Chapter 1

Module 1 Basic Concepts

1.1 Introduction

Module 1
Karnaugh maps: minimum forms of switching functions, two and three variable Karnaugh maps, four variable karnaugh maps, determination of minimum expressions using essential prime implicants, Quine-McClusky Method: determination of prime implicants, The prime implicant chart, petricks method, simplification of incompletely specified functions, simplification using map-entered variables
Two Variable K Map

1.1.1 Pairs, Quads, and Octets:

Pairs: Consider Four variable K-map as shown in Figure in which it contains a pair of 1's that are horizontally adjacent. These are called adjacent 1's, and these 1's can paired. This pairing eliminates one variable. Similarly it contains a pair of 1's that are vertically adjacent.

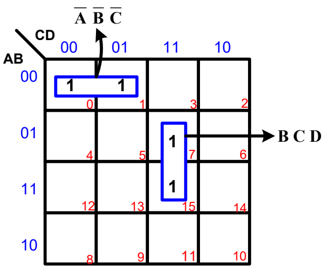


Figure 1.1

$$
\begin{aligned}
y_1 &= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,D \\
&= \overline{A}\,\overline{B}\,\overline{C}\,(\overline{D} + D) \\
&= \overline{A}\,\overline{B}\,\overline{C} \\
y_2 &= \overline{A}\,B\,C\,D + A\,B\,C\,D \\
&= B\,C\,D\,(\overline{A} + A) \\
&= B\,C\,D
\end{aligned}
$$

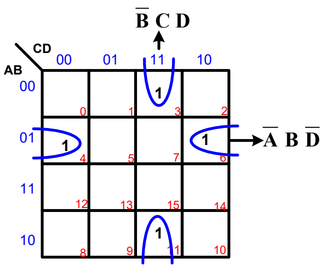


Figure 1.2

$$
\begin{aligned}
y_1 &= \overline{A}\,B\,\overline{C}\,\overline{D}\,\overline{D} + \overline{A}\,B\,C\,\overline{D} \\
&= \overline{A}\,B\,\overline{D}\,(\overline{C} + C) \\
&= \overline{A}\,B\,\overline{D} \\
y_2 &= \overline{A}\,\overline{B}\,C\,D + A\,\overline{B}\,C\,D \\
&= \overline{B}\,C\,D\,(\overline{A} + A) \\
&= \overline{B}\,C\,D
\end{aligned}
$$

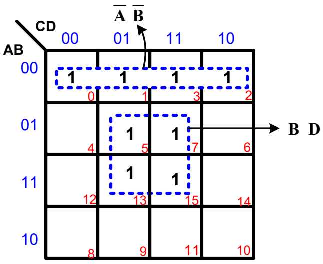


Figure 1.3

$$y_1 = \overline{A}\,\overline{B}$$
$$y_2 = B\,D$$

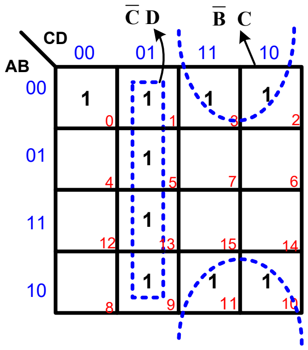


Figure 1.4

$$y_1 = \overline{C}\,D$$
$$y_2 = \overline{B}\,C$$

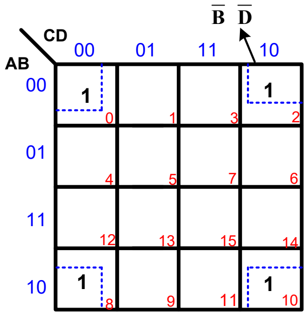


Figure 1.5

$$y = \overline{C}\,\overline{D}$$

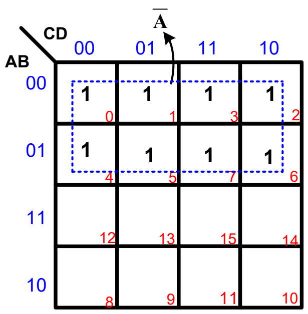
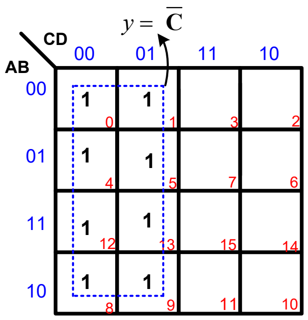


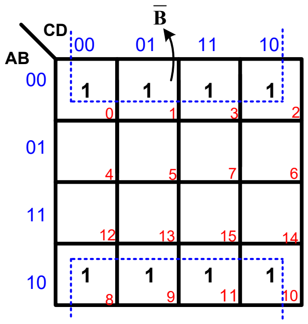
Figure 1.6

$$y = \overline{A}$$



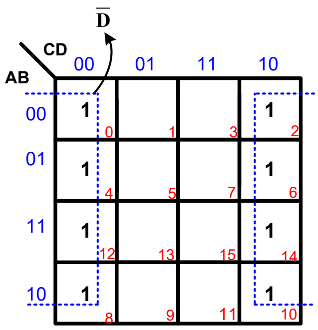
$$y = \overline{C}$$

Figure 1.7



$$y = \overline{B}$$

Figure 1.8



$$y = \overline{D}$$

Figure 1.9

Prime implicants

- Prime Implicant: A prime implicant is a rectangle of 1, 2, 4, 8, . . . 1s or Xs not included in any one larger rectangle.
- Essential Prime Implicant: An essential prime implicant is a prime implicant that covers at least one 1 not covered by any other prime implicant. Dont cares (Xs) do not make a prime implicant essential.
- Redundant Prime Implicants: The prime implicants for which each of its minterm is covered by some essential prime implicant are redundant prime implicants (RPI)
- Selective Prime Implicants: The prime implicants for which are neither essential nor redundant prime implicants are called selective prime implicants (SPI)

