0.1 Module 2

Gram-schmidt-procedure

0.1.1 Model of digital communication system

- Conceptualized model of digital communication system is as shown in Figure ??
- Consider a source that emits one symbol for every T seconds, with the symbols belonging to an alphabet of M symbols which are denoted as $m_1, m_2, \ldots m_M$.
- Consider an example of quaternary signaling scheme with an alphabet consists of four possible symbols: 00,01,10,11.
- By assuming that, all M symbols of the alphabet are equally likely. Then a priori probability of the message source output as

$$p_i = P(m_i \ emitted) = \frac{1}{M} \quad for\ all\ i$$

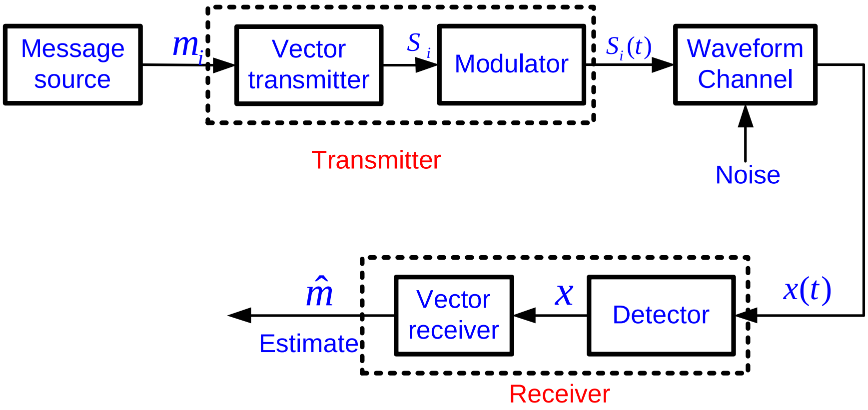


Figure 1: Model of Digital Communication System

- The output of the message source is presented to a vector transmitter producing vector of real number and source output is represented as

$$\mathrm{s_i} = \begin{bmatrix} s_{i1} \\ s_{i2} \\ . \\ . \\ . \\ s_{iN} \end{bmatrix}, i = 1, 2, \ldots, \mathrm{M}$$

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt, \left\{ \begin{array}{c} \mathrm{i} = 1, 2, \ldots, \mathrm{M} \\ \mathrm{j} = 1, 2, \ldots, \mathrm{N} \end{array} \right\}$$

- The signal $s_i(t)$ is necessarily of finite energy i.e., $E_i = \int_0^T s_i^2(t)dt \quad i = 1, 2, \ldots M$

The Channel is assumed to have two characteristics:

1. Channel is linear, with a bandwidth that is large enough to accommodate the transmission of the modulator output $s_i(t)$ without distortion.
2. The transmitted signal $s_i(t)$ is perturbed by an additive, zero-mean, stationary, white, Gaussian noise process $W(t)$. Such a channel is referred as AWGN (additive white Gaussian noise) channel

If $s_i(t)$ is transmitted, then the received signal $x(t)$ is represented as

$$x(t) = s_i(t) + w(t) \left\{ \begin{array}{c} 0 \leq t \leq T \\ 1 \leq i \leq M \end{array} \right\}$$

where $w(t)$ is the white noise process

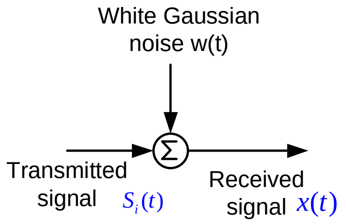


Figure 2: Model of AWGN Channel

0.1.2 Geometric Representation of Signals

- Real value energy signals $s_1(t), s_2(t), ..s_M(t)$, each of duration T sec

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \left\{ \begin{array}{l} 0 \leq \text{t} \leq \text{T} \\ \text{i} == 1, 2,...., \text{M} \end{array} \right\} (5.5)$$

- where the coefficients can be defined using:

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt, \left\{ \begin{array}{l} \text{i} = 1, 2,...., \text{M} \\ \text{j} = 1, 2,...., \text{M} \end{array} \right\}$$

- Real-valued basis functions

$$\int_0^{\text{T}} \phi_{\text{i}}(t)\phi_j(t)dt = \delta_{ij} = \left\{ \begin{array}{l} 1 \ \ \text{if} i = j \\ 0 \ \text{if} i \neq j \end{array} \right\} (5.7)$$

- i.e $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$, are orthogonal to each other in the interval $0 \leq t \leq T$. The set of coefficients can be viewed as a N-dimensional vector, denoted by S_i, where S_i has a one-to-one relationship with the transmitted signal $S_i(t)$

- Each signal in the set $s_1(t)$ is completely determined by the vector of its coefficients

$$\text{s}_{\text{i}} = \begin{bmatrix} s_{i1} \\ s_{i2} \\ . \\ . \\ . \\ s_{iN} \end{bmatrix}, i = 1, 2,...., \text{M}$$

For the N elements of vectors s_{i1}, s_{i2}, s_{iN} are the input $s_i(t)$ is generated based on Equation 6 it is represented as shown in Figure 3

Similarly for the given signals $s_i(t)$ $i = 1, 2, , , M$ as input and based on Equation 7 Figure **??** maybe used to generate coefficients

