Chapter 1

Resonance

1.0.1 Introduction:

Most of the transmission lines, electrical circuits and communication networks are made up of network elements like resistor R,inductor, L and Capacitor C. The impedance of the inductor and capacitor depends on the frequency of the applied sinusoidal voltage to the network. As we vary the frequency of the supply the network impedance is purely resistive in which the impedance of the inductor is equal to the impedance of the capacitor. ***The phenomenon in which the the complex circuit behaves like a pure resistive is called resonance.*** The frequency at which resonance takes place is called the frequency of resonance ω_r (radians/sec) or f_r. (Hz)

Resonance may occur by varying frequency of the applied voltage to the complex network or by varying inductance, L or Capacitance, C, by keeping the frequency constant. Under resonance the following conditions occur in the circuit.

- The impedance of the inductance is equal to the impedance Capacitance.
- The phase of the current in the circuit is in phase with the applied voltage.
- Maximum current will flow in the circuit.
- The voltage across the capacitor or inductor is $I \times X_C$ or $I \times X_L$ where I is the current at resonance and X_C *or* X_L is the impedance of the circuit.
- The total power is dissipated in the resistor and the absorbed average power is maximum.

1.1 Series Resonance

Consider a series circuit consists of resistor, inductor and capacitor as shown in Figure 1.1.

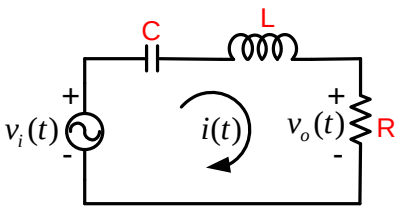

Figure 1.1: Series resonance circuit

The impedance of the circuit is

$$Z = R + j(X_L - X_C)$$

At resonance the imaginary part is zero

$$\begin{aligned} X_L - X_C &= 0 \\ \omega_r L - \frac{1}{\omega_r C} &= 0 \\ \omega_r L &= \frac{1}{\omega_r C} \\ \omega_r^2 &= \frac{1}{LC} \\ \omega_r &= \frac{1}{\sqrt{LC}} \quad radians/sec \end{aligned}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad Hz$$

The plot of the frequency response of series circuit is as shown in Figure 1.2. At resonant frequency ω_r the current is maximum.

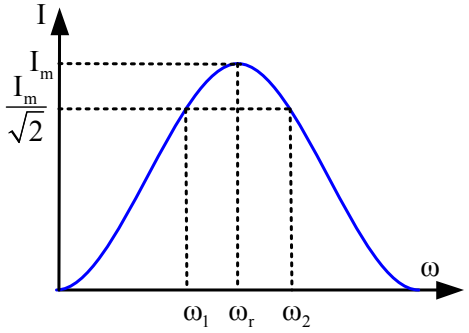

Figure 1.2: Frequency response of series circuit

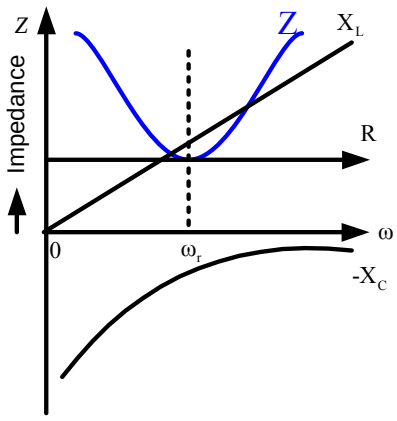

Figure 1.3: Frequency response of impedance of series circuit

At resonance frequency f_r $Z = R$ and current is I_m
At half power frequencies f_1 and f_2 the current is $\frac{I_m}{\sqrt{2}}$

$$Z = \sqrt{2}R$$

$$Z = R + jX_L - JX_C = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\begin{aligned}
\sqrt{R^2 + (X_L - X_C)^2} &= \sqrt{2}R \\
R^2 + (X_L - X_C)^2 &= 2R^2 \\
(X_L - X_C)^2 &= R^2 \\
X_L - X_C &= R
\end{aligned}$$

At frequency ω_1 the circuit impedance $X_C > X_L$

$$\begin{aligned}
X_C - X_L &= R \\
\frac{1}{\omega_1 C} - \omega_1 L &= R \\
\frac{1 - \omega_1^2 LC}{\omega_1 C} &= R \\
R\omega_1 C - 1 + \omega_1^2 LC &= 0 \\
\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} &= 0
\end{aligned}$$

$$a = 1, \quad b = \frac{R}{L}, \quad c = -\frac{1}{LC}$$

$$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Frequency is always positive

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

In terms of frequency f_1

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

At frequency ω_2 the circuit impedance $X_L > X_C$

$$\begin{aligned}
X_L - X_C &= R \\
\omega_2 L - \frac{1}{\omega_2 C} - &= R \\
\frac{\omega_2^2 LC - 1}{\omega_2 C} &= R \\
\omega_2^2 LC - R\omega_2 C - 1 &= 0 \\
\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} &= 0
\end{aligned}$$

$$a = 1, \quad b = -\frac{R}{L}, \quad c = -\frac{1}{LC}$$

$$\omega_2 = \frac{\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}}{2} = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Frequency is always positive

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

In terms of frequency f_2

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

Relation between ω_r , ω_1 and ω_2

$\omega_1 \times \omega_2 =$

$$= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \times \left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$
$$= \frac{1}{LC}$$

$$\begin{aligned} \omega_r &= \sqrt{\frac{1}{LC}} \\ \omega_r^2 &= \frac{1}{LC} = \omega_1.\omega_2 \\ \omega_r &= \sqrt{\omega_1.\omega_2} \\ & f_r = \sqrt{f_1 f_2} \end{aligned}$$

Relation between Bandwidth Quality factor

Bandwidth is $B = \omega_2 - \omega_1$

$$\begin{aligned} B &= \omega_2 - \omega_1 \\ &= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] - \left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ &= \frac{R}{L} \quad radians \end{aligned}$$

$$B = \frac{1}{2\pi}.\frac{R}{L} = \frac{R}{2\pi L} \; Hz$$
$$\omega_r = \sqrt{\frac{1}{LC}}$$
$$\omega_1 = -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}$$
$$\omega_2 = \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}$$

Quality factor Q

The quality factor Q is defined as the ratio of resonant frequency to the bandwidth

$$Q = \frac{\omega_r}{B} = \frac{\sqrt{\frac{1}{LC}}}{\frac{R}{L}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

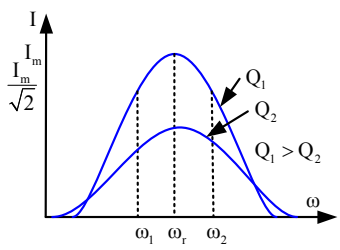

Figure 1.4: Plot of frequency verses Q

Resonance by varying circuit inductance

Consider a series RLC circuit as shown in Figure 1.5 is become resonant by varying inductance of the circuit.

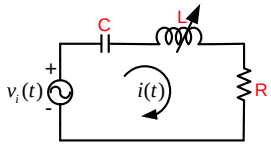

Figure 1.5: Resonance by varying inductance

Let L_1 is the inductance at ω

$$\begin{aligned} X_C - X_L &= R \\ \frac{1}{\omega C} - \omega L_1 &= R \\ L_1 &= \frac{1}{\omega^2 C} - \frac{R}{\omega} \end{aligned}$$

Let L_2 is the inductance at ω

$$\begin{aligned} X_L - X_C &= R \\ \omega L_2 - \frac{1}{\omega C} &= R \\ L_2 &= \frac{1}{\omega^2 C} + \frac{R}{\omega} \end{aligned}$$

Resonance by varying circuit capacitance

Consider a series RLC circuit as shown in Figure **??** is become resonant by varying capacitance of the circuit.