1. Given F $= \sum$ (1, 5, 6, 7, 11, 12, 13, 15), find number of implicant, PI, EPI, RPI and SPI.

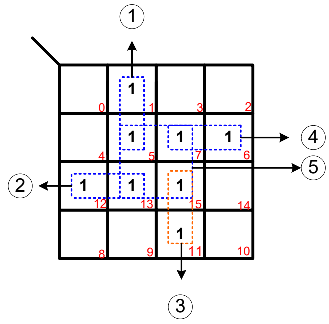


Figure 1.10

From the above Figure 1.10

- No. of Implicants = 8 (1,2,3,4,5,6,7,8)
- No. of Prime Implicants(PI) = 5 (1,2,3,4,5)
- No. of Essential Prime Implicants(EPI) = 4 (1,2,3,4)
- No. of Redundant Prime Implicants(RPI) = 1 (5)
- No. of Selective Prime Implicants(SPI) = 0

2. Given F = $\sum$ (0, 1, 5, 8, 12, 13), find number of implicant, PI, EPI, RPI and SPI.

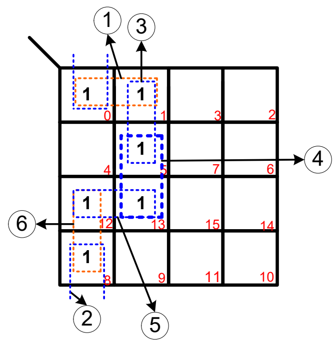


Figure 1.11

From the above Figure 1.11

- No. of Implicants = 6 (1,2,3,4,5,6)
- No. of Prime Implicants(PI) = 5 (1,2,3,4,5,6)
- No. of Essential Prime Implicants(EPI) = 0
- No. of Redundant Prime Implicants(RPI) = 0
- No. of Selective Prime Implicants(SPI) = 0

3. Given F = $\sum$ (0, 1, 5, 7, 15, 14, 10), find number of implicant, PI, EPI, RPI and SPI.

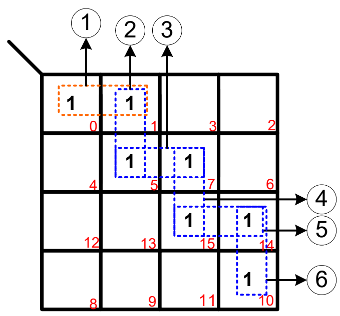


Figure 1.12

From the above Figure **??**

- No. of Implicants = 3 (1,2,3)
- No. of Prime Implicants(PI) = 5 (1,2,3,4,5,6)
- No. of Essential Prime Implicants(EPI) = 2 (1,6)
- No. of Redundant Prime Implicants(RPI) = 2
- No. of Selective Prime Implicants(SPI) = 2

4. Given F = $\sum$ (0, 1, 4, 5, 9, 11, 13, 15), find number of implicant, PI, EPI, RPI and SPI.

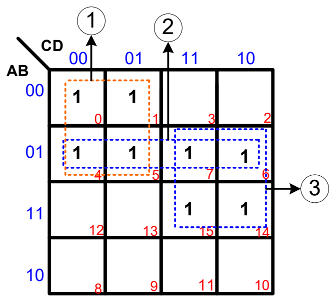


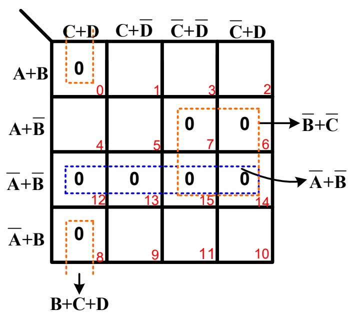
Figure 1.13

From the above Figure **??**

- No. of Implicants = 6 (1,2,3,4,5,6,7)
- No. of Prime Implicants(PI) = 5 (1,2,3,4,5,6)
- No. of Essential Prime Implicants(EPI) = 2 (1,6)
- No. of Redundant Prime Implicants(RPI) = 2
- No. of Selective Prime Implicants(SPI) = 2

Minimize the following function for POS using K- map and realize it by using basic gates

$$F(A, B, C, D) = \Pi M(0, 6, 7, 8, 12, 13, 14, 15)$$



$$f(A, B, C, D) = (\overline{A} + \overline{B})(\overline{B} + \overline{C})(B + C + D)$$

Figure 1.14

Minimize the following function for POS using K- map and realize it by using basic gates

$$f(a, b, c, d) = \Pi M(0, 1, 6, 8, 11, 12) + d(3, 7, 4, 15)$$

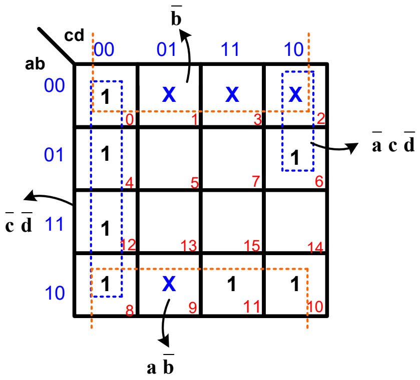
1.2 Module -1 VTU Question Papers

2020 Aug 18CS33
Q3 a) **b.Minimize the following function for SOP using K- map and implement using basic gates:**

$$f(a,b,c,d) = \Pi M(5,7,13,14,15) + d(1,2,3,9)$$

Solution:

$$f(a,b,c,d) = \Pi M(5,7,13,14,15) + d(1,2,3,9)$$
$$= \sum M(0,4,6,8,10,11,12) + d(1,2,3,9)$$



$$f(a,b,c,d) = \overline{b} + \overline{c}\,\overline{d} + \overline{a}\,c\,\overline{d}$$