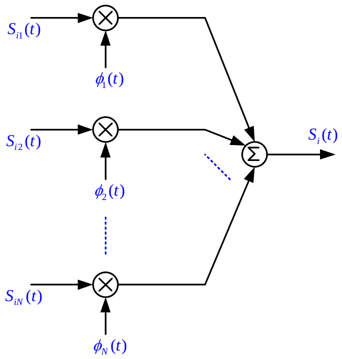


Figure 3: Scheme for generating signal $s_i(t)$

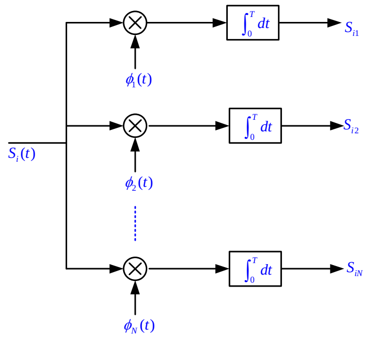


Figure 4: Scheme for generating set of coefficients $\{s_{ij}\}$

Relation between signal energy and its vector

The inner product or dot product of signal s_i with itself is

$$||s_i||^2 = s_i^T s_i = [s_{i1} s_{i2} \dots s_{iN}] \begin{bmatrix} s_{i1} \\ s_{i2} \\ . \\ . \\ . \\ s_{iN} \end{bmatrix} = s_{i1}^2, + s_{i2}^2 \dots + s_{in}^2] = \sum_{j=1}^{N} s_{ij}^2$$

$$\begin{aligned} E_i &= \int_0^T s_i^2(t)dt = \int_0^T \left[\sum_{j=1}^{N} s_{ij}(t)\phi_j(t)\right]\left[\sum_{k=1}^{N} s_{ik}(t)\phi_k(t)\right] dt \\ &= \sum_{j=1}^{N}\sum_{k=1}^{N} s_{ij}s_{ik} \int_0^T \phi_j(t)\phi_k(t) \end{aligned}$$

$$E_i = \sum_{j=1}^{N} s_{ij}^2 = \|s_i\|^2$$

- Assume we have a pair of signals: $s_i(t)$ and $s_j(t)$, each represented by its vector, Then:
- The Euclidean distance between two points represented by vectors (signal vectors) is equal to $||s_i - s_k||$ and the squared value is given by:

$$\|s_i - s_k\|^2 = \sum_{j=1}^{N} (s_{ij} - s_{kj})^2 = \int_0^T [s_i(t) - s_k(t)]^{1/2} dt$$

0.1.3 Gram-schmidt Orthogonalization procedure

- Transmitter takes the symbol (data) m_i $i = 1, 2 \dots M$ (digital message source output) and encodes it into a distinct signal $s_i(t)$.
- The set of M energy signals $\{si(t)\}$ can be represented by a linear combination of N orthonormal basis functions where $N \le M$.
- The given set of real valued energy signals $s_1(t), s_2(t), ..s_M(t)$, each of duration T sec are represented as

$$\begin{aligned} s_1(t) &= s_{11}\phi_1(t) + s_{12}\phi_2(t) \dots s_{1N}\phi_N(t) && (1) \\ s_2(t) &= s_{21}\phi_1(t) + s_{22}\phi_2(t) \dots s_{2N}\phi_N(t) && (2) \\ s_3(t) &= s_{31}\phi_1(t) + s_{32}\phi_2(t) \dots s_{3N}\phi_N(t) && (3) \\ s_M(t) &= s_{M1}\phi_1(t) + s_{M2}\phi_2(t) \dots s_{MN}\phi_N(t) && (4) \\ & && (5) \end{aligned}$$

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \left\{ \begin{array}{c} 0 \le \text{t} \le \text{T} \\ \text{i} = 1, 2, \dots, \text{M} \end{array} \right\} \quad (6)$$

where the coefficients are defined as:

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)dt, \left\{ \begin{array}{c} \text{i} = 1, 2, \dots, \text{M} \\ \text{j} = 1, 2, \dots, \text{N} \end{array} \right\} \quad (7)$$

Gram-Schmidt Orthogonalization Procedure

Step 1:

For Equation set all coefficients $s_{ij} = 0$ except s_{11}. Then

$$s_1(t) = s_{11}\phi_1(t) \qquad \therefore \phi_1(t) = \frac{s_1(t)}{s_{11}} \tag{8}$$

$$\int_0^T \phi^2{}_1(t)dt = 1 = \int_0^T \frac{s_1^2(t)}{s_{11}^2}dt \qquad \therefore s_{11} = \sqrt{\int_0^T s_1^2(t)dt} \tag{9}$$

s_{11} $\phi_1(t)$ are estimated.