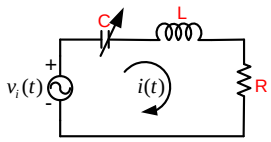

Figure 1.6: Resonance by varying capacitance

Let C_1 is the capacitance at ω_1

$$\begin{aligned} X_C - X_L &= R \Rightarrow \frac{1}{\omega_1 C_1} - \omega_1 L = R \\ \frac{1}{\omega_1 C_1} &= R + \omega_1 L \\ C_1 &= \frac{1}{\omega_1^2 L + \omega_1 R} \end{aligned}$$

Let C_2 is the capacitance at ω_2

$$X_L - X_C = R \Rightarrow \omega_2 L - \frac{1}{\omega_2 C_2} = R$$
$$\frac{1}{\omega_2 C_2} = \omega_2 L - R$$
$$C_2 = \frac{1}{\omega_2^2 L - \omega_2 R}$$

Table 1.1: Important Formulae

Parameter	Formula
At resonance	$Z = R, \quad X_L = X_C \;\; Current\ I_r = \frac{E}{R}$
Resonance	$\omega_r = \frac{1}{\sqrt{LC}} \qquad f_r = \frac{1}{2\pi\sqrt{LC}}$
Half power frequency	$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad f_1 = \frac{\omega_1}{2\pi}$ $\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad f_2 = \frac{\omega_2}{2\pi}$
Half power frequency	$\omega_1 = \frac{-B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2} \qquad f_1 = \frac{\omega_1}{2\pi}$ $\omega_2 = \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2} \qquad f_2 = \frac{\omega_2}{2\pi}$
Bandwidth	$B = \omega_2 - \omega_1 = \frac{R}{L}$ Radians $B = f_2 - f_1 = \frac{R}{2\pi L}$ Hz
Quality factor	$Q = \frac{\omega_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$
$\omega_r\ \omega_1\ \omega_2$	$\omega_r = \sqrt{\omega_1\omega_2} \quad OR\ f_r = \sqrt{f_1 f_2}$
Voltage across capacitor/inductor	$V_{Lr} = V_{Cr} = IX_{Lr}$
Value of inductor at f_1, f_2	$L_1 = \frac{1}{\omega^2 C} - \frac{R}{\omega},\ L_2 = \frac{1}{\omega^2 C} + \frac{R}{\omega}$
Value of capacitor at f_1, f_2	$C_1 = \frac{1}{\omega^2 L} - \frac{R}{\omega},\ C_2 = \frac{1}{\omega^2 L} + \frac{R}{\omega}$

Selectivity: is property of circuit in which the circuit is allowed to select a band of frequencies between f_1 and f_2.

1: For the circuit shown in Figure Find (a) The resonant and half power frequencies (b) Calculate the quality factor and bandwidth (c) Determine the amplitude of the current at $\omega_o, \omega_1, \omega_2$

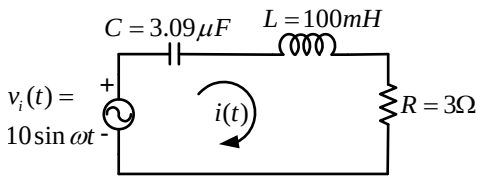

Figure 1.7: Example 1

$LC = 100 \times 10^{-3} \times 3.09 \times 10^{-6} = 3.09 \times 10^{-7}$

The resonant frequency ω_o is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.09 \times 10^{-7}}} = 1800\ rad/s$$

The half power frequency ω_1, ω_2 is

$$\frac{R}{2L} = \frac{3}{2 \times 100 \times 10^{-3}} = 15$$

$$\begin{aligned}
\omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\
&= -15 + \sqrt{225 + \frac{1}{3.09 \times 10^{-7}}} \\
&= -15 + \sqrt{225 + 3.236 \times 10^6} \\
&= -15 + 1798.96 = 1784\ rad/s \\
\omega_2 &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\
&= 15 + 1798.96 = 1814\ rad/s
\end{aligned}$$

Frequency in Hz is

$$\begin{aligned}
f_1 &= \frac{\omega_1}{2\pi} = \frac{1784}{2\pi} = 284\ Hz \\
f_2 &= \frac{\omega_2}{2\pi} = \frac{1814}{2\pi} = 289\ Hz
\end{aligned}$$

Bandwidth B is

$$B = \omega_2 - \omega_1 = 1814 - 1784 = 30\ rad/s$$

Also B is

$$B = \frac{R}{L} = \frac{3}{100 \times 10^{-3}} = 30\ rad/s$$

Quality factor Q is

$$Q = \frac{\omega_o}{B} = \frac{1800}{30} = 60$$

The amplitude of the current at ω_o is

$$I = \frac{V}{R} = \frac{10}{3} = 3.33A$$

The amplitude of the current at ω_1, ω_2 is

$$I = \frac{V}{\sqrt{2}R} = \frac{10}{\sqrt{2} \times 3} = 2.36A$$

2: For the circuit shown in Figure find the resonant frequency, quality factor and bandwidth for the circuit. Determine the change in Q and the bandwidth if R is changed from $R = 2\ \Omega$ to $R = 0.4\ \Omega$

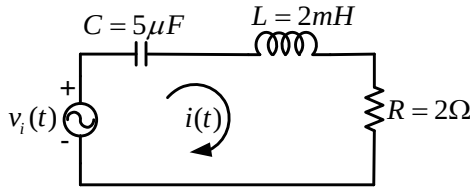

Figure 1.8: Example

$LC = 2 \times 10^{-3} \times 5 \times 10^{-6} = 10 \times 10^{-9}$

The resonant frequency ω_o is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 10^{-9}}} = 10 \times 10^3\ rad/s$$

B is

$$B = \frac{R}{L} = \frac{2}{2 \times 10^{-3}} = 1000\ rad/s$$

Quality factor Q is

$$Q = \frac{\omega_o}{B} = \frac{10 \times 10^3}{1000} = 10$$

When $R = 0.4\Omega$ B is

$$B = \frac{R}{L} = \frac{0.4}{2 \times 10^{-3}} = 200\ rad/s$$

Quality factor Q is

$$Q = \frac{\omega_o}{B} = \frac{10 \times 10^3}{200} = 50$$

3: For the circuit shown in Figure find the following (a) The resonant frequency f_o (b) Quality factor Q (c) f_{c1}, f_{c2}(d) Bandwidth B

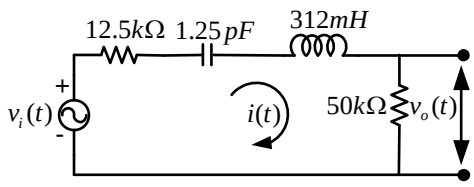

Figure 1.9: Example

$LC = 312 \times 10^{-3} \times 1.25 \times 10^{-12} = 0.39 \times 10^{-12}$

The resonant frequency ω_o is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.39 \times 10^{-12}}} = 1.6 \times 10^6 \ rad/s$$

$$f_o = \frac{\omega_0}{2\pi} = \frac{1.6 \times 10^6}{2\pi} = 254 \times 10^3 \ Hz$$

B is

$$B = \frac{R}{L} = \frac{62.5 \times 10^3}{312 \times 10^{-3}} = 200 \times 10^3$$

Quality factor Q is

$$Q = \frac{\omega_o}{B} = \frac{1.6 \times 10^6}{200 \times 10^3} = 8$$

The half power frequency ω_1, ω_1 is

$$\frac{R}{2L} = \frac{62.5 \times 10^3}{2 \times 312 \times 10^{-3}} = 1 \times 10^5$$

$$\begin{aligned}
\omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\
\omega_1 &= -1 \times 10^5 + \sqrt{(1 \times 10^5)^2 + \frac{1}{0.39 \times 10^{-12}}} \\
&= -1 \times 10^5 + \sqrt{1 \times 10^{10} + 2.5641 \times 10^{12}} \\
&= -1 \times 10^5 + 1.6 \times 10^6 = 1.5 \times 10^6 \ rad/s \\
\omega_2 &= \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}} \\
\omega_2 &= 1 \times 10^5 + 1.6 \times 10^6 = 1.7 \times 10^6 \ rad/s
\end{aligned}$$

Frequency in Hz is

$$\begin{aligned}
f_{c1} &= \frac{\omega_1}{2\pi} = \frac{1.5 \times 10^6}{2\pi} = 238.73 \times 10^3 \ Hz \\
f_{c2} &= \frac{\omega_2}{2\pi} = \frac{260 \times 10^3}{2\pi} = 271.56 \times 10^3 \ Hz
\end{aligned}$$

Bandwidth B is

$$B = \omega_2 - \omega_1 = 1.7 \times 10^6 - 1.5 \times 10^6 = 200 \times 10^3 \ rad/s$$

Bandwidth B in Hz is

$$B = \frac{200 \times 10^3}{2 \times \pi} = 31.83 \times 10^3 \ Hz$$

4: A variable frequency voltage source drives the network shown in Figure. Find the resonant frequency f_o, Quality factor Q, Bandwidth B and the average power dissipated by the network at resonance

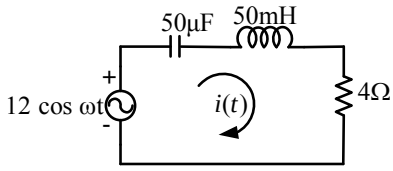

Figure 1.10: Example

$LC = 50 \times 10^{-3} \times 5 \times 10^{-6} = 2.5 \times 10^{-6}$

The resonant frequency ω_o is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.5 \times 10^{-6}}} = 632.45 \ rad/s$$

$$f_o = \frac{\omega_0}{2\pi} = \frac{632.45}{2\pi} = 100 \ Hz$$

B is

$$B = \frac{R}{L} = \frac{4}{50 \times 10^{-3}} = 80$$

Quality factor Q is

$$Q = \frac{\omega_o}{B} = \frac{632.45}{80} = 7.9$$

The average power dissipated at resonance

$$\begin{aligned}
P &= \frac{1}{2} \times \frac{V^2}{R} \\
P &= \frac{1}{2} \times \frac{12^2}{4} = 18 \ Watts
\end{aligned}$$