Figure 1.15

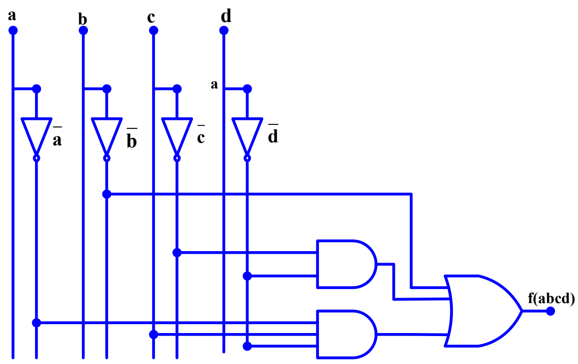
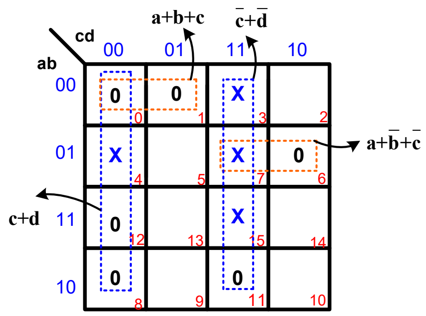


Figure 1.16

Q4 a) 2020 Aug 18CS33 **b.Minimize the following function for POS using K- map and realize it using basic gates:**

$$f(a,b,c,d) = \Pi M(0,1,6,8,11,12) + d(3,7,4,15)$$

Solution:



$$f(a,b,c,d) = (a+b+c)(c+d)(\overline{c}+\overline{d})(a+\overline{b}+\overline{c})$$

Figure 1.17

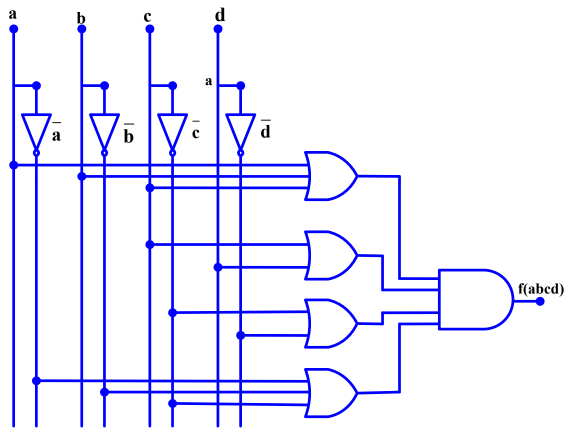


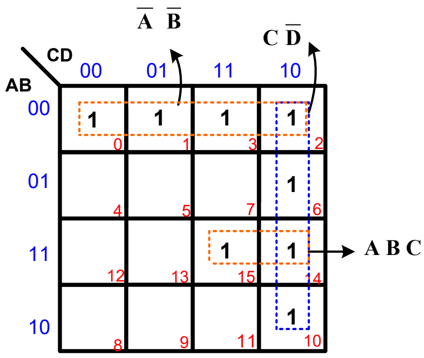
Figure 1.18

Q4 b) 2020 Aug 18CS33

Plot the following function on k-map (do not expand to minterm before plotting)

$$f(A,B,C,D) = \overline{A}\ \overline{B} + C\overline{D} + ABC + \overline{A}\ \overline{B}\ \overline{C}\ \overline{D} + ABC\overline{D}$$

Solution:



$$f(A,B,C,D) = \overline{A}\ \overline{B} + C\ \overline{D} + ABC$$

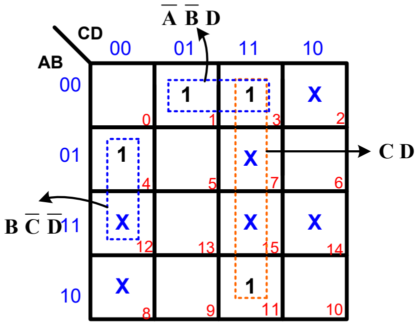
Figure 1.19

Q3 A) **2019 JAN 18CS33**

Find minimum SOP and minimum POS expression for the following function using k-map

$$f(A,B,C,D) = \sum m(1,3,4,11) + \sum d(2,7,8,12,14,15)$$

Solution:



$$f(A,B,C,D) = \overline{A}\,\overline{B}\,D + C\,D + B\,\overline{C}\,\overline{D}$$

Figure 1.20

$$f(A,B,C,D) = \sum m(1,3,4,11) + \sum d(2,7,8,12,14,15)$$
$$= \prod m(0,5,6,9,10,13) + \sum d(2,7,8,12,14,15)$$

Q3 A) **2019 July 17CS32**
Use a Karnaugh map to find minimum SOP form for the following Boolean function

$$f(A,B,C,D) = \sum m(0,2,3,5,6,7,8,9) + \sum d(10,11,12,13,14,15)$$

Also draw the logic circuit diagram for the simplified SOP

Solution:

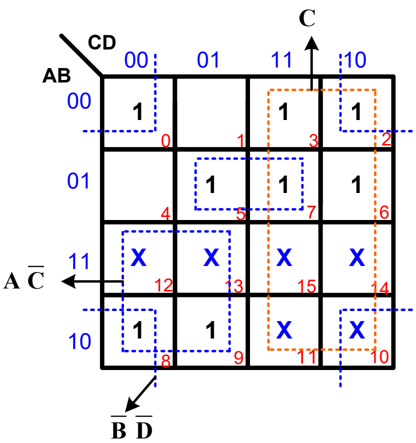


Figure 1.21

$$f(A,B,C,D) = C + A\,\overline{C} + \overline{B}\,\overline{D}$$

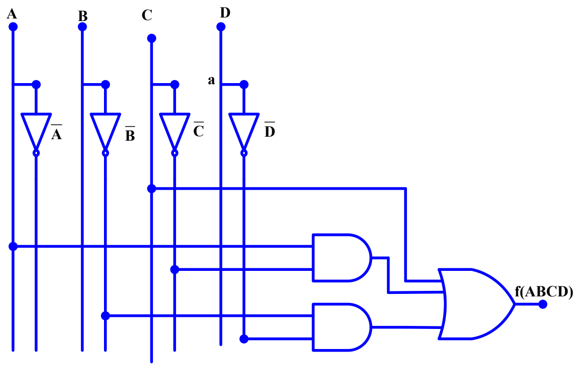


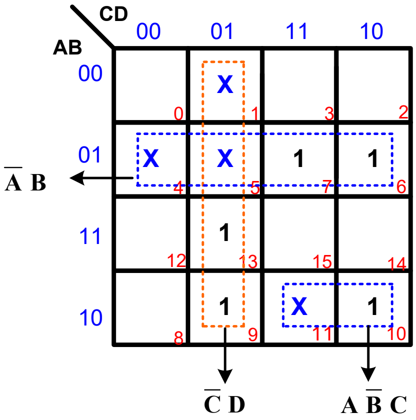
Figure 1.22

Q3 b) **2020 Aug 17CS32**
Find the minimal sum and minimal product using a Karnaugh map

$$f(A,B,C,D) = \sum m(6,7,9,10,13) + \sum d(1,4,5,11)$$

Solution:

Minimal Sum

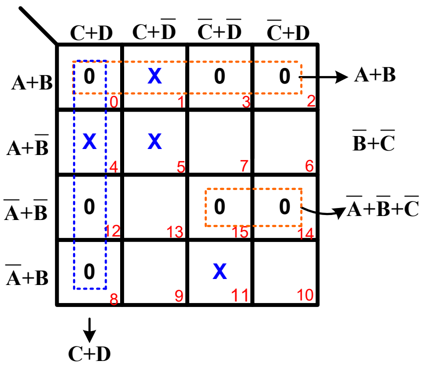


$$f(A,B,C,D) = \overline{A}\ B + \overline{C}\ D + A\ \overline{B}\ C$$

Figure 1.23

Minimal Product

$$f(A,B,C,D) = \sum m(6,7,9,10,13) + \sum d(1,4,5,11)$$
$$= \prod m(0,2,3,8,12,14,15) + \prod d(1,4,5,11)$$



$$f(A,B,C,D) = \overline{A}\ B + \overline{C}\ D + A\ \overline{B}\ C$$

Figure 1.24

1.3 Quine McCluskey method

Quine McCluskey method

The minimization of boolean expressions is important to reduce the number of logic gates required to implement digital logic circuits. The K map method is used to simplify the boolean expressions. When the number of variables is greater than 5 its difficult to use K Map method. Quine-McCluskey algorithm is classical method for simplifying boolean expressions, which can handle any number of variables. When the number of input variables is greater than 5, the tabular method for simplifying boolean expressions developed by Quine and McCluskey is used.

- Step 1: Convert the given minterms into its binary form.
- Step 2: Group the minterms according to the number of 1s
- Step 3: Compare elements of Group N with Group N+1. While comparing if they differ in only one position, then put a check mark and copy the terms in the next column. Place hyphen or dash in position of a minterm where they differ with each other.
- Step 4: Repeat the above Step 3 until no merging possible
- Step 5: Put all prime implicants in a cover table
- Step 6: Identify essential minterms, and hence essential prime implicants
- Step 7: Add prime implicants to the minimum expression until all minterms of are covered

1. Find the minimum SOP for the function $f(A, B, C, D) = \sum m(5, 7, 9, 11, 13, 15)$ using Quine McCluskey method.

Solution:

Determination of prime implicants

Step 1:

- Given SOP contains maximum number of 15, hence requires 4 digit binary number to represent the given SOP.
- Four variables A, B, C,D are used to represent in terms of binary.
- Given min terms are 5, 7, 9, 11, 13, 15, the details of the binary representation is as shown in Table 1.

Table 1

Min terms	A	B	C	D
5	0	1	0	1
7	0	1	1	1
9	1	0	0	1
11	1	0	1	1
13	1	1	0	1
15	1	1	1	1

Step: 2

- Minterms 5 and 9 contains 2 1s, and are grouped into group no 2.
- Minterms 7, 11 and 13 contains 3 1s, and are grouped into group no 3.
- Minterm 15 contains 4 1s, and are grouped into group no 4.