Step 2:

Set all coefficients $s_{ij} = 0$ except s_{21} and s_{22} to zero. Then

$$s_2(t) = s_{21}\phi_1(t) + s_{22}\phi_2(t) \tag{10}$$

Multiply both sides by $\phi_1(t)$ and integrating

$$s_2(t)\phi_1(t) = s_{21}\phi_1(t)\phi_1(t) + s_{22}\phi_2(t)\phi_1(t) \tag{11}$$

$$s_{21} = \int_0^T s_2(t)\phi_1(t)dt \tag{12}$$

$$s_2(t) - s_{21}\phi_1(t) = s_{22}\phi_2(t) \tag{13}$$

Squaring and integrating

$$\int_0^T [s_2(t) - s_{21}\phi_1(t)]^2 dt = \int_0^T s_{22}^2\phi_2^2(t) = s_{22}^2 \tag{14}$$

$$s_{22} = \sqrt{\int_0^T [s_2(t) - s_{21}\phi_1(t)]^2 dt} \tag{15}$$

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{s_{22}} \tag{16}$$

Step 3:

Set all coefficients $s_{ij} = 0$ except s_{31},s_{32} and s_{33} to zero. Then

$$s_3(t) = s_{31}\phi_1(t) + s_{32}\phi_2(t) + s_{33}\phi_3(t) \tag{17}$$

Multiply both sides by $\phi_1(t)$ and integrating

$$s_3(t)\phi_1(t) = s_{31}\phi_1(t)\phi_1(t) + s_{32}\phi_2(t)\phi_1(t) + s_{33}\phi_3(t)\phi_1(t) \tag{18}$$

$$s_{31} = \int_0^T s_3(t)\phi_1(t)dt \;\; similarly \quad s_{32} = \int_0^T s_3(t)\phi_2(t)dt \tag{19}$$

$$s_{33}^2 = \int_0^T [s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)]^2 \, dt \tag{20}$$

$$s_{33} = \sqrt{\int_0^T [s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)]^2 \, dt} \tag{21}$$

$$\phi_3(t) = \frac{s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)}{s_{33}} \tag{22}$$

Continuing in the same steps to find N orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$, and estimating the coefficients s_{ij}. Based on N orthonormal basis functions and coefficients the signals $s_i(t)$ are expressed in linear combination of orthonormal basis functions.

0.2 Response of bank of correlators to noisy input

Received Signal X(t) is given by

$$X(t) = s_i(t) + W(t) \quad 0 \le t \le T \quad 1 \le i \le M$$

where W(t) is AWGN with Zero Mean and PSD $N_0/2$. Output of each correlator is a random variable defined by

$$X_j = \int_0^T X(t)\phi_j(t)dt = S_{ij} + w_j(t) = \int_0^T [s_i(t) + W(t)]\phi_j(t)dt = S_{ij} + w_j(t) \quad 1 \le j \le N$$

The first Component $S_{ij}(t)$ is deterministic quantity contributed by the transmitted signal $S_i(t)$, it is defined by

$$S_{ij} = \int_0^T S_i(t)\phi_j(t)dt$$

The second Component W_j is a random variable due to the presence of the noise at the input, it is defined by

$$W_j = \int_0^T W(t)\phi_j(t)dt$$

Mean and variance:

The noise process W(t) has zero mean. The mean value of he jth correlator output depends only on S_{ij} hence mean is

$$m_{X_j} = E[X_j] = E[S_{ij} + W_j] = S_{ij}E[W_j] = S_{ij}$$

Variance:

$$\sigma^2_{X_j} = Var[X_j] = E[(X_j - S_{ij})^2] = E[W_j^2]$$

$$\begin{aligned}
\sigma^2_{X_j} &= E\left[\int_0^T W(t)\phi_j(t)dt \int_0^T W(u)\phi_j(u)du\right] \\
&= E\left[\int_0^T \int_0^T \phi_j(t)\phi_j(u)W(t)W(u)dt\ du\right] \\
&= \int_0^T \int_0^T \phi_j(t)\phi_j(u)E\left[W(t)W(u)\right]dt\ du \\
&= \int_0^T \int_0^T \phi_j(t)\phi_j(u)R_w(t,u)dt\ du
\end{aligned}$$

where $R_w(t,u)$ is autocorrelation function of the noise process which is stationary and depends only on the time difference $(t-u)$ Noise process is is $W(t)$ is white with power spectral density $\frac{N_0}{2}$, which is expressed as

$$R_w(t,u) = \frac{N_0}{2}\delta(t-u)$$

where $\delta(t-u))$ is a Dirac delta function

$$\sigma^2_{X_j} = \frac{N_0}{2}\int_0^T \int_0^T \phi_j(t)\phi_j(u)\delta(t-u)dt\ du = \frac{N_0}{2}\int_0^T \phi_j^2(t)dt$$

$\phi_j(t)$ has unit energy i.e., $\phi_j^2(t) = 1$ Then Variance becomes

$$\sigma^2_{X_j} = \frac{N_0}{2}$$

The set $\phi_j(t)$ is orthonormal set, then the X_j are mutually uncorrelated and is given by

$$\begin{aligned}
Cov[X_j, X_k] &= E[(X_j - m_{X_j})(X_k - m_{X_j})] = E[(X_j - s_{ij})(X_k - s_{ij})] = E[W_j W_k] \\
&= E\left[\int_0^T W(t)\phi_j(t)dt \int_0^T W(u)\phi_k(u)du\right] = \int_0^T \int_0^T \phi_j(t)\phi_k(u)R_w(t,u)(u)dt\ du \\
&= \frac{N_0}{2}\int_0^T \int_0^T \phi_j(t)\phi_k(u)\delta(t-u)dt\ du = \frac{N_0}{2}\int_0^T \phi_j(t)\phi_k(u)dt \\
&= 0 \quad j \neq k
\end{aligned}$$