5: For the circuit shown in Figure the maximum amplitude of current is 10A, circuit quality factor Q is 100 and L=$0.1H$. If the applied voltage is 100 V find the value of capacitance

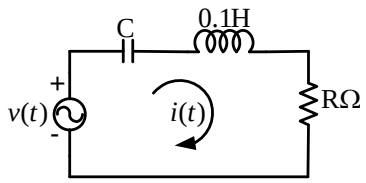

Figure 1.11: Example

Solution:

The maximum value of current flows in a circuit when the circuit is in resonance and the impedance of the circuit is pure resistor and its value is

$$R = \frac{V}{I_m} = \frac{100}{10} = 10 \ \Omega$$

The relation between Q and Bandwidth B is

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow \sqrt{\frac{L}{C}} = Q \times R$$

$$\frac{L}{C} = (Q \times R)^2 = (100 \times 10)^2 = 1 \times 10^6$$

$$C = \frac{L}{1 \times 10^6} = \frac{0.1}{1 \times 10^6} = 0.1\mu F$$

6: The Q of a series circuit network is 10. The maximum amplitude of current at resonance is 1 A when applied voltage is 10 V. If L=$0.1H$ find the value of capacitance

Solution:

The maximum value of current flows in a circuit when the circuit is in resonance and the impedance of the circuit is pure resistor and its value is

$$R = \frac{V}{I_m} = \frac{10}{1} = 10\ \Omega$$

The relation between Q and Bandwidth B is

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow \sqrt{\frac{L}{C}} = Q \times R$$

$$\frac{L}{C} = (Q \times R)^2 = (10 \times 10)^2 = 10 \times 10^3$$

$$C = \frac{L}{10 \times 10^3} = \frac{0.1}{10 \times 10^3} = 10\mu F$$

7: A coil of inductance 9H and resistance 50 Ω in series with a capacitor is supplied at constant voltage from a variable frequency source. If the maximum current is 1A at 75 Hz, find the frequency when the current is 0.5A

Solution:

8: Design a series resonant circuit to have $\omega_r = 2500\ rad/sec$ $Z(\omega_r) = 100\ \Omega$ and $B = 500\ rad/sec$

Solution:

$$B = \omega_r - \omega_1 = \frac{R}{L}$$

$$L = \frac{R}{B} = \frac{100}{500} = 0.2H$$

The resonant frequency ω_r is

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\sqrt{LC} = \frac{1}{\omega_r} = \frac{1}{2500} = 4 \times 10^{-4}$$

$$LC = 1.6 \times 10^{-7}$$

$$C = \frac{1.6 \times 10^{-7}}{0.2 \times 10^{-3}} = 0.8 \times 10^{-6} = 0.8\mu F$$

$$R = 100\Omega\ L = 0.2H\ C = 0.8\mu F$$

9: A series resonant RLC circuit has resonant frequency of 80 K rad/sec and a quality factor of 8. Find the bandwidth, the upper cutoff frequency and lower cutoff frequency

Solution:

$$Q = \frac{\omega_r}{B}$$

$$B = \frac{\omega_r}{Q} = \frac{80 \times 10^3}{8} = 10 \times 10^3\ rad/sec$$

Bandwidth in Hertz

$$B = \frac{B}{2\pi} = \frac{10 \times 10^3}{2 \times \pi} = 1.59 \times 10^3\ Hz$$

$$B = \omega_2 - \omega_1 = \frac{R}{L}$$

$$\begin{aligned}
\omega_1 &= -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2} \\
&= \frac{-10 \times 10^3}{2} + \sqrt{\left(\frac{10 \times 10^3}{2}\right)^2 + (80 \times 10^3)^2} \\
&= -5 \times 10^3 + \sqrt{25 \times 10^6 + 6.4 \times 10^9} \\
&= -5 \times 10^3 + 80.156 \times 10^3 = 75156\ rad/sec \\
\omega_2 &= \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2} \\
&= 5 \times 10^3 + 80.156 \times 10^3 = 85156\ rad/sec
\end{aligned}$$

Frequency in Hertz

$$\begin{aligned}
f_1 &= \frac{\omega_1}{2\pi} = \frac{75156}{2 \times \pi} = 11.96 \times 10^3\ Hz \\
f_2 &= \frac{\omega_1}{2\pi} = \frac{85156}{2 \times \pi} = 13.55 \times 10^3\ Hz
\end{aligned}$$

10: A series RLC circuit has R=10 Ω, L=0.1H, and C=100 $\mu F$and is connected across 200 V, variable frequency source. Find (a) the resonant frequency (b)impedance at this frequency (c) the voltage across drop across

inductance and capacitance at this frequency (d) quality factor and (e) bandwidth.

Solution: (a) Resonant frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 100 \times 10^{-6}}} = 50.33Hz$$

(b)Impedance at resonance $Z = R = 10\Omega$

$$I = \frac{V}{R} = \frac{200}{10} = 20A$$

(c) Voltage across inductance/capacitance

$$X_L = X_C = 2\pi f_r \times L = 2\pi \times 50.33 \times 0.1 = 31.62\Omega$$

$$V_L = IX_L = 20 \times 31.62 = 632.46V$$

(d) Quality factor

$$Q = \frac{X_L}{R} = \frac{31.62}{10} = 3.162$$

(e) Bandwidth

$$B = \frac{R}{2\pi L} = \frac{10}{2 \times \pi \times 0.1} = 15.9\ Hz$$

11: A coil of resistance R=20 Ω and inductance L=0.2 H is connected in series with a capacitance across 230 V supply Find (a) the value of the capacitance for which resonance occurs at 100 Hz frequency (b) the current through and voltage across the capacitor (c) Q factor of the coil

Solution: (a) Resonant frequency

$$\begin{aligned} f_r &= \frac{1}{2\pi\sqrt{LC}} \\ \sqrt{LC} &= \frac{1}{2\pi f_r} = \frac{1}{2\pi \times 100} = 1.6 \times 10^{-3} \\ LC &= 2.53 \times 10^{-6} \\ C &= \frac{2.53 \times 10^{-6}}{0.2} = 12.66\mu F \end{aligned}$$

Impedance at resonance is $Z = R = 10\Omega$

(b) Current through and voltage across the capacitor

$$I = \frac{V}{R} = \frac{230}{20} = 11.5A$$

$$X_C = \frac{1}{2\pi f_r \times C} = \frac{1}{2\pi \times 100 \times 12.66 \times 10^{-6}} = 125.7\Omega$$

Voltage drop across capacitance

$$V_C = IX_C = 11.5 \times 125.7 = 1445.7V$$

(c) Quality factor

$$Q = \frac{X_L}{R} = \frac{2\pi f_r \times L}{R} = \frac{2\pi \times 100 \times 0.2}{20} = 6.28$$

12: An RLC series circuit has a resistance of R=10 Ω and a variable inductance. is connected in series with a capacitance across 230 V supply Find (a) the value of the inductance for which the voltage across the resistance is maximum. resonance occurs at 100 Hz frequency (b) Q factor of the coil (c) Voltage across R, L, C. The applied voltage is 230 V 50 Hz.

Solution: (a) Resonant frequency

$$\begin{aligned} f_r &= \frac{1}{2\pi\sqrt{LC}} \\ \sqrt{LC} &= \frac{1}{2\pi f_r} = \frac{1}{2\pi \times 50} = 3.18 \times 10^{-3} \\ LC &= 1 \times 10^{-5} \\ L &= \frac{1 \times 10^{-5}}{100 \times 10^{-6}} = 101\ \ mH \end{aligned}$$

(b) Quality factor

$$Q = \frac{X_L}{R} = \frac{2\pi f_r \times L}{R} = \frac{2\pi \times 100 \times 100 \times 10^{-3}}{10} = 3.14$$

Impedance at resonance is $Z = R = 10\Omega$

(c) Current through resistance is

$$I = \frac{V}{R} = \frac{230}{10} = 23A$$

Voltage drop across resistor is

$$V = IR = 23 \times 10 = 230V$$

Voltage drop across Inductor/capacitance is

$$X_L = 2\pi f_r \times L = 2\pi \times 50 \times 101 \times 10^{-3} = 31.714\Omega$$

$$X_L = X_C$$

$$V_L = V_C = IX_L = 23 \times 31.714 = 729.2V$$

1.2 Parallel Resonance

Consider a parallel circuit consists of resistor and inductor in one branch and capacitor in another branch which is as shown in Figure 1.27.