Table 2

Group	Minterms	A	B	C	D	
2	5	0	1	0	1	✓
	9	1	0	0	1	✓
3	7	0	1	1	1	✓
	11	1	0	1	1	✓
	13	1	1	0	1	✓
4	15	1	1	1	1	✓

Step: 3

- Compare the Group 2 minterms with Group 3 minterms.
- First compare minterm 5(0101) with Group 3 minterm 7(0111), it is differ only in one position i.e, 01–1, hence minterms are grouped.
- Next minterm 5(0101) and 11(1011) is differ in two positions hence it is not possible group them.
- minterm 5(0101) with Group 3 minterm 13(1101), it is differ only in one position i.e, –101
- Compare the Group 3 minterms with Group 4 minterms.
- Minterm 7(0111) and 15(1111) is differ only in one position i.e, –111, hence minterms are grouped.
- Minterm 11(1011) and 15(1111) is differ only in one position i.e, 1–11, hence minterms are grouped.
- Minterm 13(1101) and 15(1111) is differ only in one position i.e, 11–1, hence minterms are grouped.
- All the minterms in Table 2 are covered hence put tick mark
- The details are as shown in Table 3

Table: 3

Group	Minterms	A	B	C	D	
2	5,7	0	1	–	1	✓
	5,13	–	1	0	1	✓
	9,11	1	0	–	1	✓
	9,13	1	–	0	1	✓
3	7,15	–	1	1	1	✓
	11,15	1	–	1	1	✓
	13,15	1	1	–	1	✓

Step: 4

- Compare the Group 2 minterms with Group 3 minterms in Table 3.
- Compare the Group 2 minterm 5, 7(01–1) with Group 3 minterm 7, 15 (–111), it is differ in two positions hence it is not possible to group them.
- Compare the Group 2 minterm 5, 7(01–1) with Group 3 minterm 11, 15 (1–01), it is differ in two positions hence it is not possible to group them.
- Compare the Group 2 minterm 5, 7(01–1) with Group 3 minterm 13, 15 (11–1), , it is differ only in one position i.e, –1–1, hence minterms are grouped.
- Compare the Group 2 minterm 5, 13(–101) with Group 3 minterm 7, 15 (–111), , it is differ only in one position i.e, –1–1, hence minterms are grouped.
- Compare the Group 2 minterm 5, 13(–101) with Group 3 minterm 11, 15 (1–11), it is differ in two positions hence it is not possible to group them.
- Compare the Group 2 minterm 5, 13(–101) with Group 3 minterm 13, 15 (11–1), it is differ in two positions hence it is not possible to group them.
- Continue the same steps for remaining Group 2 minterms with Group 3 minterms.
- All the minterms in Table 3 are covered hence put tick mark
- The details are as shown in Table 4

Table: 4

Group	Minterms	A	B	C	D
2	5,7,13,15	–	1	–	1
	5,13,7,15	–	1	–	1
	9,11,13,15	1	–	–	1
	9,13,11,15	1	–	–	1

Step: 5

Minterms (5,7,13,15) (5,13,7,15) are having variables as –1–1, this is represented as BD and similarly minterms (9,11,13,15) (9,13,11,15) are having variables as 1– – 1, this is represented as AD.

Put all prime implicants in a table as shown in Table 5. Column 1 contains Prime implicants, column 2 corresponding decimal numbers and column given minterms. Put X mark wherever the minterms are covered.

The prime implicant chart

Table: 5

P.I	Decimal	Minterms					
		5	7	9	11	13	15
$B\ D$	5,7,13,15	X	X			X	X
$A\ D$	9,11,13,15			X	X	X	X

Step 6: Identify the essential minterms, and hence essential prime implicants. 5, 7, 9,11 are the essential minterms.

Table: 6

P.I	Decimal	Minterms					
		5	7	9	11	13	15
$B\ D$	5,7,13,15	(X)	(X)			X	X
$A\ D$	9,11,13,15			(X)	(X)	X	X

Step 7: Based on essential prime implicants write the expression.

$$Y = B\ D + A\ D$$
$$= D(A + B)$$

Step 8: Result can also be verified by K map method. Given function is $f(A, B, C, D) = \sum m(5, 7, 9, 11, 13, 15)$

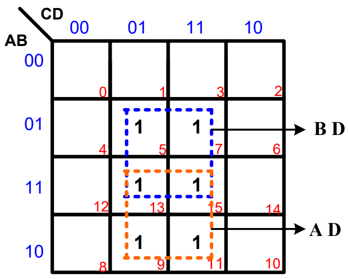


Figure 1.25

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2. Find the minimum SOP for the function $f(A, B, C, D) = \sum m(0, 1, 3, 7, 8, 9, 11, 15)$ using Quine McCluskey method.

Solution:

Table 1

Min terms	A	B	C	D	
0	0	0	0	0	
1	0	0	0	1	
3	0	0	1	1	
7	0	1	1	1	
8	1	0	0	0	
9	1	0	0	1	
11	1	0	1	1	
15	1	1	1	1	

Step: 2

Group	Minterms	A	B	C	D	
0	0	0	0	0	0	✓
1	1	0	0	0	1	✓
	8	1	0	0	0	✓
2	3	0	0	1	1	✓
	9	1	0	0	1	✓
3	7	0	1	1	1	✓
	11	1	0	1	1	✓
4	15	1	1	1	1	✓

Table: 3

Group	Minterms	A	B	C	D
0	0,1	0	0	0	–
	0,8	–	0	0	0
1	1,3	0	0	–	1
	1,9	–	0	0	1
	8,9	1	0	0	–
2	3,7	0	–	1	1
	3,11	–	0	0	1
	9,11	–	0	1	1
3	7,15	–	1	1	1
	11,15	1	–	1	1