The correlator output is defined as N random variables

$$X = \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_N \end{bmatrix}$$

whose elements are independent Gaussian random variables with mean s_{ij} and variance $N_0/2$. The elements of the vector X are statistically independent, and can be expressed as the conditional probability density function of the vector X, given that the signal $s_i(t)$ was transmitted as the product of conditional probability density function of its individual elements and is written as

$$f_X(X|m_i) = \prod_{j=1}^{N} f_{X_j}(X_j|m_i)$$

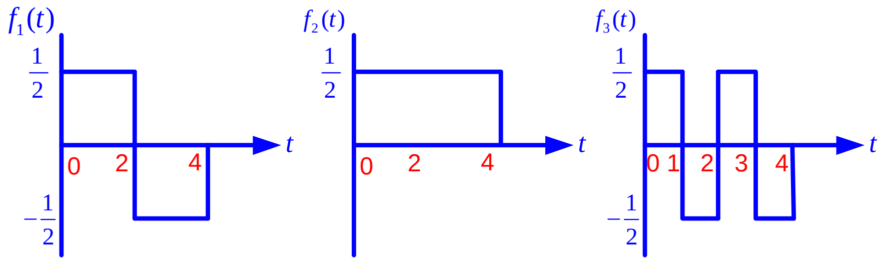
$$f_{X_j}(X_j|m_i) = \frac{1}{\sqrt{\pi N_0}} exp\left[-\frac{1}{N_0}(x_j - s_{ij})\right] \quad 1 \leq j \leq N \quad 1 \leq i \leq M$$

$$f_X(X|m_i) = (\pi N_0)^{-N_0/2}\left[-\frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{ij})^2\right] \quad 1 \leq i \leq M$$

0.3 Problems

0.3.1 Problems

Problem 1: Consider the three waveforms $f_n(t)$shown in Fig Show that these waveforms are orthonormal.



Solution:

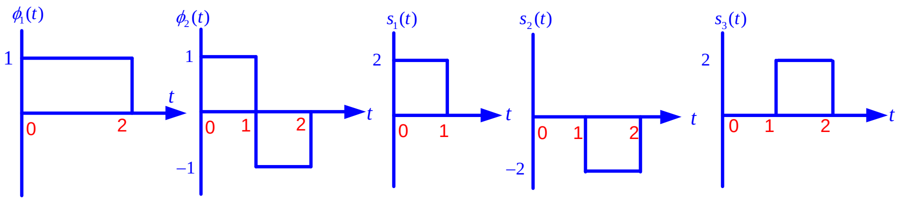
$$\begin{aligned}
C_{12} &= \int_{-\infty}^{\infty} f_1(t)f_2(t)dt = \int_0^4 f_1(t)f_2(t)dt \\
&= \int_0^2 f_1(t)f_2(t)dt + \int_2^4 f_1(t)f_2(t)dt \\
&= \int_0^2 \frac{1}{2}\frac{1}{2}dt + \int_2^4 \frac{1}{2} \times -\frac{1}{2}dt \\
&= \frac{1}{4}\int_0^2 dt - \frac{1}{4}\int_2^4 dt \\
&= \frac{1}{4} \times 2 - \frac{1}{4} \times (4-2) = 0
\end{aligned}$$

$$\begin{aligned}
C_{13} &= \int_0^4 f_1(t)f_3(t)dt \\
&= \int_0^1 \frac{1}{2}dt - \int_1^2 \frac{1}{4}dt - \int_2^3 \frac{1}{4}dt4 + \int_3^4 \frac{1}{4}dt \\
&= 0
\end{aligned}$$

$$\begin{aligned}
C_{23} &= \int_0^4 f_2(t)f_3(t)dt \\
&= \int_0^1 \frac{1}{2}dt - \int_1^2 \frac{1}{4}dt + \int_2^3 \frac{1}{4}dt - \int_3^4 \frac{1}{4}dt \\
&= 0
\end{aligned}$$

$$\int_{-\infty}^{\infty} |f_i(t)|^2 dt = 1 \quad i = 1, 2, 3$$

Problem 2: Check whether the signals $\phi_1(t)$ and $\phi_2(t)$ are orthogonal. Obtain corresponding orthonormal functions. Express the given signals $s_1(t)$ $s_2(t)$ and $s_3(t)$ in terms of $\phi_1(t)$ and $\phi_2(t)$



Solution:

$$\begin{aligned}
C_{12} &= \int_0^2 \phi_1(t)\phi_2(t)dt \\
&= \int_0^1 (1)(1)dt + \int_1^2 (1)(-1)dt = 0
\end{aligned}$$

$$\int_0^2 \phi_1^2(t)dt = \int_0^2 \phi_2^2(t)dt = 2$$

$$\int_0^2 \phi_i(t)\phi_j(t) = \begin{cases} 0 & i \neq j \\ 2 & i = j \end{cases}$$

