0.1 AC-Fundamentals

Generation of sinusoidal AC voltage:

Consider a conductor of length l placed in a uniform magnetic field with flux density $\mathbf{B}$ Wb/m^2 is as shown in the Figure. 1. The conductor is rotating in the anticlockwise direction at a uniform angular velocity of ω rad/sec.

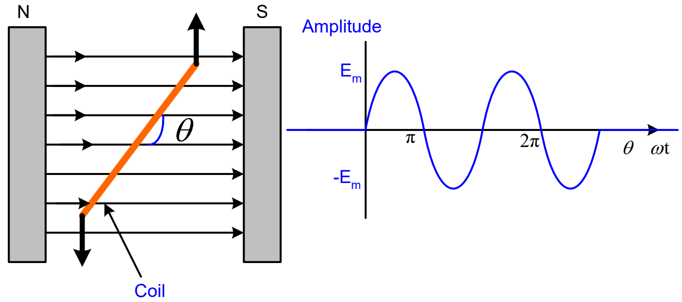

Figure 1

- Consider a position of conductor which is in horizontal position ($\theta = 0$), then the flux cut by the coil is zero because the the conductor is in parallel with the direction of the magnetic field. The induced emf in the conductor becomes is zero.

- When the conductor slowly moves in the anticlockwise direction, then there is a rate of change of flux linking the conductor and hence an emf is induced in the conductor. When the conductor reaches the vertical position, with $\theta = 90°$ then the conductor cuts maximum flux and emf is induced in the conductor is maximum E_m.

- When the conductor continues to moves in the anticlockwise direction, then the induced emf in the coil reduces and becomes zero when it is in horizontal position ($\theta = \pi$).

- Again if the conductor continues to move and angle $\theta = 3\pi/2$ induced emf in the conductor is in the opposite direction.

Important terms associated with an alternating quantity:

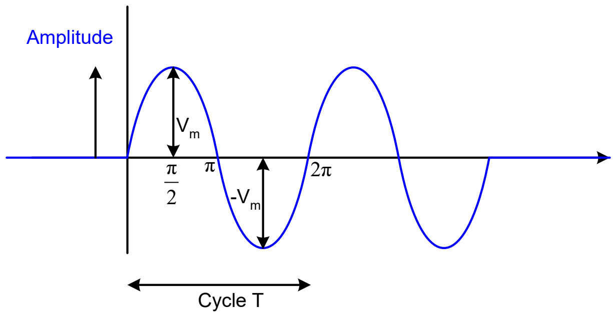

Figure 2

- **Amplitude:** "It is the maximum value attained by an alternating quantity. Also called as maximum or peak value.

- **Instantaneous Value:** It is the value of the e.m.f. induced in the conductor at any instant.

- **Time Period (T):** It is the Time Taken in seconds to complete one cycle of an alternating quantity.

- **Cycle of e.m.f:** A set of positive values along with a set of negative values of the e.m.f induced in the conductor is called a cycle of e.m.f. induced.

- **Frequency (f):** It is defined as the number of cycles of e.m.f. induced in the conductor per second. The unit for frequency is Hz or cycles/sec.

- **Time Period (T):** It is the time taken to complete one cycle of the e.m.f. induced. The relationship between frequency and time period is as follows.

$$T = \frac{1}{f}$$

- **Angular Frequency (ω):** Angular frequency is defined as the number of radians covered in one second (ie the angle covered by the rotating coil). The unit of angular frequency is rad/sec.

Phase & Phase Difference of an alternating quantity:

From the Figure 3 it is observed the magnitude of alternating voltage is continuously changes with time. It is important to know the magnitude of alternating at differen time intervals or at different phases. When the two alternating voltages have the same frequency, and their maximum and minimum point occurs at the same point, then the quantities are said to have in the same phase.