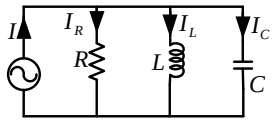

Figure 1.12: General parallel resonant circuit

The total admittance of the circuit is

$$Y = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

When the circuit is at resonance the imaginary part is zero

$$\omega_r C - \frac{1}{\omega_r L} = 0$$

$$\omega_r^2 = \frac{1}{LC}$$

$$\omega_r = \sqrt{\frac{1}{LC}}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Consider a parallel circuit consists of resistor and inductor in one branch and capacitor in another branch which is as shown in Figure 1.13.

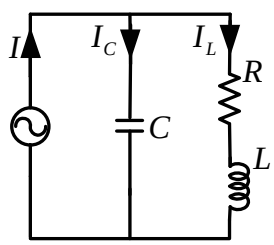

Figure 1.13: General parallel resonant circuit

The impedance of the inductor branch is

$$Z_L = R + j\omega L$$

The admittance of the inductor branch is

$$Y_L = \frac{1}{Z_L} = \frac{1}{R + j\omega L}$$

$$\begin{aligned} Y_L &= \frac{1}{Z_L} = \frac{1}{R + j\omega L} \\ &= \frac{1}{R + j\omega L} \times \frac{R - j\omega L}{R - j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} \end{aligned}$$

Similarly the impedance of the capacitor branch is

$$Z_C = \frac{1}{j\omega C}$$

The admittance of the capacitor branch is

$$Y_C = \frac{1}{Z_C} = j\omega C$$

Total admittance of the circuit is

$$Y = Y_L + Y_C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

Separating real and imaginary parts

$$Y = \frac{R}{R^2 + \omega^2 L^2} + j\left[\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right]$$

$$\omega_r C - \frac{\omega_r L}{R^2 + \omega_r^2 L^2} = 0$$

$$\omega_r C = \frac{\omega_r L}{R^2 + \omega_r^2 L^2}$$

$$R^2 + \omega_r^2 L^2 = \frac{\omega_r L}{\omega_r C} = \frac{L}{C}$$

$$\omega_r^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 = \frac{\frac{L}{C} - R^2}{L^2} = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Consider a parallel circuit consists of resistor and inductor in one branch and capacitor in another branch which is as shown in Figure 1.14.

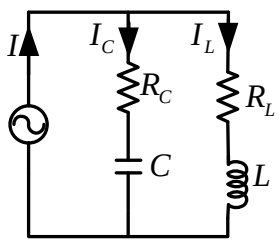

Figure 1.14: General parallel resonant circuit

The impedance of the inductor branch is

$$Z_L = R_L + j\omega L$$

The admittance of the inductor branch is

$$Y_L = \frac{1}{Z_L} = \frac{1}{R_L + j\omega L}$$

$$\begin{aligned} Y_L &= \frac{1}{Z_L} = \frac{1}{R_L + j\omega L} \\ &= \frac{1}{R_L + j\omega L} \times \frac{R_L - j\omega L}{R_L - j\omega L} = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} \end{aligned}$$

Similarly the impedance of the capacitor branch is

$$Z_C = R_C - j\frac{1}{\omega C}$$

The admittance of the inductor branch is

$$\begin{aligned} Y_C &= \frac{1}{Z_C} = \frac{1}{R_C - j\frac{1}{\omega C}} \\ &= \frac{1}{R_C - j\frac{1}{\omega C}} \times \frac{R_C + j\frac{1}{\omega C}}{R_C + j\frac{1}{\omega C}} = \frac{R_C + j\frac{1}{\omega C}}{R_c^2 + \frac{1}{\omega^2 C^2}} \end{aligned}$$

Total admittance of the circuit is

$$Y = Y_L + Y_C = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + j\frac{1}{\omega C}}{R_c^2 + \frac{1}{\omega^2 C^2}}$$

Separating real and imaginary parts

Real part is

$$\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_c^2 + \frac{1}{\omega^2 C^2}}$$

Imaginary part is

$$\frac{\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} - \frac{\omega L}{R_L^2 + \omega^2 L^2}$$

$$\frac{\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} - \frac{\omega L}{R_L^2 + \omega^2 L^2} = 0$$

$$\frac{\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} = \frac{\omega L}{R_L^2 + \omega^2 L^2}$$

$$\frac{1}{\omega_r C}\left(R_L^2 + \omega_r^2 L^2\right) = \omega_r L\left(R_C^2 + \frac{1}{\omega_r^2 C^2}\right)$$

$$\frac{1}{LC}\left(R_L^2 + \omega_r^2 L^2\right) = \omega_r^2\left(R_C^2 + \frac{1}{\omega_r^2 C^2}\right)$$

$$\frac{R_L^2}{LC} + \omega_r^2 \frac{L}{C} = \omega_r^2 R_C^2 + \frac{1}{C^2}$$

$$\omega_r^2\left(R_C^2 - \frac{L}{C}\right) = \frac{R_L^2}{LC} - \frac{1}{C^2}$$

$$\omega_r^2\left(R_C^2 - \frac{L}{C}\right) = \frac{1}{LC}\left(R_L^2 - \frac{L}{C}\right)$$

$$\omega_r^2 = \frac{1}{LC} \times \frac{\left(R_L^2 - \frac{L}{C}\right)}{\left(R_C^2 - \frac{L}{C}\right)}$$

$$\omega_r = \sqrt{\frac{1}{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$$

The frequency response of parallel graph is as shown in Figure 1.15. From the figure it is observed that the current is minimum at resonance. The parallel circuit is called as a rejector circuit. The circuit impedance is maximum at the resonance. The half power frequencies are at $\sqrt{2}I_r$.

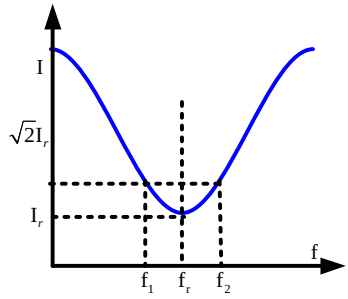

Figure 1.15: plot of parallel resonant circuit

Calculate the resonant frequency of the circuit shown in Figure 1.16

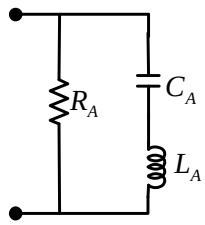

Figure 1.16: General parallel resonant circuit

$$Z_{LC} = jX_L + jX_C = j\omega L + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C}$$

$$Z_{LC} = \frac{j(\omega^2 LC - 1)}{\omega C}$$

$$\omega^2 LC = 1$$

$$\omega^2 = \frac{1}{\sqrt{LC}}$$

JAN-2018-CBCS 8 c) Show that a two-branch parallel circuit is resonant at all frequencies if $R_L = R_C = \sqrt{\frac{L}{C}}$ where R_L is resistor in the inductor branch and R_C is resistor in the capacitor branch.

Solution

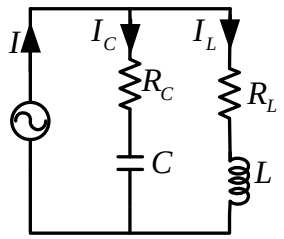

Figure 1.17: 2018-CBCS-Question Paper

$$\begin{aligned} Z_L &= R_L + jX_L \\ Y_L &= \frac{1}{R_L + jX_L} \\ Y_L &= \frac{1}{R_L + j\omega L} \\ Y_L &= \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} \end{aligned}$$

$$\begin{aligned} Z_C &= R_C - jX_C \\ Y_C &= \frac{1}{R_C - jX_C} \\ Y_C &= \frac{1}{R_C j\frac{1}{\omega C}} \\ Y_C &= \frac{R_C + j\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} \end{aligned}$$

$$Y = Y_L + Y_C = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + j\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}$$

Separating imaginary parts and equated to zero

$$\begin{aligned} \frac{\omega L}{R_L^2 + \omega^2 L^2} + \frac{\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} &= 0 \\ \omega L(1 + R_C^2\omega^2 C^2) &= \omega C(R_L^2 + \omega^2 L^2) \\ L(1 + R_C^2\omega^2 C^2) &= C(R_L^2 + \omega^2 L^2) \\ L + \omega^2 L R_C^2 C^2 &= C R_L^2 + \omega^2 C L^2 \\ \textit{Divide by } LC & \\ \frac{1}{C} + \omega^2 R_C^2 C &= \frac{R_L^2}{L} + \omega^2 L \\ \textit{Given that } R_L = R_C = \sqrt{\frac{L}{C}} & \\ R_L^2 = R_C^2 = \frac{L}{C} & \\ \frac{1}{C} + \omega^2 \frac{L}{C} C &= \frac{\frac{L}{C}}{L} + \omega^2 L \\ \frac{1}{C} + \omega^2 L &= \frac{1}{C} + \omega^2 L \end{aligned}$$

From the above equation it is observed that on both sides it contains ω. As we vary ω it varies on both sides and hence the circuit is resonant for all frequencies.