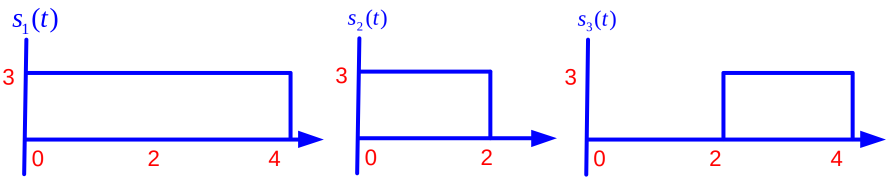
Corresponding orthonormal functions are

$$\frac{\phi_1(t)}{\sqrt{2}} \quad and \quad \frac{\phi_2(t)}{\sqrt{2}}$$

From the figure it is observed that

$$\begin{aligned} s_1(t) &= \phi_1(t) + \phi_2(t) \\ s_2(t) &= -\phi_1(t) + \phi_2(t) \\ s_3(t) &= \phi_1(t) - \phi_2(t) \end{aligned}$$

Problem 3: Three signals $s_1(t)$ $s_2(t)$ and $s_3(t)$ are as shown in Figure. Apply Gram-Schmidt procedure to obtain and orthonormal basis functions for the signals. Express the signals $s_1(t)$ $s_2(t)$ and $s_3(t)$ in terms of orthonormal basis functions. Also give the signal constellation diagram.



Solution:

$$s_1(t) = s_{11}\phi_1(t)$$

$$\int_0^4 s_1^2(t)dt = 9 \times 4 = 36 = s_{11}^2$$

$$\phi_1(t) = \frac{s_1(t)}{s_{11}} = \frac{s_1(t)}{6} = \frac{1}{2}$$

$$s_2(t) = s_{21}\phi_1(t) + s_{22}\phi_2$$

$$\begin{aligned} \int_0^T s_2 t\phi_1(t)dt &= \int_0^2 3 \times \frac{1}{2}dt \\ &= \frac{3}{2} \times 2 = 3 = s_{21} \end{aligned}$$

$$s_{22}\phi_2(t) = s_2(t) - s_{21}\phi_1(t)$$

$$\int_0^T s_{22}^2\phi_2^2(t)dt = s_{22}^2 = \int_0^T (s_2(t) - 3\phi_1(t))^2 dt$$

$$s_{22}^2 = \int_0^2 s_2^2(t)dt + \int_0^4 9\phi_1^2(t)dt - 2\int_0^2 3s_2(t)\phi_1(t)dt = 9$$

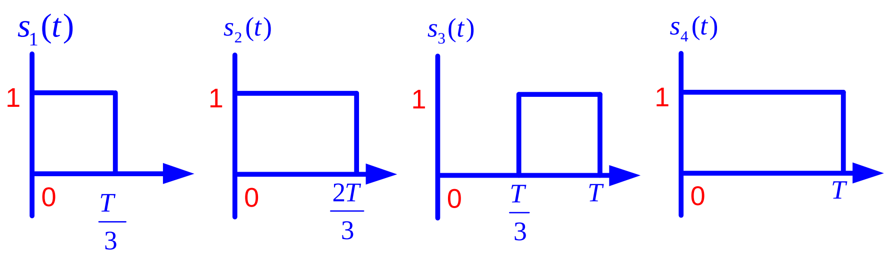
$$s_{22} = 3$$

$$\phi_2(t) = \frac{1}{3}[s_2(t) - s_{22}\phi_1(t)] = \frac{1}{3}[3 - 3\frac{1}{2}] = \frac{1}{2}$$

From the figure it is observed that $s_3(t) = s_1(t) - s_2(t)$ Expressing the signals in terms of basis functions

$$\begin{aligned} s_1(t) &= \phi_1(t) \times 6 \\ s_2(t) &= \phi_1(t) \times 6 + \phi_2(t) \times 3 \\ s_3(t) &= 3 \times \phi_1(t) - 3 \times \phi_2(t) \end{aligned}$$

Problem 4: Consider the signals $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$ are as shown in Figure. Apply Gram-Schmidt procedure to obtain and orthonormal basis functions for the signals.



Solution:

$$E_1 = \int_0^T s_1^2(t)dt = \int_0^{T/3} 1^2 dt = T/3$$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \frac{1}{\sqrt{T/3}} = \sqrt{3/T}$$

$$E_2 = \int_0^T s_2^2 tdt = \int_0^{2T/3} (1)2dt = \frac{2T}{3}$$

s_{21} is

$$\begin{aligned} s_{21} &= \int_0^T s_2 t(t)\phi_1(t)dt \\ &= \int_0^{T/3} 1\sqrt{3/T}dt = \sqrt{3/T} \end{aligned}$$

$$\begin{aligned} \phi_2(t) &= \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}} \\ &= \sqrt{3/T} \end{aligned}$$

$$\begin{aligned} s_{31} &= \int_0^T s_3 t(t)\phi_1(t)dt \\ &= 0 \end{aligned}$$

$$\begin{aligned} g_3(t) &= s_3 t - s_{31}\phi_1(t) - s_{32}(t)\phi_1(t) \\ &= 1 \end{aligned}$$

$$\begin{aligned} s_{32} &= \int_0^T s_3 t(t)\phi_2(t)dt \\ &= \int_{T/3}^{2T/3} (1)\sqrt{3/T}dt = \sqrt{T/3} \end{aligned}$$

$$\begin{aligned} \phi_3(t) &= \frac{g_3(t)}{\sqrt{\int_0^T g_3^2(t)}} dt \\ &= \sqrt{T/3} \end{aligned}$$

