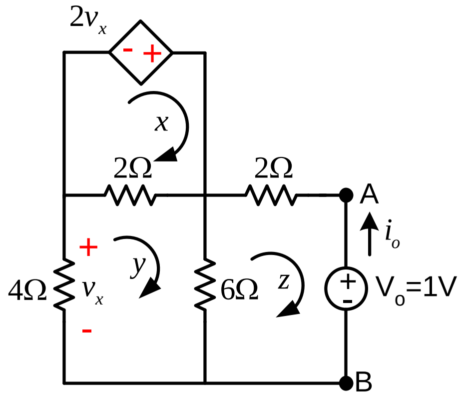


Figure 81

It is observed that

$$v_x = -4y$$

Apply KVL for the mesh x

$$\begin{aligned} -2v_x+2(x-y) &= 0 \\ v_x &= x-y \\ -4y &= x-y \\ x &= -3y \end{aligned}$$

Apply KVL for the mesh y

$$\begin{aligned} -2x+12y-6z &= 0 \\ -2(-3y)+12y-6z &= 0 \\ 18y-6z &= 0 \end{aligned}$$

Apply KVL for the mesh z

$$-6y+8z = -1$$

Solving the following simultaneous equations

$$\begin{aligned} 18y-6z &= 0 \\ -6y+8z &= -1 \end{aligned}$$

$$y = -0.055, \ z = -0.166$$

$$i_O = -z = 0.166A$$

$$R_{TH} = \frac{V_O}{i_O} = \frac{1V}{0.166A} = 6\Omega$$

The Thevenin and Norton circuits are as shown in Figure 82

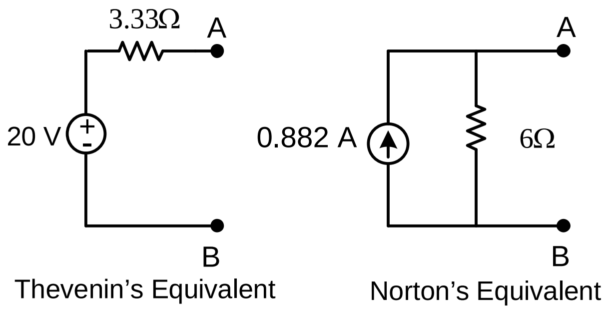


Figure 82

Q 13) Find the Thevenin equivalent for the circuit shown in Figure 83 with respect terminals a-b

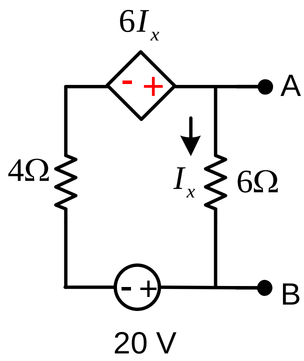


Figure 83

Solution:

$$\begin{aligned} 10I_x - 6I_x + 20 &= 0 \\ 4I_x &= -20 \\ I_x &= \frac{-20}{4} = -5A \\ V_{OC} &= -5A \times 6 = -30V \end{aligned}$$

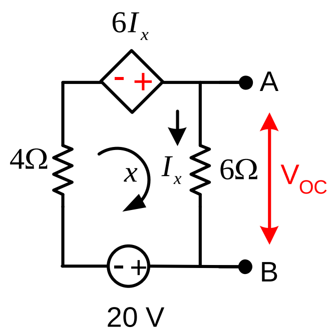


Figure 84

$$I_x = 0$$

$$\begin{aligned} 4I_x + 20 &= 0 \\ I_x &= \frac{20}{4} = 5A \\ I_{SC} &= 5A \end{aligned}$$

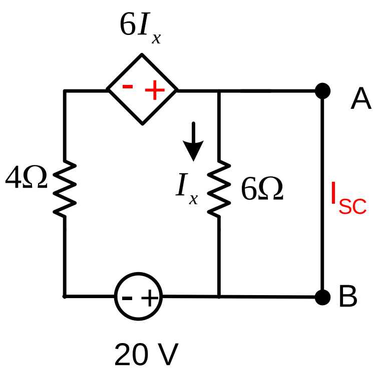


Figure 85

$$Z_N = \frac{V_{OC}}{I_{SC}} = \frac{30}{5} = 6\Omega$$

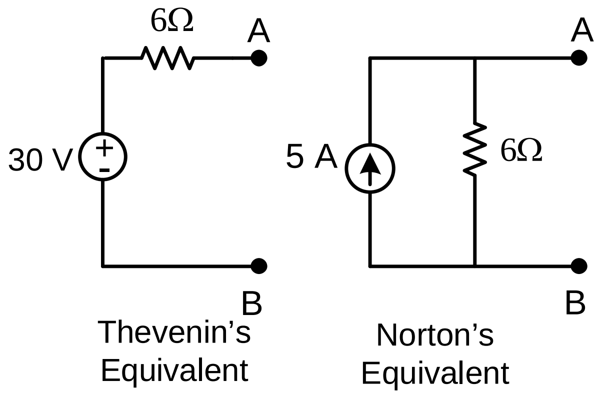


Figure 86

Q 14) Find the Thevenin equivalent for the circuit shown in Figure 83 with respect terminals a-b