Selectivity: It is defined as the ratio of bandwidth to resonant frequency of a series resonant circuit.

$$\begin{aligned} \textit{Selectivity} &= \frac{B}{f_r} \\ &= \frac{f_2 - f_1}{f_r} \end{aligned}$$

1: For the circuit as shown in Figure 1.18 find the resonant frequency and the corresponding current in each branch.

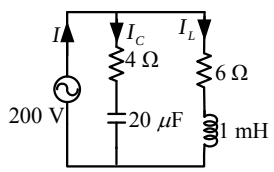

Figure 1.18: General parallel resonant circuit

Solution:

$$\frac{L}{C} = \frac{1 \times 10^{-3}}{20 \times 10^{-6}} = 50$$

$$\begin{aligned} f_r &= \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2-\frac{L}{C}}{R_C^2-\frac{L}{C}}} \\ &= \frac{1}{2\pi\sqrt{2\times10^{-8}}}\sqrt{\frac{6^2-50}{4^2-50}} \\ &= 1125.4\times0.641 = 721\ Hz \end{aligned}$$

$$X_L = 2\pi\times f\times L = 2\pi\times721\times1\times10^{-3} = 4.53\Omega$$

$$X_C = \frac{1}{2\pi\times f\times C} = \frac{1}{2\pi\times721\times20\times10^{-6}} = 11.03\Omega$$

$$\begin{aligned} I_L &= \frac{V}{Z_L} = \frac{200}{R+jX_L} = \frac{200}{6+j4.53} \\ &= \frac{200}{7.51,\angle37} = 26.63\angle-37 \end{aligned}$$

$$\begin{aligned} I_C &= \frac{V}{Z_C} = \frac{200}{R+jX_C} = \frac{200}{4-j11.03} \\ &= \frac{200}{11.73,\angle-70} = 17.05\angle70 \end{aligned}$$

2: Find the value of L for which the circuit as shown in Figure 1.19 is resonance at $\omega = 5000\ rad/sec$.

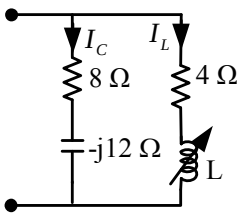

Figure 1.19: General parallel resonant circuit

Solution: The admittance of circuit is given by

$$\begin{aligned} Y &= \frac{1}{4+jX_L}+\frac{1}{8-j12} \\ &= \frac{4-jX_L}{4^2+X_L^2}+\frac{8+12}{8^2+12^2} \end{aligned}$$

At resonance

$$\begin{aligned} \frac{X_L}{4^2+X_L^2} &= \frac{12}{8^2+12^2} \\ 12(4^2+X_L^2) &= X_L(8^2+12^2) \\ 192+12X_L^2 &= 208X_L \\ 12X_L^2-208X_L+192 &= 0 \\ 3X_L^2-52X_L+48 &= 0 \end{aligned}$$

$$X_{L1} = \frac{52\pm\sqrt{52^2-(4\times3\times38)}}{2\times3}$$

$$X_{L1} = 16.36\ \Omega$$

$$X_{L2} = 0.978\ \Omega$$

$$L1 = \frac{X_{L1}}{\omega} = \frac{16.36}{5000} = 3.27mH$$

$$L2 = \frac{X_{L2}}{\omega} = \frac{0.978}{5000} = 0.196mH$$

3: Find the value of L for which the circuit as shown in Figure 1.20 is resonance at 1000 Hz.

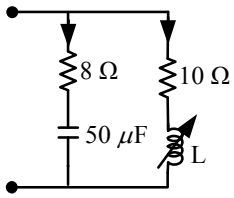

Figure 1.20: General parallel resonant circuit

Solution: The admittance of circuit is given by

$$X_C = \frac{1}{2\pi f_r C} = \frac{1}{2\pi1000\times50\times10^{-6}} = 3.18\ \Omega$$

$$\begin{aligned} Y &= \frac{1}{10+jX_L}+\frac{1}{8-j3.18} \\ &= \frac{10-jX_L}{10^2+X_L^2}+\frac{8+j3.18}{8^2+3.18^2} \end{aligned}$$

At resonance

$$\begin{aligned} \frac{X_L}{10^2+X_L^2} &= \frac{3.18}{8^2+3.18^2} \\ 3.18(100+X_L^2) &= X_L(64+10.11) \\ 318+3.18X_L^2 &= 74X_L \\ 3.18X_L^2-74X_L+318 &= 0 \end{aligned}$$

$$\begin{aligned} X_L &= \frac{74\pm\sqrt{74^2-(4\times3.18\times318)}}{2\times3.18} \\ &= \frac{74\pm\sqrt{5476-(4045)}}{6.36} \\ &= \frac{74\pm37.8}{6.36} \end{aligned}$$

$$X_{L1} = 17.57\ \Omega$$

$$X_{L2} = 5.69\ \Omega$$

$$L1 = \frac{X_L}{\omega} = \frac{17.57}{2\pi\times1000} = 2.797mH$$

$$L2 = \frac{X_L}{\omega} = \frac{5.69}{2\pi\times1000} = 0.9mH$$

4: Find the value of C for which the circuit as shown in Figure 1.28 is resonance at 750 Hz.

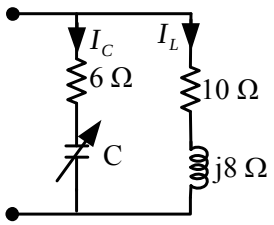

Figure 1.21: General parallel resonant circuit

Solution: The admittance of circuit is given by

$$\begin{aligned} Y &= \frac{1}{10+j8} + \frac{1}{6-jX_C} \\ &= \frac{10+j8}{10^2+8^2} + \frac{6-jX_C}{6^2+X_C^2} \\ &= \left(\frac{10}{164} + \frac{6}{36+X_C^2}\right) + j\left(\frac{X_C}{36+X_C^2} - \frac{8}{164}\right) \end{aligned}$$

At resonance

$$\begin{aligned} \frac{X_C}{36+X_C^2} - \frac{8}{164} &= 0 \\ 164X_C - 8X_C^2 - 288 &= 0 \\ 2X_C^2 - 41X_C + 78 &= 0 \end{aligned}$$

$$X_{C1} = \frac{41 \pm \sqrt{41^2 - (4\times 2\times 72)}}{2\times 2}$$

$$X_{C1} = 18.56\ \Omega$$

$$X_{C2} = 1.94\ \Omega$$

$$C_1 = \frac{1}{\omega X_{C1}} = \frac{1}{2\times\pi\times 750\times 18.56} = 11.44\ \mu F$$

$$C_2 = \frac{1}{\omega X_{C2}} = \frac{1}{2\times\pi\times 750\times 1.94} = 109.45\ \mu F$$

5: Find the value of C for which the circuit as shown in Figure 1.22 is resonance at 3000 rad/sec.

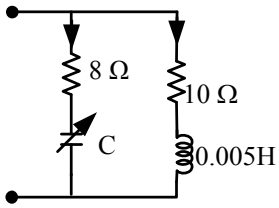

Figure 1.22: General parallel resonant circuit

Solution:

$$X_L = \omega \times L = 3000 \times 0.005 = 15\ \Omega$$

The admittance of circuit is given by

$$\begin{aligned} Y &= \frac{1}{10+j15} + \frac{1}{8-jX_C} \\ &= \frac{10-j8}{10^2+15^2} + \frac{8+jX_C}{8^2+X_C^2} \\ &= \left(\frac{10}{325} + \frac{8}{64+X_C^2}\right) + j\left(\frac{X_C}{64+X_C^2} - \frac{8}{325}\right) \end{aligned}$$

At resonance

$$\begin{aligned} \frac{X_C}{64+X_C^2} - \frac{8}{325} &= 0 \\ 325X_C &= 512 + 8X_C^2 \\ 8X_C^2 - 325X_C + 512 &= 0 \end{aligned}$$

$$\begin{aligned} 325X_C &= \frac{325 \pm \sqrt{325^2 - (4\times 8\times 512)}}{2\times 8} \\ &= \frac{325 \pm \sqrt{105625 - 16384}}{16} \\ &= \frac{325 \pm 298.7}{16} \end{aligned}$$

$$X_{C1} = 38.98\ \Omega$$

$$X_{C2} = 1.64\ \Omega$$

$$C_1 = \frac{1}{\omega C} = \frac{1}{3000\times 38.98} = 8.55\ \mu F$$

$$C_2 = \frac{1}{\omega C} = \frac{1}{3000\times 1.64} = 203\ \mu F$$

6: Find the value of R_L for which the circuit as shown in Figure 1.23 is resonant .