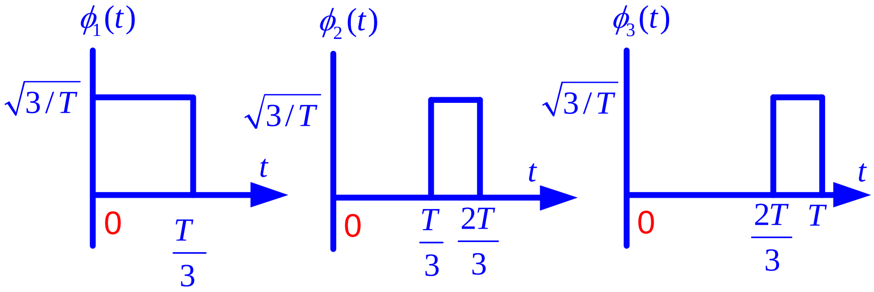


Figure 5: The set of orthonormal functions

0.4 Module 2

0.5 Optimum receivers using coherent detection

0.5.1 Maximum Likelihood Decoding

- In each time slot of duration T seconds, one of the M possible signals $s_1(t), s_2(t), \ldots, s_M(t)$ is transmitted with equal probability, $1/M$.
- For geometric signal representation, the signal $s_i(t), i = 1, 2, \ldots, M$, is applied to a bank of correlators with a common input and with an appropriate set of N orthonormal basis functions, as depicted in Figure 7.2b.
- The outputs of correlator contains the signal vector s_i.
- The received signal $s_i(t)$ is represented by a point in a Euclidean space of dimension$N \leq M$.
- This point is a transmitted signal point, or message point for short.
- The set of message points corresponding to the set of transmitted signals and is called as a message constellation.
- The received signal $x(t)$ consists of transmitted signal and additive noise $w(t)$.

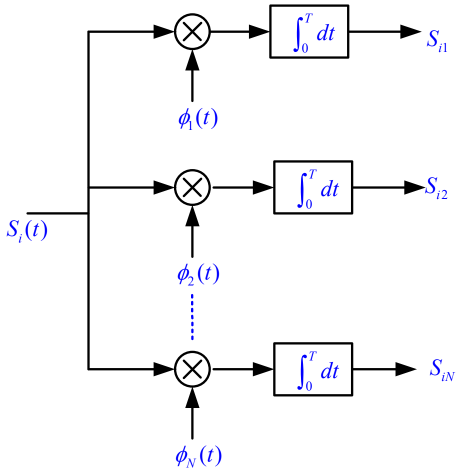


Figure 6: Detector or demodulator

- The noise vector w is completely characterized by the channel noise $w(t)$.
- The noise vector w is the portion of the noise $w(t)$ that will interfere with the detection process.
- The remaining portion of this noise, $w'(t)$, is tuned out by the bank of correlators.

- Based on the observation vector $\mathbf{x}$, the received signal $x(t)$ is represented in the Euclidean space and this point is referred as the received signal point.
- Due to the presence of noise, the received signal point wanders about the message point in a completely random fashion, and it may lie anywhere inside a Gaussian-distributed "cloud" centered on the message point.
- This is illustrated in Figure 8 for the case of a three-dimensional signal space.

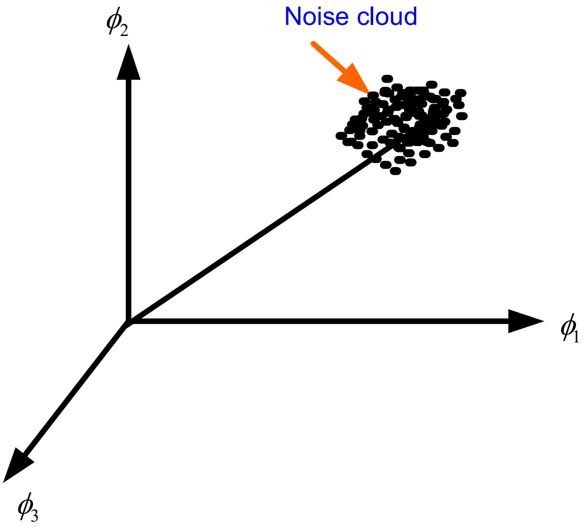


Figure 7

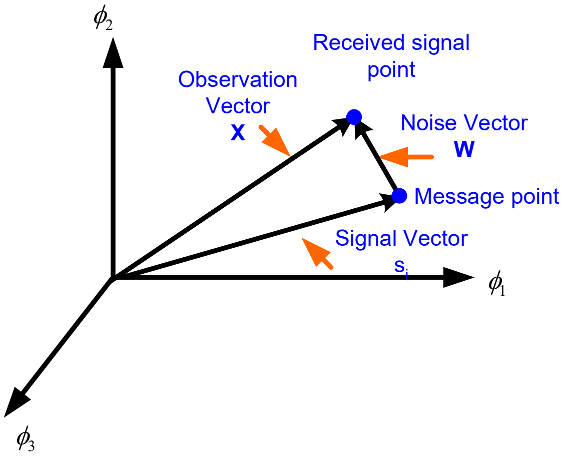


Figure 8

Given the observation vector $\mathbf{x}$, then decision is made such as.