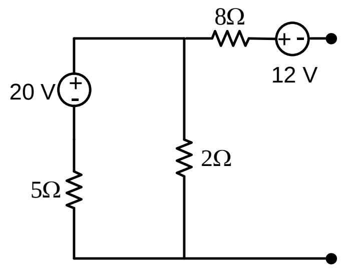


Figure 87

Solution:

$$\begin{aligned} 7x - 20 &= 0 \\ x &= \frac{20}{7} = 2.857A \\ V_{OC} &= x \times 2 - 12 \\ &= 2.857 \times 2 - 12 \\ &= 6.29V \end{aligned}$$

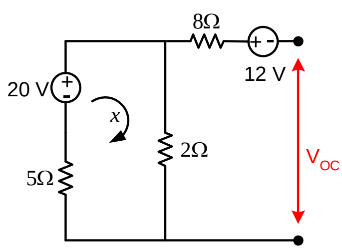


Figure 88

$$\begin{aligned} Z_{TH} &= 8 + \frac{2 \times 5}{2 \times 5} \\ &= 8 + \frac{10}{7} = 2.857A \\ &= 9.43\Omega \end{aligned}$$

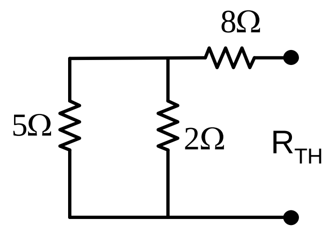


Figure 89

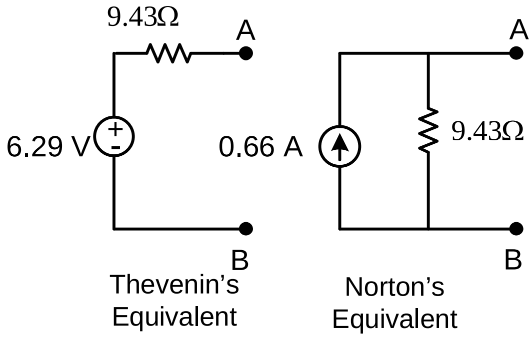


Figure 90

Q 15) **For the circuit shown in Figure 91 Find the Thevenin and Norton equivalent circuit with respect terminals A-B**

Solution:

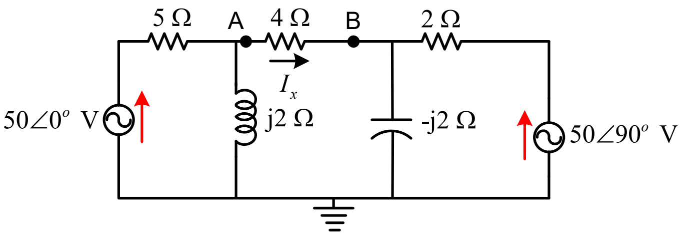


Figure 91

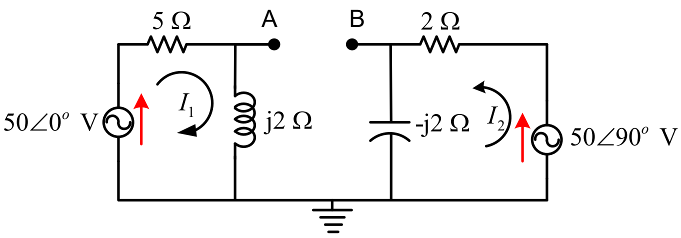


Figure 92

$$\begin{aligned} V_A &= \frac{50\angle 0^\circ}{5+j2} \times j2 \\ &= 18.57\angle 68.2^\circ \end{aligned}$$

$$\begin{aligned} V_B &= \frac{50\angle 90^\circ}{2-j2} \times -j2 \\ &= 35.35\angle 45^\circ \end{aligned}$$

$$\begin{aligned} V_{AB} = V_A - V_B &= 18.57\angle 68.2^\circ - 35.35\angle 45^\circ \\ &= 19.69\angle -156^\circ \end{aligned}$$

$$\begin{aligned} Z_{AB} &= \frac{5\times j2}{5+j2} + \frac{2\times -j2}{2-j2} \\ &= (0.689 + j1.724) + (1 - j1) = 1.689 + j0.724 \end{aligned}$$

$$\begin{aligned} I_{AB} &= \frac{V_{AB}}{Z_{AB}+4} \\ &= \frac{16.69\angle 0^{-156}}{1.689 + j0.724 + 4} \\ &= 2.91\angle -163.25^\circ \end{aligned}$$