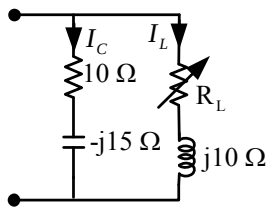

Figure 1.23: General parallel resonant circuit

Solution: The admittance of circuit is given by

$$\begin{aligned} Y &= \frac{1}{R_L+j10} + \frac{1}{10-j15} \\ &= \frac{R_L-j10}{R_L^2+100} + \frac{10+j15}{100+225} \\ &= \left(\frac{R_L}{R_L^2+100} + \frac{10}{325}\right) + j\left(\frac{15}{325} - \frac{10}{R_L^2+100}\right) \end{aligned}$$

At resonance

$$\begin{aligned}
\frac{15}{325} - \frac{10}{R_L^2 + 100} &= 0 \\
15(R_L^2 + 100) &= 3250 \\
R_L^2 &= 116.6 \\
R_L &= 10.8
\end{aligned}$$

7: Two impedances (10+j12) Ω and (20-j15)Ω are connected in parallel and this combination is connected in series with an impedance $(5 - jX_C)$ Ω. Find the value of X_C, for which resonance occurs.

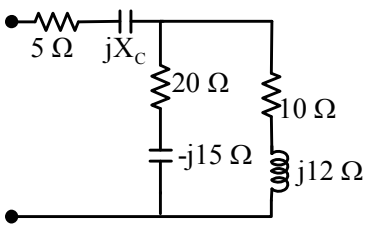

Figure 1.24: General parallel resonant circuit

Solution: The impedance of circuit is given by

$$\begin{aligned}
Z &= (5 - jX_C) + [(10 + j12)||(20 - j15)] \\
&= (5 - jX_C) + \frac{(10 + j12)(20 - j15)}{(10 + j12) + (20 - j15)} \\
&= (5 - jX_C) + \frac{(10 + j12)(20 - j15)}{(30 - j3)} \\
&= (5 - jX_C) + \frac{380 + j90}{30 - j3} \\
&= (5 - jX_C) + \frac{390.51\angle 13.325}{30.14\angle -5.71} \\
&= (5 - jX_C) + 13\angle 19.03 \\
&= (5 - jX_C) + 12.78 + j2.33 \\
&= (5 - jX_C) + 17.78 + j(2.33 - jX_C)
\end{aligned}$$

$$\begin{aligned}
Z &= (5 - jX_C) + [(10 + j12)||(20 - j15)] \\
&= (5 - jX_C) + \frac{(10 + j12)(20 - j15)}{(10 + j12) + (20 - j15)} \\
&= (5 - jX_C) + \frac{(10 + j12)(20 - j15)}{(30 - j3)} \\
&= (5 - jX_C) + \frac{380 + j90}{30 - j3} \\
&= (5 - jX_C) + \frac{390.51\angle 13.325}{30.14\angle -5.71} \\
&= (5 - jX_C) + 13\angle 19.03 \\
&= (5 - jX_C) + 12.78 + j2.33 \\
&= (5 - jX_C) + 17.78 + j(2.33 - jX_C)
\end{aligned}$$

At resonance imaginary part is zero

$$X_C = 2.33$$

8: (2015-JAN) Find the value of C for which the circuit as shown in Figure 1.25 is resonance at 50 Hz.

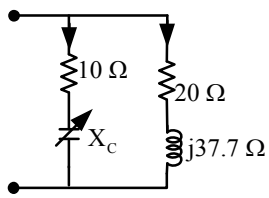

Figure 1.25: General parallel resonant circuit

Solution:

The admittance of circuit is given by

$$\begin{aligned}
Y &= \frac{1}{20 + j37.7} + \frac{1}{10 - jX_C} \\
&= \frac{20 - j37.7}{20^2 + 37.7^2} + \frac{10 + jX_C}{10^2 + X_C^2} \\
&= \frac{20 - j37.7}{1821.29} + \frac{10 + jX_C}{10^2 + X_C^2}
\end{aligned}$$

At resonance imaginary part is

$$\begin{aligned}
\frac{X_C}{100+X_C^2} - \frac{37.7}{1821.29} &= 0 \\
\frac{X_C}{100+X_C^2} &= \frac{37.7}{1821.29} \\
1821.29X_C &= 37.7X_C^2 + 3770 \\
37.7X_C^2 - 1821.29X_C + 3770 &= 0
\end{aligned}$$

$$\begin{aligned}
X_C &= \frac{1821.29 \pm \sqrt{1821.29^2 - (4 \times 37.7 \times 3770)}}{2 \times 37.7} \\
&= \frac{1821.29 \pm \sqrt{3317097 - 568516}}{75.4} \\
&= \frac{1821.29 \pm 1657.8}{75.4} \\
X_{C1} &= 46.14\ \Omega \\
X_{C2} &= 2.16\ \Omega
\end{aligned}$$

$$\omega = 2 \times \pi \times f = 2 \times \pi \times 50 = 314.16$$

$$C_1 = \frac{1}{\omega X_{C1}} = \frac{1}{314.16 \times 46.14} = 68.98\ \mu F$$

$$C_2 = \frac{1}{\omega X_{C2}} = \frac{1}{314.16 \times 2.16} = 1.47 \times 10^{-3}\ F$$

9: (2012-DEC) Determine the value of R_L R_C for which the circuit as shown in Figure 1.26 is resonates at all frequencies.

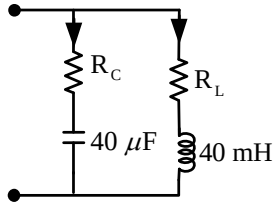

Figure 1.26: General parallel resonant circuit

Solution:

The admittance of circuit is given by

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}}$$

The circuit will resonate at any frequency if

$$R_L^2 = R_C^2 = \frac{L}{C}$$

$$R_L = R_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{40 \times 10^{-3}}{40 \times 10^{-6}}} = 31.6\ \Omega$$

JAN 2018 CBCS Q 7 a) What is resonance? Derive an expression for half power frequencies **8 Marks** JAN 2018 CBCS Q 7 b) Define the Q factor Selectivity and **3 Marks** JAN 2018 CBCS Q 7 c) **A series RLC circuit of $R = 4\ \Omega$ L=$1mH$ and C=$10\mu F$. Calculate the resonant frequency Q factor half power frequencies and bandwidth 8 Marks**

Solution:

i) $LC = 1 \times 10^{-3} \times 10 \times 10^{-6} = 10 \times 10^{-9}$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10 \times 10^{-9}}} = 1.59 \times 10^3 Hz$$

ii) $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{2\pi f_o L}{R}$

$$Q = \frac{2\pi f_o L}{R} = \frac{2\pi \times 1.59 \times 10^3 \times 1 \times 10^{-3}}{4} = 2.49$$

iii) Bandwidth BW

$$B = \frac{R}{L} = \frac{4}{1 \times 10^{-3}} = 4 \times 10^3 rad/sec$$

Bandwidth in Hz

$$B = \frac{B}{2\pi} = \frac{4 \times 10^3}{2\pi} = 636 Hz$$

iv) f_1 and f_2

$$\frac{R}{2L} = \frac{4}{2 \times 1 \times 10^{-3}} = 2 \times 10^3$$

$$\begin{aligned}
f_1 &= \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\
&= \frac{1}{2\pi}\left[-2 \times 10^3 + \sqrt{(2 \times 10^3)^2 + \frac{1}{10 \times 10^{-9}}}\right] \\
&= \frac{1}{2\pi}\left[-2 \times 10^3 + \sqrt{4 \times 10^6 + 0.1 \times 10^9}\right] \\
&= \frac{1}{2\pi}\left[-2 \times 10^3 + 10.19 \times 10^3\right] = 8.198 \times 10^3 Hz \\
f_2 &= \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\
&= \frac{1}{2\pi}\left[2 \times 10^3 + 10.19 \times 10^3\right] = 12.198 \times 10^3 Hz
\end{aligned}$$

JAN 2018 CBCS Q 8 a) Obtain an expression for resonant frequency in a parallel resonant circuit **6 Marks** JAN 2018 CBCS Q 8 b) Obtain an expression for resonant frequency in a parallel resonant circuit **6 Marks**

JAN 2018 CBCS Q 8 c) Find the value of R_L for which the circuit shown in Figure resonance condition **4 Marks**

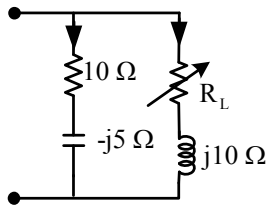

Figure 1.27: General parallel resonant circuit

JAN 2017 CBCS Q 7 a) Prove that $f_0 = \sqrt{f_1 f_2}$ where f_1 and f_2 are the two half power frequencies of a resonant circuits. **8 Marks**