$$\hat{m} = m_i \tag{1}$$

The probability of error in this decision is denoted as $Pe(m_i|\mathbf{x})$, is expressed as

$$Pe(m_i|\mathbf{x}) = 1 - \mathbb{P}(m_i\ sent|\mathbf{x})$$

Minimize the average probability of error in mapping each given observation vector $\mathbf{x}$ into a decision. Based on equation 1, the optimum decision rule is:

$$\mathbb{P}(m_i\ sent|\mathbf{x}) \geq \mathbb{P}(m_k\ sent|\mathbf{x}) \quad for\ all\ k \neq i \quad k = 1, 2...m \tag{2}$$

The decision rule described in 2 is referred to as the ***maximum a posteriori probability (MAP) rule***. Correspondingly, the system used to implement this rule is called a maximum a ***posteriori decoder.***

In terms of the prior probabilities of the transmitted signals and the likelihood functions, by Bayes rule the MAP rule is as follows:

$$Pe(m_i|\mathbf{x}) = \frac{\pi_k f_{\mathbf{x}}(\mathbf{x}|m_i)}{f_{\mathbf{x}}}$$

where $\pi_k$is the prior probability of transmitting symbol m_k, $f_{\mathbf{x}}(\mathbf{x}|m_i)$ is the conditional probability density function of the random observation vector $\mathbf{x}$ given the transmission of symbol m_k, and $f_{\mathbf{x}}(\mathbf{x})$ is the unconditional probability density function of $\mathbf{x}$.

0.5.2 Correlation receiver

The optimum receiver for an AWGN channel when the transmitted signals $s_1(t), s_2(t), \ldots s_M(t)$ s1(t) are equally likely is called a correlation receiver. It consists of two subsystems, which are Detector and Maximum-likelihood decoder.

1. Detector consists of M correlators supplied with a set of locally generated orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$. Bank of correlators operates on the received signal $x(t)$, $0 \leq t < T$, to produce the observation vector X. The details are as shown in Figure 9.

2. Maximum-likelihood decoder operates on the observation vector X to produce an estimate of the transmitted symbol m_i, $i = 1, 2, \ldots, M$, in such a way that the average probability of symbol error is minimized. The details are as shown in Figure 10.

In accordance with the maximum likelihood decision rule, the decoder multiplies the N elements of the observation vector x by the corresponding N elements of each of the M signal vectors $s_1, s_2, \ldots s_M$. Then, the resulting products are successively summed in accumulators to form the corresponding set of inner products $\{x^T s_k | k = 1, 2, \ldots, M\}$.

The inner products are corrected for the fact that the transmitted signal energies may be unequal. Finally, the largest one in the resulting set of numbers is selected, and an appropriate decision on the transmitted message is thereby made.