Table: 4

Group	Minterms	A	B	C	D
0	0,1,8,9	–	0	0	–
	0,8,1,9	–	0	0	–
1	1,3,9,11	-	0	–	1
	1,9,3,11	–	0	–	1
2	3,7,11,15	–	–	1	1
	3,11,7,15	–	–	1	1

Table: 5

P.I	Decimal	Minterms							
		0	1	3	7	8	9	11	15
$\overline{B}\ \overline{C}$	0,1,8,9	(X)	X			(X)	X		
$\overline{B}\ D$	1,3,9,11		X	X			X	X	
$C\ D$	3,7,11,15			X	(X)			X	(X)

$$Y = \overline{B}\ \overline{C} + C\ D$$

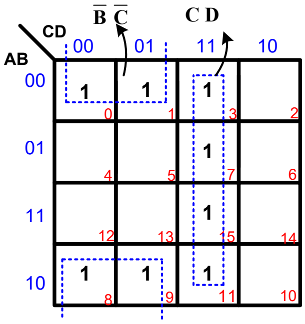

Figure 1.26

3. Find the minimum SOP for the function $f(A, B, C, D) = \sum m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)$ using Quine McCluskey method.

Solution:

Table 1

Min terms	A	B	C	D	
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0		1	✓
5	0	0	1	1	✓
6	0	0	0	1	✓
7	0	1	1	1	✓
8	1	0	0	0	✓
9	1	0	0	1	✓
10	1	0	1	1	✓
14	1	1	1	1	

Step: 2

Group	Minterms	A	B	C	D
0	0	0	0	0	0
1	1	0	0	0	1
	2	0	0	1	0
	8	1	0	0	0
2	5	0	0	0	1
	6	0	1	1	0
	9	1	0	0	1
	10	1	0	1	0
3	7	0	1	1	1
	14	1	1	1	0

Table: 3

Group	Minterms	A	B	C	D	
0	0,1	0	0	0	–	✓
	0,2	0	0	–	0	✓
	0,8	–	0	0	0	✓
1	1,5	0	–	0	1	
	1,9	–	0	0	1	✓
	2,6	0	–	1	0	✓
	2,10	0	–	1	0	✓
	8,9	1	0	0	–	✓
	8,10	1	0	–	0	✓
2	5,7	0	1	–	1	
	6,7	0	1	1	–	
	6,14	–	1	1	0	✓
	10,14	1	–	1	0	✓

Table: 4

Group	Minterms	A	B	C	D
0	0,1,8,9	–	0	0	–
	0,2,8,10	–	0	–	0
	0,8,1,9	–	0	0	–
	0,8,2,10	–	0	–	0
1	2,6,10,14	–	0	1	0
	2,10,6,14	–	0	1	0

P.I	Decimal	Minterms									
		0	1	2	5	6	7	8	9	10	14
$\overline{B}\ \overline{C}$	0,1,8,9	X	X					X	(X)		
$\overline{B}\ \overline{D}$	0,2,8,10	X		X				X		X	
$C\ \overline{D}$	2,6,10,14			X		X				X	(X)
$\overline{A}\ \overline{C}\ \overline{D}$	1,5		X		X						
$\overline{A}\ B\ D$	5,7				(X)		X				
$\overline{A}\ B\ C$	6,7					X	X				

$$Y = \overline{B}\ \overline{C} + C\ \overline{D} + \overline{A}\ BD$$

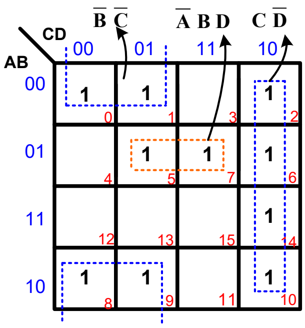

Figure 1.27

4. Find the minimum SOP for the function $f(A, B, C, D) = \sum m(0, 2, 3, 6, 7, 8, 10, 12, 13)$ using Quine McCluskey method.

Solution:

Table 1

Min terms	A	B	C	D	
0	0	0	0	0	✓
2	0	0	1	0	✓
3	0	0	1	1	✓
6	0	1	1	0	✓
7	0	1	1	1	✓
8	1	0	0	0	✓
10	1	0	1	1	✓
12	1	1	0	0	✓
13	1	1	0	1	

Step: 2

Group	Minterms	A	B	C	D	
0	0	0	0	0	0	✓
1	2	0	0	0	1	✓
	8	1	0	0	0	✓
2	3	0	0	0	1	✓
	6	0	1	1	0	✓
	10	1	0	1	0	✓
	12	1	1	0	0	✓
3	7	0	1	1	1	✓
	13	1	1	0	1	✓

Table: 3

Group	Minterms	A	B	C	D	
0	0,2	0	0	–	0	✓
	0,8	–	0	0	0	✓
1	2,3	0	0	1	–	✓
	2,6	0	–	1	0	✓
	2,10	–	0	1	0	✓
	8,10	1	0	–	0	✓
	8,12	1	–	0	0	
2	3,7	0	–	1	1	✓
	6,7	0	1	1	–	✓
	12,13	1	1	0	–	

Table: 4

Group	Minterms	A	B	C	D
0	0,2,8,10	–	0	–	0
1	2,3,6,7	0	1	1	0
	8,12	1	–	0	0
	12,13	1	1	0	–