JAN 2017 CBCS Q 7 b) **A series RLC circuit of $R = 100\ \Omega$ L=$0.01H$ and C=$0.01\mu F$ is connected across supply of 10 mV. Determine i) f_0 ii) Q factor, iii) BW iv) f_1 and f_2 iv) I_0 8 Marks**

Solution:

i) $LC = 0.01 \times 0.01 \times 10^{-6} = 1 \times 10^{-10}$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1 \times 10^{-10}}} = 15.9 \times 10^3 Hz$$

ii) $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{2\pi f_o L}{R}$

$$Q = \frac{2\pi f_o L}{R} = \frac{2\pi \times 15.9 \times 10^3 \times 0.01}{100} = 10$$

iii) Bandwidth BW

$$B = \frac{R}{L} = \frac{100}{0.01} = 10 \times 10^3 rad/sec$$

Bandwidth in Hz

$$B = \frac{B}{2\pi} = \frac{10 \times 10^3}{2\pi} = 1.59 \times 10^3 Hz$$

iv) f_1 and f_2

$$\frac{R}{2L} = \frac{100}{2 \times 0.01} = 5 \times 10^3$$

$$
\begin{aligned}
f_1 &= \frac{1}{2\pi}\left[-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}\right] \\
&= \frac{1}{2\pi}\left[-5\times10^3+\sqrt{(5\times10^3)^2+\frac{1}{1\times10^{-10}}}\right] \\
&= \frac{1}{2\pi}\left[-5\times10^3+\sqrt{25\times10^6+1\times10^{10}}\right] \\
&= \frac{1}{2\pi}\left[-5\times10^3+100\times10^3\right]=15.12\times10^3Hz \\
f_2 &= \frac{1}{2\pi}\left[\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}\right] \\
&= \frac{1}{2\pi}\left[5\times10^3+100\times10^3\right]=16.71\times10^3Hz
\end{aligned}
$$

iv) I_0

$$I_0=\frac{V}{R}=\frac{10\times10^{-3}}{100}=0.1\times10^{-3}Ampere$$

2017 JAN 5 a) **A series R-L-C circuit is fed with 50 V rms supply. At resonance the current through circuit is 25 A and the voltage across inductor is 1250 volts. If $C=4\mu F$ determine the values of R, L, Q, resonant frequency, bandwidth and half power frequencies. (12 M)**

Solution:

$$R=\frac{V}{I}=\frac{50}{25}=2\ \Omega$$

At resonance

$$X_L=X_C$$

Quality factor Q is

$$Q=\frac{V_L\ or\ V_C}{E}=\frac{1250}{50}=25$$

$$Q=\frac{1}{R}\sqrt{\frac{L}{C}}\quad\Rightarrow\quad\sqrt{\frac{L}{C}}=QR$$

$$L=(QR)^2C=(25\times2)^2\times4\times10^{-6}=0.01H$$

$$f_o=\frac{1}{2\pi\sqrt{LC}}$$

$$f_o=\frac{1}{2\pi\sqrt{0.01\times4\times10^{-6}}}=795.7\ Hz$$

iii) Bandwidth BW

$$B=\frac{R}{L}=\frac{2}{0.01}=200rad/sec$$

Bandwidth in Hz

$$B=\frac{B}{2\pi}=\frac{200}{2\pi}=31.83Hz$$

iv) f_1 and f_2

$$\frac{R}{2L}=\frac{2}{2\times0.01}=100\quad LC=0.01\times4\times10^{-6}=4\times10^{-8}$$

$$
\begin{aligned}
f_1 &= \frac{1}{2\pi}\left[-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}\right] \\
&= \frac{1}{2\pi}\left[-100+\sqrt{(100)^2+\frac{1}{4\times10^{-8}}}\right] \\
&= \frac{1}{2\pi}\left[-100+\sqrt{10\times10^3+25\times10^6}\right] \\
&= \frac{1}{2\pi}\left[-100+5\times10^3\right]=779.8\ Hz \\
f_2 &= \frac{1}{2\pi}\left[\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\frac{1}{LC}}\right] \\
&= \frac{1}{2\pi}\left[100+5\times10^3\right]=811.69Hz
\end{aligned}
$$

JULY 2016 Q 5 a) With respect to series resonant circuit define resonant frequency f_r and half power frequencies f_1 and f_2. Also show that the resonant frequency is equal to the geometric mean of half power frequencies **10 Marks**

JULY 2016 Q 5 B)) **A series circuit is energized by constant voltage and constant frequency supply. Resonance takes place due to variation of inductance and the supply frequency is 300 Hz. The capacitance in the circuit is $10\mu F$. Determine the value of resistance in the circuit if the quality factor is 5. Also find the value of the inductance at half power frequencies. 10 Marks**

Solution: $LC=0.01\times0.01\times10^{-6}=1\times10^{-10}$
The resonant frequency f_r is

$$f_r=\frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

$$\sqrt{LC}=\frac{1}{2\pi}\frac{1}{f_r}=\frac{1}{2\pi}\frac{1}{300}=5.3\times10^{-4}$$

$$LC=2.8\times10^{-7}$$

$$L=\frac{2.8\times10^{-7}}{10\times10^{-6}}=28.1\times10^{-3}Henry$$

$$Q=\frac{2\pi f_rL}{R}$$

$$R = \frac{2\pi f_r L}{Q} = \frac{2\pi \times 300 \times 0.0281}{5} = 10.6\ \Omega$$

The value of the inductance at half power frequencies is

$$\omega_r = 2\pi f_r = 2 \times \pi \times 300 = 1.88 \times 10^3$$

$$\begin{aligned} L_1 &= \frac{1}{\omega^2 C} - \frac{R}{\omega} \\ &= \frac{1}{(1.88 \times 10^3)^2.10 \times 10^{-6}} - \frac{10.6}{1.88 \times 10^3} \\ &= 0.02829 - 5.638 \times 10^{-3} = 22.65 \times 10^{-3}\ H \end{aligned}$$

$$\begin{aligned} L_2 &= \frac{1}{\omega^2 C} + \frac{R}{\omega} \\ &= \frac{1}{(1.88 \times 10^3)^2.10 \times 10^{-6}} + \frac{10.6}{1.88 \times 10^3} \\ &= 0.02829 + 5.638 \times 10^{-3} = 33.93 \times 10^{-3}\ H \end{aligned}$$

DEC/JAN 2015 5 a) **What is resonance? Derive an expression for cut-off frequencies 8 Marks**

DEC/JAN 2015 5 b) **Calculate the half power frequencies of series resonant circuit where the resonance frequency is 150 KHz and bandwidth is 75 KHz 4 Marks**

Solution:

Frequency in radians is

$$\omega_r = 2\pi f_r = 2 \times \pi \times 150 \times 10^3 = 942.5 \times 10^3\ radians$$

Bandwidth in radians is

$$B = 2\pi B = 2 \times \pi \times 75 \times 10^3 = 471.2 \times 10^3\ radians$$

$$\begin{aligned} \omega_1 &= -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2} \\ &= -235.6 \times 10^3 + \sqrt{55.5 \times 10^9 + 888.3 \times 10^9} \\ &= -235.6 \times 10^3 + 971.5 \times 10^3 \\ &= 736 \times 10^3\ Radians \\ \omega_2 &= \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2} \\ &= 235.6 \times 10^3 + 971.5 \times 10^3 = 1.2 \times 10^6 \\ &= 1.2 \times 10^6\ Radians \end{aligned}$$

Frequency in Hertz

$$\begin{aligned} f_1 &= \frac{\omega_1}{2\pi} = \frac{736 \times 10^3}{2 \times \pi} = 117.13 \times 10^3\ Hz \\ f_2 &= \frac{\omega_1}{2\pi} = \frac{1.2 \times 10^6}{2 \times \pi} = 191 \times 10^3\ Hz \end{aligned}$$

DEC 2015 5 a) **It is required that a series RLC circuit should resonate at 500 KHz. Determine the values of R, L, and C if the bandwidth of the circuit is 10 KHz and its impedance is 100 Ω at resonance. Also find the voltages across L and C resonance if the applied voltage is 75 volts 10 Marks**

Solution:

At resonance the circuit acts as a pure resistive hence the value of R=100 Ω

$$B = \frac{R}{2\pi L}$$

$$L = \frac{R}{2\pi B} = \frac{100}{2 \times \pi \times 10 \times 10^3} = 1.59 \times 10^{-3} H$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\sqrt{LC} = \frac{1}{2\pi f_0} = \frac{1}{2\pi \times 500 \times 10^3} = 0.318 \times 10^{-6}$$

$$LC = (0.318 \times 10^{-6})^2 = 0.1 \times 10^{-12}$$

$$C = \frac{0.1 \times 10^{-12}}{1.59 \times 10^{-3}} = 62.83 \times 10^{-12} F$$

$$\begin{aligned} X_L &= 2\pi f_0 L = 2\pi \times 500 \times 10^3 \times 1.59 \times 10^{-3} \\ &= 5 \times 10^3 \Omega \end{aligned}$$