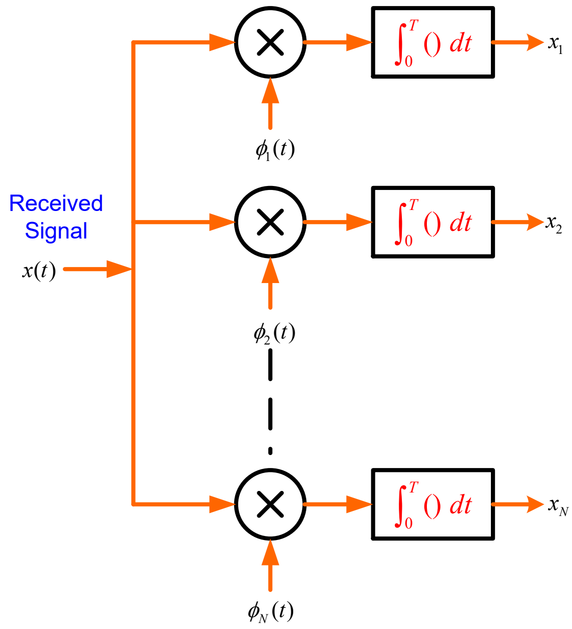


Figure 9: Detector or demodulator

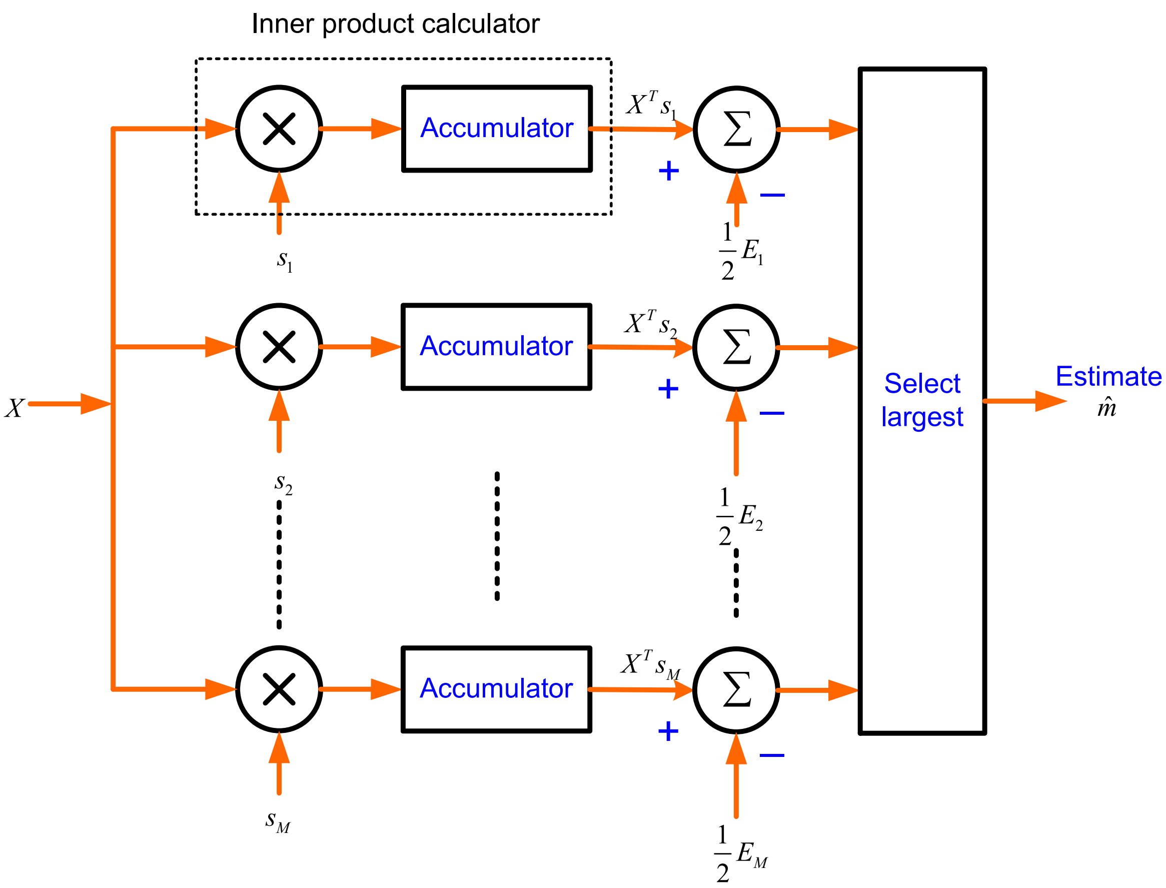


Figure 10: Signal transmission decoder.

0.5.3 Matched Filter Receiver

Matched Filter Receiver consists of a linear time-invariant filter with impulse response $h_j(t)$. With the received signal $x(t)$ operating as input, the resulting filter output is defined by the convolution integral

$$y_j(t) = \int_{-\infty}^{\infty} x(\tau)h_j(t-\tau)d\tau$$

The integral is evaluated over the duration of a transmitted symbol, $0 \leq t \leq T$. Replace the variable τ with t.

$$y_j(T) = \int_0^T x(t)h_j(T-t)dt$$

The output of the j^{th} correlator is represented as:

$$x_j = \int_0^T x(t)\phi_j(t)dt$$

$y_j(t) = x_j$ when the following condition is satisfied

$$h_j(T-t) = \phi_j(t) \qquad 0 \leq t \leq T \;\; and \; j = 1, 2, \ldots M$$

The condition on the desired impulse response of the filter as

$$h_j(t) = \phi_j(T-t) \qquad 0 \leq t \leq T \;\; and \; j = 1, 2, \ldots M$$

Given a pulse signal $\phi(t)$ occupying the interval $0 \leq t \leq T$, a linear time-invariant filter is said to be matched to the signal $\phi(t)$ if its impulse response h(t) satisfies the following condition

$$h(t) = \phi(T-t) \qquad 0 \leq t \leq T$$

A time-invariant filter defined in this way is called a matched filter. An optimum receiver using matched filters in place of correlators is called a matched-filter receiver. The details of the receiver is depicted in Figure 11.

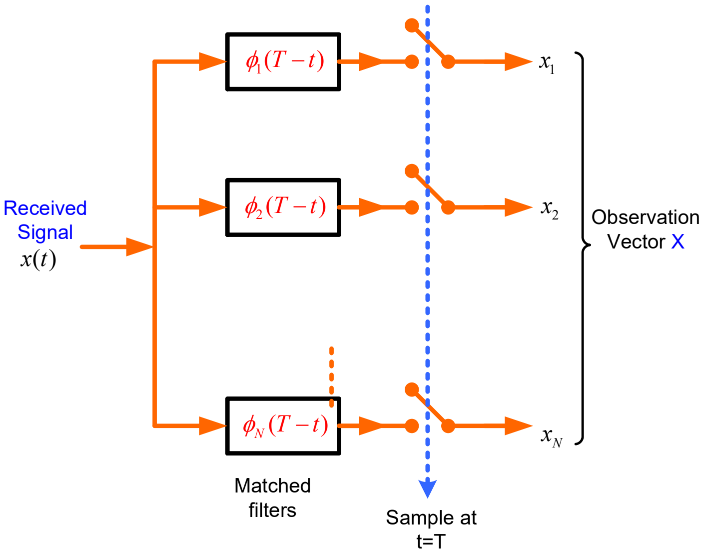


Figure 11: Matched filter