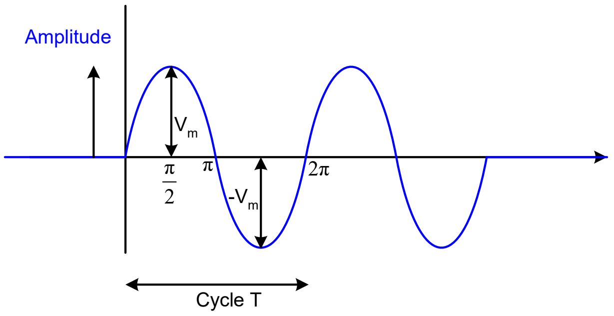

Figure 3

Consider the two alternating voltages with same frequency with different magnitudes $Vm1$ and $Vm2$ shown in the Figure 4. Both the voltages attains their maximum and minimum peak point at the same time. And the zero value of both the voltages occurs at the same time.

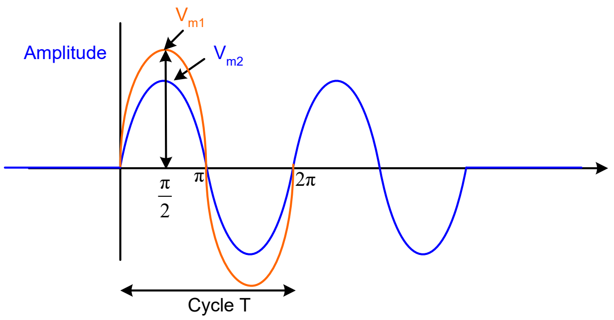

Figure 4

Phase Difference

Phase Difference of an alternating quantity:

If the two alternating quantities with same frequency but have their zero value at the different instant is said to be phase difference. The angle between two alternating quantities is called angle of phase difference.

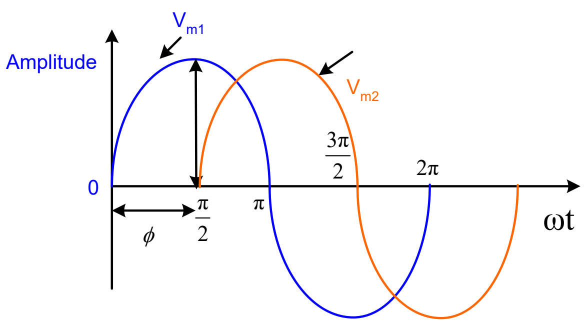

Figure 5

Consider the two alternating voltages of magnitudes $Vm1$ and $Vm2$ shown in the Figure 5. The maximum value of $Vm1$ occurs at $\frac{\pi}{2}$ and The maximum value of $Vm2$ occurs at 180°. They have phase difference of angle $\frac{\pi}{2}$.

The quantity $Vm1$ which attains its +ve maximum value before the $Vm2$ is called a leading quantity, whereas the quantity which reaches its maximum positive value after the other, is known as a lagging quantity. The voltage $Vm1$ is leading the voltage on $Vm2$.

Advantages of AC system over DC system

- AC voltages can be efficiently stepped up/down using transformer.
- AC motors are cheaper and simpler in construction than
- Switch gear for AC system is simpler than DC system.

Average Value of an Alternating Current

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value

$$Average\ value = \frac{Area\ under\ one\ cycle}{Base}$$

The average value of symmetrical AC waveforms over one cycle is zero because the presence of positive negative area. The average value of symmetrical waveforms is calculated for half cycle.

$$i = I_m sin\theta$$

$$\begin{aligned}
i_{av} &= \frac{1}{\pi}\int_0^{\pi} I_m sin\theta d\theta \\
&= \frac{I_m}{\pi}[-cos\theta]_0^{\pi} \\
&= \frac{I_m}{\pi}[-cos(\pi) + (cos(0))] \\
&= \frac{2I_m}{\pi} = 0.637 I_m
\end{aligned}$$

The Effective value of current is

$$\begin{aligned}
I^2 &= \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta = \frac{1}{2\pi}\int_0^{2\pi} I_m^2 sin^2\theta d\theta \\
&= \frac{I_m^2}{2\pi}\int_0^{2\pi}\frac{1 - cos2\theta}{2} d\theta \\
&= \frac{I_m^2}{4\pi}\left[\theta - \frac{sin2\theta}{\theta}\right]_0^{2\pi} \\
&= \frac{I_m^2}{4\pi}[2\pi - 0 - (0 - 0)] \\
I^2 &= \frac{I_m^2}{2} \\
I &= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m
\end{aligned}$$