P.I	Decimal	Minterms								
		0	2	3	6	7	8	10	12	13
$\overline{B}\ \overline{D}$	0,2,8,10	(X)	X				X	(X)		
$\overline{A}\ C$	2,3,6,7		X	(X)	(X)	(X)				
$A\ C\ \overline{D}$	8,12						X		X	
$A\ B\ \overline{C}$	12,13								X	(X)

$$Y = \overline{B}\ \overline{D} + \overline{A}\ C + A\ B\ \overline{C}$$

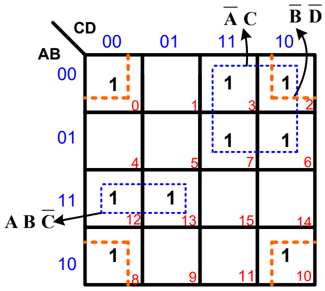

Figure 1.28

1.4 Simplification of incompletely specified functions

Simplification of incompletely specified functions

1. Find the minimum SOP for the function $f(A,B,C,D) = \sum m(0,3,5,6,7,10,12,13) + \sum d(2,9,15)$ using Quine McCluskey method.

Solution:

Determination of prime implicants
Step 1:

- Given SOP contains maximum number of 15, hence requires 4 digit binary number to represent the given SOP.
- Four variables A, B, C,D are used to represent in terms of binary.
- Given min terms are 0, 3, 5, 6, 7, 10, 12, 13. The details of the binary representation is as shown in Table 1.
- Given Don't cares are 2, 9, 15. The details of the binary representation is as shown in Table 2.

Table 1 for minterms

Min terms	A	B	C	D
0	0	0	0	0
3	0	0	1	1
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
10	1	0	1	0
12	1	0	1	0
13	1	1	0	1

Table 2 for don't cares

Min terms	A	B	C	D
2	0	0	1	0
9	1	0	0	1
15	1	1	1	1

Step: 2

- Club the minterms and don't cares and grouped together based on the content of number of 1s
- Minterms 0 is grouped into group no 0.
- Minterms 2 contains 1 1s, and is grouped into group no 1.
- Minterms 3, 5, 6, 9, 10 and 12 contains 2 1s, and are grouped into group no 2.
- Minterms 7 and 13 contains 3 1s, and are grouped into group no 3.
- Minterm 15 contains 4 1s, and are grouped into group no 4.
- The details are as shown in Table 3

Table 3

Group	Minterms	A	B	C	D	
0	0	0	0	0	0	✓
1	2	0	0	1	0	✓
2	3	0	0	1	1	✓
	5	0	1	0	1	✓
	6	0	1	1	0	✓
	9	1	0	0	1	✓
	10	1	0	1	0	✓
	12	1	1	0	0	✓
3	7	0	1	1	1	✓
	13	1	1	0	1	✓
4	15	1	1	1	1	✓

Table 4

Group	Minterms	A	B	C	D	
0	0, 2	0	0	–	0	
1	2,3	0	0	1	–	✓
	2,6	0	–	1	0	✓
	2,10	–	0	1	0	
2	3,7	0	–	1	1	✓
	5,7	0	1	–	1	✓
	5,13	–	1	0	1	✓
	6,7	0	1	1	–	✓
	9,13	1	–	0	1	
	12,13	1	1	1	–	
	12	1	1	0	–	✓
3	7,15	–	1	1	1	✓
	13,15	1	1	–	1	✓

Table: 5

Group	Minterms	A	B	C	D
1	2,3,6,7	0	–	1	–
	2,6,3,7	0	–	1	–
2	5,7,13,15	–	1	–	1
	5,13,7,15	–	1	–	1

Table: 4

Group	Minterms	A	B	C	D
0	0,2	0	0	–	0
1	2,3,6,7	0	–	1	–
	2,10	–	0	1	0
2	5,7,13,15	–	1	–	1
	9,13	1	–	0	1
	12,13	1	1	0	–

Step: 5

Minterms (0,2) is having variable $\overline{A}\ \overline{B}\ \overline{D}$ (2,3,6,7) is having variables as 0 –1–, this is represented as $\overline{A}\ C$ (2,10) is having variables as –0 10 is represented as $\overline{B}\ C\ \overline{D}$ (5,7,13,15) is having variables as –1–1 is represented as BD, 9,13 is having variables as 1 – 0 1 is represented as $A\overline{C}\ D$ and similarly minterms (12,13) is having variables as 110 –, this is represented as $AB\overline{C}$.

Put all prime implicants in a table as shown in Table 5. Column 1 contains Prime implicants, column 2 corresponding decimal numbers and column given minterms. Put X mark wherever the minterms are covered.

The prime implicant chart

Table: 6

P.I	Decimal	Minterms							
		0	3	5	6	7	10	12	13
$\overline{A}\ \overline{B}\ \overline{D}$	0,2	X							
$\overline{A}\ C$	2,3,6,7		X		X	X			
$\overline{B}\ C\ \overline{D}$	2,10						X		
$B\ D$	5,7,13,15			X		X			X
$A\ \overline{C}\ D$	9,13								X
$AB\overline{C}$	12,13							X	X

Step 6: Identify the essential minterms, and hence essential prime implicants. 5, 7, 9,11 are the essential minterms.