Q 16) **Find the Thevenin and Norton equivalent circuit between terminals A-B for the circuit shown in Figure 93.**

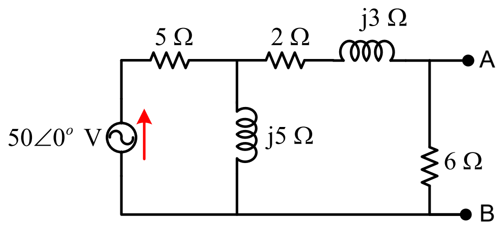


Figure 93

Solution:

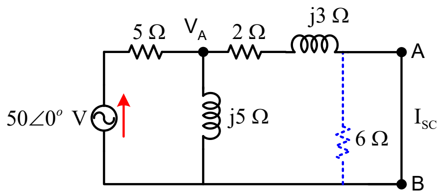


Figure 94

$$\begin{aligned} \frac{V_A - 50}{5} + \frac{V_A}{j5} + \frac{V_A}{2+j3} &= 0 \\ V_A[0.2 - j0.2 + 0.153 - j0.23] &= 10 \\ 0.353 - j0.43 &= 10 \\ V_A &= \frac{10}{0.353 - j0.43} \\ &= 18\angle 50.6^\circ \end{aligned}$$

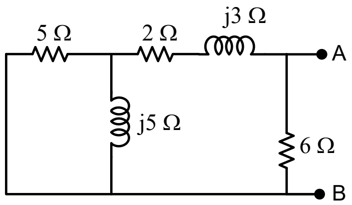


Figure 95

$$\begin{aligned} Z_{AB} &= \left[\left[\frac{5\times j5}{5+j5}\right] + (2+j3)\right] \| 6 \\ &= [(2.5 + j2.5) + (2+j3)] \| 6 \\ &= [(4.5 + j5.5)] \| 6 \\ &= 3.31 + j1.4 = 3.6\angle 23^\circ \end{aligned}$$

$$
\begin{aligned}
I_N &= \frac{V_A}{2+j3} = \frac{18\angle 50.6^\circ}{2+j3} \\
&= 14.99\angle -5.71^\circ
\end{aligned}
$$

Q 17) **Find the Thevenin and Norton equivalent circuit between terminals A-B for the circuit shown in Figure 96.**

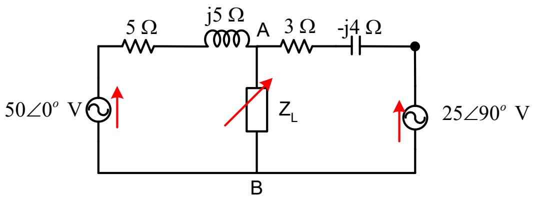


Figure 96

Solution:

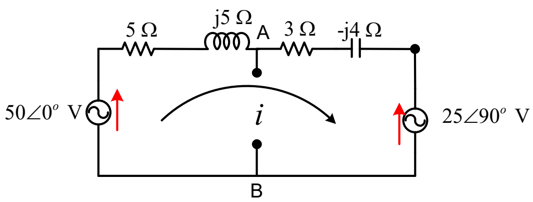


Figure 97

$$
I = \frac{50\angle 0^\circ - 25\angle 90^\circ}{8+j1} = 6.933\angle -33.7^\circ
$$

$$
\begin{aligned}
V_{AB} &= 50\angle 0^\circ - 6.933\angle -33.7^\circ \times (5+j5) \\
&= 9.79\angle -78.65^\circ
\end{aligned}
$$

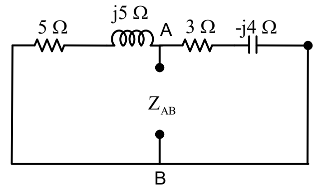


Figure 98

$$
\begin{aligned}
Z_{AB} &= \frac{(5+j5)\times(3-j4)}{8+j1} \\
&= 4.23 - j1.15\ \Omega
\end{aligned}
$$