Form Factor K_f: The ratio of RMS value to the average value of an alternating quantity is known as Form Factor.

$$K_f = \frac{rms\ value}{average\ value} = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

Peak Factor K_P: The ratio of maximum value to the RMS value of an alternating quantity is known as the peak factor.

$$K_P = \frac{maximum\ value}{rms\ value} = \frac{I_m}{I} = \frac{I_m}{0.707 I_m} = 1.414$$

2019-DEC (2018 scheme ECE) 1 c) Define RMS value of alternating current, show that its value is proportional to maximum value

2019-DEC (2018 scheme ECE) 2 b) The equation for an AC voltage is given as $V = 0.04sin(2000t + 60°)V$. Determine the frequency, the angular frequency, instantaneous voltage when $t = 160\mu s$. What is the time represented by a 60°phase angle

Solution:

$$V = V_m sin(2\pi f t + \theta°)$$

$$V = 0.04 sin(2000t + 60°)$$

The Frequency in Hz is

$$\begin{aligned} 2\pi f &= 2000 \\ f &= 318Hz \end{aligned}$$

The angular Frequency in ω is

$$\omega = 2\pi f = 2000$$

Instantaneous voltage when $t = 160\mu s$

$$\begin{aligned} V &= 0.04sin(2000 \times 160 \times 10^{-6} + 60°) \\ &= 0.04sin(2\pi \times 318 \times 160 \times 10^{-6} + 60°) \\ &= 0.04sin(360° \times 318 \times 160 \times 10^{-6} + 60°) \\ &= 0.04sin(18.316° + 60°) \\ &= 0.04sin(78.316° = 0.04V \end{aligned}$$

Q The equation for an alternating current is given by $i = 28.28sin(314t + 30°)$. Find its r.m.s. value, frequency and phase angle.

Solution:

$$I = \frac{I_m}{\sqrt{2}} = \frac{28.28}{\sqrt{2}} = 20A$$

$$\begin{aligned} \omega &= 2\pi f \\ f &= \frac{\omega}{2\pi} = \frac{314}{2\pi} = 50Hz \end{aligned}$$

Phase angle=30°

Q The equation for an alternating current is given by $i = 100sin(396.8t - 30°)$ A. Find its r.m.s. value, frequency and phase angle of the current.

Solution:

$$I = \frac{I_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71A$$

$$\begin{aligned} \omega &= 2\pi f \\ f &= \frac{\omega}{2\pi} = \frac{396.8}{2\pi} = 63.185Hz \end{aligned}$$

Phase angle=−30°

Q The alternating current carrying sinusoidally with a frequency of 50 Hz has an r.m.s. value of 20 A

i) write down the equation for the instantaneous value of current.

ii) Find the value a the instant 0.0125s after passing through a +ve maximum value and

ii) At what time measured from the +ve maximum value will be the instantaneous current be 14.14 A

Solution:

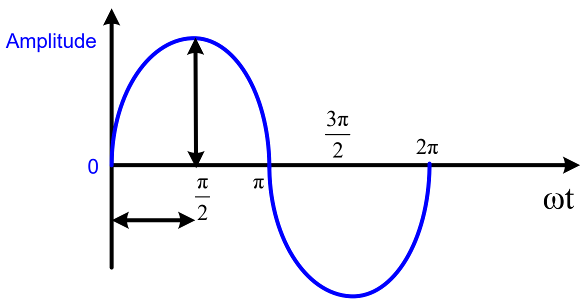

Figure 6

The current is maximum at $\phi = \frac{\pi}{2}$

$$\begin{aligned} i &= I_m sin(\omega t + \frac{\pi}{2}) = I_m sin(2\pi f t + \frac{\pi}{2}) \\ i &= 20\sqrt{2} sin(2\pi \times 50 \times 0.0125 + \frac{\pi}{2}) \\ i &= 20\sqrt{2} sin(360^\circ \times 50 \times 0.0125 + 90^\circ) \\ &= -20A \end{aligned}$$

$$\begin{aligned} 14.14 &= 20\sqrt{2} sin(360^\circ \times 50 \times t + 90^\circ) \\ t &= 3.333 \times 10^{-3} sec \end{aligned}$$