Table: 7

P.I	Decimal	Minterms							
		0	3	5	6	7	10	12	13
$\overline{A}\ \overline{B}\ \overline{D}$	0,2	(X)							
$\overline{A}\ C$	2,3,6,7		(X)		X	X			
$\overline{B}\ C\ \overline{D}$	2,10						(X)		
$B\ D$	5,7,13,15			(X)		X			X
$A\ \overline{C}\ D$	9,13								X
$AB\overline{C}$	12,13							(X)	X

Step 7: Based on essential prime implicants write the expression.

$$Y = \overline{A}\ \overline{B}\ \overline{D} + \overline{A}\ C + \overline{B}\ C\ \overline{D} + BD + AB\overline{C}$$

Step 8: Result can also be verified by K map method. Given function is $f(A,B,C,D) = \sum m(0,3,5,6,7,10,12,13) + \sum d(2,9,15)$

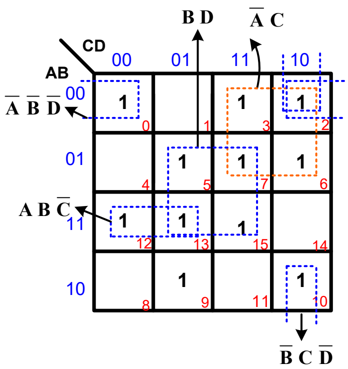

Figure 1.29

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2. Find the minimum SOP for the function $f(A,B,C,D) = \sum m(2,3,7,9,11,13) + \sum d(1,10,15)$ using Quine McCluskey method.

Solution:

Determination of prime implicants

Step 1:

- Given SOP contains maximum number of 15, hence requires 4 digit binary number to represent the given SOP.
- Four variables A, B, C,D are used to represent in terms of binary.
- Given min terms are 2, 3, 7, 9, 11, 13. The details of the binary representation is as shown in Table 1.
- Given Don't cares are 1, 10, 15. The details of the binary representation is as shown in Table 2.

Table 1 for minterms

Min terms	A	B	C	D
2	0	0	1	0
3	0	0	1	1
7	0	1	1	1
9	1	0	0	1
11	1	0	1	1
13	1	1	0	1

Table 2 for don't cares

Min terms	A	B	C	D
1	0	0	0	1
10	1	0	1	0
15	1	1	1	1

Step: 2

- Club the minterms and don't cares and grouped together based on the content of number of 1s
- Minterms 0 is grouped into group no 0.
- Minterms 2 contains 1 1s, and is grouped into group no 1.
- Minterms 3, 5, 6, 9, 10 and 12 contains 2 1s, and are grouped into group no 2.
- Minterms 7 and 13 contains 3 1s, and are grouped into group no 3.
- Minterm 15 contains 4 1s, and are grouped into group no 4.
- The details are as shown in Table 3

Table 3

Group	Minterms	A	B	C	D	
1	1	0	0	0	1	✓
	2	0	0	1	0	✓
2	3	0	0	1	1	✓
	9	1	0	0	1	✓
	10	1	0	1	0	✓
3	7	0	1	1	1	✓
	11	1	0	1	1	✓
	13	1	1	0	1	✓
4	15	1	1	1	1	✓

Table 4

Group	Minterms	A	B	C	D	
1	1,3	0	0	–	1	✓
	1,9	–	0	0	1	✓
	2,3	0	0	1	–	✓
	2,10	–	0	1	0	✓
2	3,7	0	–	1	1	✓
	3,11	–	0	1	1	✓
	9,11	1	0	–	1	✓
	9,13	1	–	0	1	✓
	10,11	1	0	1	–	✓
3	7,15	–	1	1	1	✓
	11,15	1	–	1	1	✓
	13,15	1	1	–	1	✓

Table: 5

Group	Minterms	A	B	C	D
1	1,3,9,11	–	0	–	1
	2,3,10,11	–	0	1	–
2	3,7,11,15	–	–	1	1
	9,11,13,15	1	–	–	1

Step: 5

Minterms (1,3,9,11) is having variable –0–1 $\overline{B}\ D$, (2,3,10,11) is having variables as –0 1–, this is represented as $\overline{B}\ C$ (3,7,11,15) is having variables as – – 11 is represented as CD (9,1,13,15) is having variables as 1– –1 is represented as AD

Put all prime implicants (excluding don't cares) in a table as shown in Table 6. Column 1 contains Prime implicants, column 2 corresponding decimal numbers and column given minterms. Put X mark wherever the minterms are covered.

The prime implicant chart

Table: 6

P.I	Decimal	Minterms					
		2	3	7	9	11	13
$\overline{B}\ D$	1,3,9,11		X		X	X	
$\overline{B}\ D$	2,3,10,11	X	X			X	
$C\ D$	3,7,11,15		X	X		X	
$A\ D$	9,11,13,15				X	X	X

Step 6: Identify the essential minterms, and hence essential prime implicants. 5, 7, 9,11 are the essential minterms.

Table: 6

P.I	Decimal	Minterms					
		2	3	7	9	11	13
$\overline{B}\ D$	1,3,9,11		X		X	X	
$\overline{B}\ C$	2,3,10,11	(X)	X			X	
$C\ D$	3,7,11,15		X	(X)		X	
$A\ D$	9,11,13,15				X	X	(X)

Step 7: Based on essential prime implicants write the expression.

$$Y = \overline{B}\ C + CD + AD$$

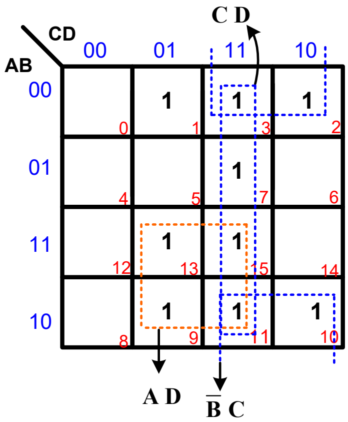

Figure 1.30