At resonance current I is

$$I = \frac{V}{R} = \frac{75}{100} = 0.75A$$

$$\begin{aligned} X_C &= \frac{1}{2\pi f_0 C} = \frac{1}{2\pi \times 500 \times 10^6 \times 62.9 \times 10^{-12}} \\ &= 5 \times 10^3 \Omega \end{aligned}$$

At resonance

$$X_L = X_C$$

Voltage across capacitor/inductor is

$$V_C = V_L = I X_C = 0.75 \times 5 \times 10^3 = 3750V$$

JULY 2014 **5 a) Define the following terms i) Resonance ii) Q factor iii) Selectivity of series RLC circuit iv) Bandwidth 4 Marks**

JULY 2014 **5 b) Prove that $f_0 = \sqrt{f_1 f_2}$ where f_1 and f_2 are the two half power frequencies of resonant circuit 8 Marks**

JULY 2014 **5 C) A series RLC circuit has R=4 Ω L=1 mH and C= 10 μF . Calculate the Q factor, band width resonant frequency and the half power frequencies f_1 and f_2 8 Marks**

Solution:

$$\begin{aligned} f_r &= \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1\times10^{-3}\times10\times10^{-6}}} \\ &= 1591.5\ Hz \end{aligned}$$

$$B = \frac{R}{2\pi L} = \frac{4}{2\pi\times1\times10^{-3}} = 636.62Hz$$

$$\frac{R}{2L} = \frac{4}{2\times1\times10^{-3}} = 2\times10^3$$

$$\begin{aligned} f_1 &= \frac{1}{2\pi}\left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ &= \frac{1}{2\pi}\left[-2\times10^3 + \sqrt{4\times10^6 + 0.1\times10^9}\right] \\ &= \frac{1}{2\pi}\left[-2\times10^3 + 10198\right] = 1305\ Hz \\ f_2 &= \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ &= \frac{1}{2\pi}\left[2\times10^3 + 10198\right] = 1941\ Hz \end{aligned}$$

JULY 2013 **5 a) For the series RLC circuit shown in Figure find the resonant frequency, half power frequencies, band width Quality factor and 10 Marks**

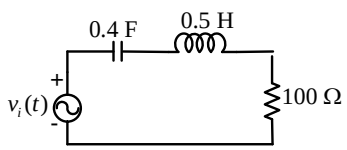

Figure 1.28: Example

Solution:

Resonant frequency is

$$\begin{aligned} f_r &= \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.4\times0.5}} \\ &= 0.356\ Hz \end{aligned}$$

Given data may wrongly printed because frequency is always more than 1 Hz

JULY 2012 **5 a) Define the following terms i) Resonance ii) Q factor iii) Selectivity of series RLC circuit iv) Bandwidth 6 Marks**

DEC 2012 **5 b) A series RLC circuit has R=10 Ω L=0.01 H and C= 0.01 μF and it is connected across 10 mV supply. Calculate i)f_0, ii)Q_0 iii)band width iv) f_1 and f_2 v) I_0 10 Marks**

Solution:

$$\begin{aligned} f_o &= \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\times0.01\times10^{-6}}} \\ &= 15915.5\ Hz \end{aligned}$$

$$Q_o = \frac{2\pi f_0 L}{R} = \frac{2\pi\times15915.5\times0.01}{10} = 100$$

$$B = \frac{R}{2\pi L} = \frac{10}{2\pi\times0.01} = 160Hz$$

$$\frac{R}{2L} = \frac{10}{2\times10\times10^{-3}} = 500$$

$$\begin{aligned} f_1 &= \frac{1}{2\pi}\left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ &= \frac{1}{2\pi}\left[-500 + \sqrt{250\times10^3 + 1\times10^{10}}\right] \\ &= \frac{1}{2\pi}\left[-500 + 100\times10^3\right] \\ &= 15836\ Hz \\ f_2 &= \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ &= \frac{1}{2\pi}\left[500 + 100\times10^3\right] \\ &= 15995\ Hz \end{aligned}$$

$$I_o = \frac{V}{R} = \frac{10\times10^{-3}}{10} = 1\times10^{-3}A$$

DEC 2011 **5 a) Define the following terms i) Resonance ii) Q factor iii) Selectivity of series RLC circuit iv) Bandwidth 6 Marks**

JULY 2010 **5 b) Derive for resonant circuit $f_0 = \sqrt{f_1 f_2}$ where f_1 and f_2 are the two half power frequencies 8 Marks**

DEC 2010 **5 C) An RLC series circuit has $R = 1K\Omega$ L=100 mH and C= 10 μF. If a voltage**

of 100 V is applied across series combination determine i)Resonant frequency ii) Q factor, and iii) Half power frequencies 4 Marks

Solution:

$$\begin{aligned} f_r &= \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times10^{-3}\times10\times10^{-6}}} \\ &= 159.15\ Hz \end{aligned}$$

ii) Q factor

$$Q = \frac{2\pi f_r L}{R} = \frac{2\pi\times159.15\times100\times10^{-3}}{1\times10^3} = 0.1$$

$$B = \frac{R}{2\pi L} = \frac{1\times10^3}{2\pi\times100\times10^{-3}} = 1592.35Hz$$

iii) Half power frequencies

$$\frac{R}{2L} = \frac{1\times10^3}{2\times100\times10^{-3}} = 5\times10^3$$

$$\begin{aligned} f_1 &= \frac{1}{2\pi}\left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ &= \frac{1}{2\pi}\left[-5\times10^3 + \sqrt{25\times10^6 + 1\times10^6}\right] \\ &= \frac{1}{2\pi}\left[-5\times10^3 + 5099\right] = 15.75\ Hz \\ f_2 &= \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ &= \frac{1}{2\pi}\left[5\times10^3 + 5099\right] = 1607\ Hz \end{aligned}$$

May/June 2010 **5 C) Determine the i) line current ii) the power factor and iii) voltage across coil when a coil of resistance 40Ω and inductance of 0.75 H forms a part of series circuit, for which the resonant frequency is 55 Hz If the supply voltage is 250 V 50 Hz 8 Marks**

Solution:

$$\omega_r L = 2\pi f_r L = \frac{1}{\omega_r C} = 2\pi\times55\times0.75 = 259.05$$

X_L at 50 Hz

$$X_L = \omega L = 2\pi f L = 2\pi\times50\times0.75 = 235.5$$

$$X_C = 259.05\times\frac{55}{50} = 284.96$$

Circuit Impedance at 50 Hz is

$$\begin{aligned} Z &= 40 + J(X_L - X_C) = 40 + J(235.5 - 284.96) \\ &= 40 - J49.46 = 63.61\angle -51.04 \end{aligned}$$

i) line current

$$I = \frac{V}{R} = \frac{250}{63.61} = 3.93\ A$$

ii) Power factor is

$$I = cos\phi = cos51.04 = 0.629 leading$$

iii) voltage across coil

$$V = IZ = I\sqrt{R^2 + X_L^2} = 3.93\sqrt{40^2 + 235.5^2} = 938.77$$

June/July 2009 **5 C) A series circuit RLC consists of $R = 1K\Omega$ and an inductance of 100 mH in series with capacitance of 10 nF. If a voltage of 100 V is applied across series combination determine i)Resonant frequency the ii) maximum current in the circuit iii) Q factor, and iii) Half power frequencies 8 Marks**

Solution:

$$\begin{aligned} f_r &= \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100\times10^{-3}\times10\times10^{-9}}} \\ &= 5\ KHz \end{aligned}$$

ii) Maximum current in the circuit

$$I = \frac{V}{R} = \frac{100}{1\times10^3} = 0.1A$$

iii) Q factor

$$Q = \frac{2\pi f_r L}{R} = \frac{2\pi\times5\times10^3\times100\times10^{-3}}{1\times10^3} = 3.14$$

iv) Half power frequencies

$$\frac{R}{2L} = \frac{1 \times 10^3}{2 \times 100 \times 10^{-3}} = 5 \times 10^3$$

$$\begin{aligned} f_1 &= \frac{1}{2\pi}\left[\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ &= \frac{1}{2\pi}\left[-5 \times 10^3 + \sqrt{25 \times 10^6 + 1 \times 10^9}\right] \\ &= \frac{1}{2\pi}\left[-5 \times 10^3 + 32 \times 10^3\right] = 27\ KHz \\ f_2 &= \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right] \\ &= \frac{1}{2\pi}\left[5 \times 10^3 + 32 \times 10^3\right] = 37\ KHz \end{aligned}$$