Q An alternating current carrying sinusoidally with a frequency of 50 Hz has a maximum value of 100 A. Reckoning time from the instant when the current is zero and is becoming positive, calculate

a) the instantaneous value after 1/300 sec .

b) the time taken for the current to reach 80 A for the first time

Solution:

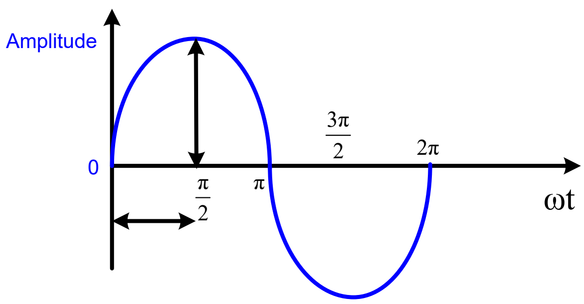

Figure 7

The current is maximum at $\phi = \frac{\pi}{2}$

$$\begin{aligned} i &= I_m sin(\omega t + \frac{\pi}{2}) = I_m sin(2\pi f t + \frac{\pi}{2}) \\ i &= 100\sqrt{2} sin(2\pi \times 50 \times 0.003 + \frac{\pi}{2}) \\ i &= 100\sqrt{2} sin(360^\circ \times 50 \times 0.003 + 90^\circ) \\ &= 83.125 \end{aligned}$$

$$\begin{aligned} 80 &= 100\sqrt{2} sin(360^\circ \times 50 \times t + 90^\circ) \\ \frac{80}{100\sqrt{2}} &= sin(360^\circ \times 50 \times t + 90^\circ) \\ 34.45 &= 18000^\circ \times t + 90^\circ) \\ t &= \frac{55.55}{18000} = 0.003s \end{aligned}$$

Q Find the sum of the electromotive forces $e_1 = 30sin\omega t$, $e_2 = 20sin(\omega t + \frac{\pi}{3})$, $e_3 = 15cos(\omega t)$, $e_4 = 10sin(\omega t - \frac{\pi}{3})$, and $e_5 = 20cos(\omega t + \frac{2\pi}{3})$. Express the result in the form of $e = E_m sin(\omega t \pm \phi)$,

Solution:

Convert the cosine functions into sine functions

$$\begin{aligned} e_3 &= 15cos(\omega t) = 15sin(\omega t + 90^\circ) \\ e_5 &= 20cos(\omega t + \frac{2\pi}{3}) = 20sin(\omega t + \frac{2\pi}{3} + 90^\circ) \\ &= 20sin(\omega t + 210^\circ) \end{aligned}$$

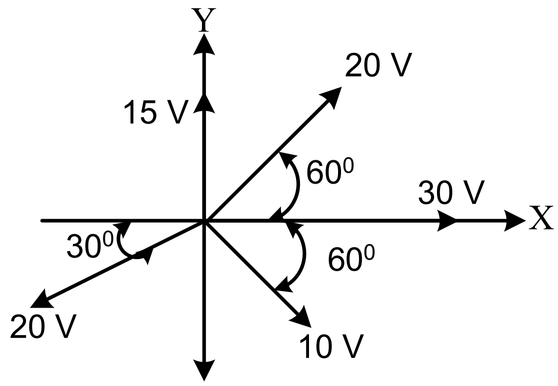

Figure 8

$$\begin{aligned} \sum X &= 30 + 20cos60^\circ + 0 - 20cos30^\circ + 10cos60^\circ \\ &= 27.68 \\ \sum Y &= 0 + 20sin60^\circ + 15 - 20sin30^\circ - 10sin60^\circ \\ &= 13.66 \end{aligned}$$

$$\begin{aligned} V &= \sqrt{(27.68)^2 + (13.66)^2} = 30.87\ Volts \\ \phi &= tan^{-1}\frac{13.66}{27.68} = 26.27^\circ \end{aligned}$$

$$\begin{aligned} e_r &= E_m sin(\omega t \pm \phi) \\ &= 30.87 sin(\omega t + 26.27^\circ) \end{aligned}$$

Q Find graphically or otherwise the following four voltages $e_1 = 25sin\omega t$, $e_2 = 30sin(\omega t + \frac{\pi}{6})$, $e_3 = 30cos(\omega t)$, $e_4 = 20sin(\omega t - \frac{\pi}{4})$. Express the result in the form of $e = E_m sin(\omega t \pm \phi)$,

Solution:

Convert the cosine functions into sine functions

$$e_3 = 30cos(\omega t) = 30sin(\omega t + 90^\circ)$$

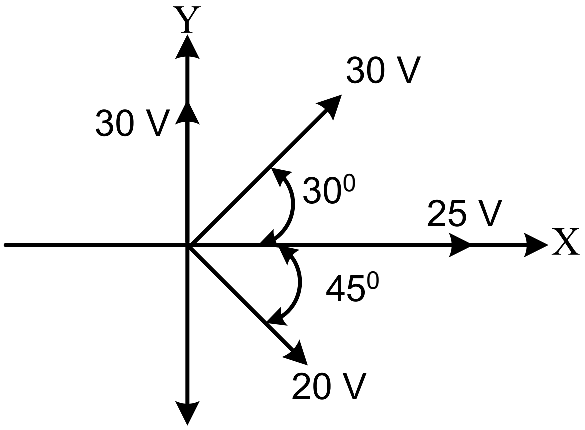

Figure 9

$$\sum X = 25 + 30cos30^\circ + 0 + 20cos45^\circ$$
$$= 65.12$$
$$\sum Y = 0 + 30sin30^\circ + 30 - 20sin45^\circ$$
$$= 30.857$$

$$V = \sqrt{(65.12)^2 + (30.857)^2} = 72.06\ Volts$$
$$\phi = tan^{-1}\frac{30.857}{65.12} = 25.36^\circ$$

$$e_r = E_m sin(\omega t \pm \phi)$$
$$= 72.06 sin(\omega t + 25.36^\circ)$$

Q Four emfs $e_1 = 100 sin\omega t$, $e_2 = 80 sin(\omega t - \frac{\pi}{6})$, $e_3 = 120 sin(\omega t + \frac{\pi}{6})$, $e_4 = 100 sin(\omega t - \frac{\pi}{4})$ are induced i four coils connected in series in such a way that the sum of the four emfs obtained. Find the emf if the connections to the coil in which emf e2 is induced are reversed, find the new resultant emf. Express the result in the form of $e = E_m sin(\omega t \pm \phi)$,

Solution:

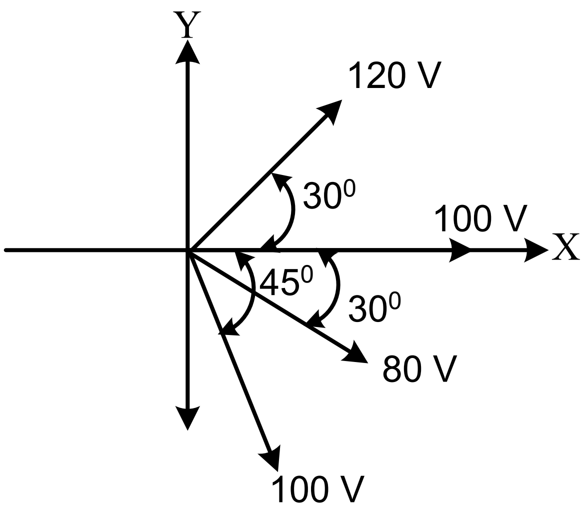

Figure 10

$$\sum X = 100 + 120cos30^\circ + 80cos30^\circ + 100cos45^\circ$$
$$= 343.9$$
$$\sum Y = 0 + 120sin30^\circ - 80sin30^\circ - 100sin45^\circ$$
$$= -50.71$$

Q Three alternating currents $i_1 = 141 sin(\omega t + \frac{\pi}{4})$, $i_2 = 30 sin(\omega t + \frac{\pi}{2})$, and $i_3 = 20 sin(\omega t - \frac{\pi}{6})$ are fed into common conductor. Find the equation for the resultant current, its rms value, form factor, and peak factor.

Solution:

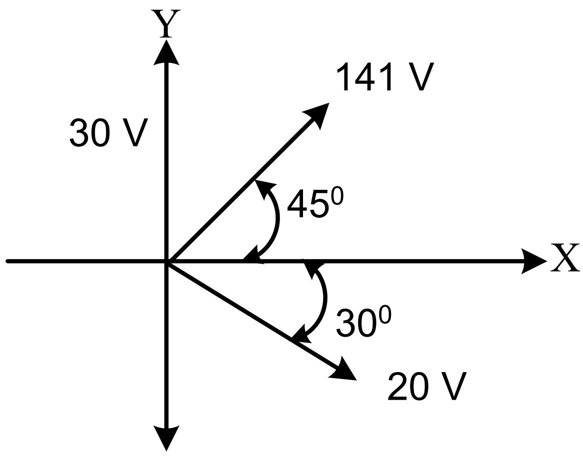

Figure 11

$$\sum X = 0 + 141cos45^\circ + 20cos30^\circ$$
$$= 117$$
$$\sum Y = 30 + 141sin45^\circ - 20sin30^\circ$$
$$= 119.7$$

$$V = \sqrt{(117)^2 + (119.7)^2} = 167.38\ Volts$$
$$\phi = tan^{-1}\frac{3119.7}{117} = 45.62^\circ$$

$$e_r = E_m sin(\omega t \pm \phi)$$
$$= 167.38 sin(\omega t + 45.62^\circ)$$

rms value, form factor, and peak factor

$$I = \frac{I_m}{\sqrt{2}} = \frac{167.38}{\sqrt{2}}$$
$$= 118.34\ A$$

$$I_{av} = 0.637 \times 167.38 = 106.62$$

Form factor

$$K_f = \frac{I}{I_{av}} = \frac{118.34}{106.62} = 1.11$$

Peak factor

$$K_P = \frac{I_m}{I} = \frac{167.38}{118.34} = 1.414$$

Q The current has the following steady values in amperes for equal intervals of time changing instantaneously from one value to the next, 0, 10,20, 30, 20, 10, 0, -10, -20, -30, -20, -10, 0 etc. Calculate the r.m.s. value, the average value and its form factor

Solution:

the r.m.s. value

$$I = \sqrt{\frac{0^2+10^2+20^2+30^2+20^2+10^2}{6}} = 17.795\ A$$

average value

$$I_{av} = \frac{0+10+20+30+20+10}{6} = 15\ A$$

form factor

$$K_f = \frac{I}{I_{av}} = \frac{17.795}{15} = 1.1863$$

2019-Jan 2 b) The instantaneous values of two alternating voltages are represented respectively by $V_1 = 60sin\theta$ and $V_2 = 40sin\left(\theta - \frac{\pi}{3}\right)$ volts. Derive an expression for instantaneous value of (i) the sum (ii) the difference of these voltages

Solution:

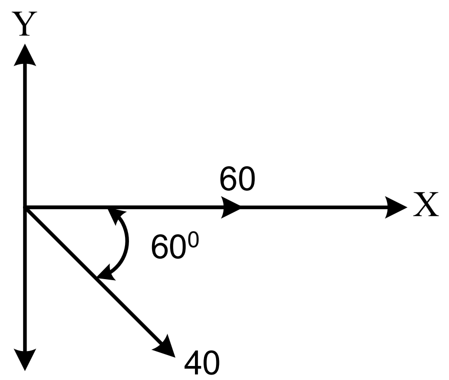

Figure 12

$$V_1 = 60sin\theta$$
$$V_2 = 40sin\left(\theta - \frac{\pi}{3}\right)$$

(i) The sum of voltages is

$$\sum X = 60 + 40cos60^\circ = 80$$
$$\sum Y = 0 - 40sin60^\circ = -34.64$$

$$V = \sqrt{80^2 + (-34.64)^2} = 87.17$$

$$\phi = tan^{-1}\left(-\frac{80}{34.64}\right) = -23.41$$

$$V = 87.17sin(\omega t - 23.41)$$

(ii) The difference of these voltages

$$\sum X = 60 - 40cos60^\circ = 40$$
$$\sum Y = 0 + 40sin60^\circ = 34.64$$

$$V = \sqrt{40^2 + (34.64)^2} = 52.9$$

$$\phi = tan^{-1}\left(\frac{40}{34.64}\right) = 49$$

$$V = 52.9sin(\omega t + 4923.41)$$

Q Calculate the r.m.s. value, the average value and its form factor peak factor of a periodic voltage having the following steady values for equal intervals of time , 0, 10,20, 40, 60, 80, 100, 80, 60, 40, 20, 10, 0, -10,-20, etc. What is the r.m.s. value, the average value and its form factor peak factor of a sine wave having the same maximum value?

Solution:

the r.m.s. value

$$\begin{aligned} I &= \sqrt{\frac{0^2+10^2+20^2+40^2+60^2+80^2+100+80+60+40+20+10}{12}} \\ &= 53.385\ A \end{aligned}$$

average value

$$\begin{aligned} I_{av} &= \frac{0+10+20+40+60+80+100+80+60+40+20+10}{12} \\ &= 43.3\ A \end{aligned}$$

form factor

$$\begin{aligned} K_f &= \frac{I}{I_{av}} = \frac{53.385}{43.3} \\ &= 1.232 \end{aligned}$$

Peak factor

$$\begin{aligned} K_P &= \frac{I_m}{I_{rms}} = \frac{100}{53.385} \\ &= 1.873 \end{aligned}$$

The r.m.s. value, the average value and its form factor peak factor of a sine wave having the same maximum value.

In the given data the maximum value is 100, The r.m.s. valu

$$I_{rms} = 0.707 I_m = 0.707 \times 100 = 70.7A$$

average value

$$I_{av} = 0.637 I_m = 0.637 \times 100 = 63.7A$$

form factor

$$\begin{aligned} K_f &= \frac{I}{I_{av}} = \frac{70.7}{63.7} \\ &= 1.111 \end{aligned}$$

Peak factor

$$\begin{aligned} K_P &= \frac{I_m}{I_{rms}} = \frac{100}{70.7} \\ &= 1.414 \end{aligned}$$

Q In a circuit supplied from 50 Hz, the voltage and current have maximum values of 500 V and 10 A respectively. At t=0 their respective values are 400 V and 4 A, both increasing positively. (i) Write expressions for instantaneous values (ii) Find the angle between V and I and (iii) I at t=0.015sec

Solution:

$$\begin{aligned} e &= E_m\, sin(\omega t + \phi_1) \\ 400 &= 500 sin(\omega t + \phi_1) \end{aligned}$$

When t=0

$$\begin{aligned} 400 &= 500sin(\phi_1) \\ \phi_1 &= 53.13^\circ \end{aligned}$$

$$e = 500sin(\omega t + 53.13^\circ)$$

$$\begin{aligned} i &= I_m \ sin(\omega t + \phi_2) \\ 4 &= 10sin(\omega t + \phi_2) \end{aligned}$$

When t=0

$$\begin{aligned} 4 &= 10sin(\phi_2) \\ \phi_2 &= 23.57^\circ \end{aligned}$$

$$i = 10sin(\omega t + 23.57^\circ)$$

ii) The angle between V and I

$$\phi = \phi_1 - \phi_2 = 53.13^\circ - 23.57^\circ = 29.55^\circ$$

(iii) I at t=0.015sec

$$\begin{aligned} i &= 10sin(\omega t + 23.57^\circ) = 10sin(2\pi \times ft + 23.57^\circ) \\ &= 10sin(360^\circ \times 50 \times 0.015 + 23.57^\circ) \\ &= 10sin(293.57^\circ) \\ &= -9.1A \end{aligned}$$