Chapter 1

Resonance

1.0.1 Introduction:
Most of the transmission lines, electrical circuits and communication networks are made up of network elements like resistor R, inductor, L and Capacitor C. The impedance of the inductor and capacitor depends on the frequency of the applied sinusoidal voltage to the network. As we vary the frequency of the supply the network impedance is purely resistive in which the impedance of the inductor is equal to the impedance of the capacitor. The phenomenon in which the complex circuit behaves like a pure resistive is called resonance. The frequency at which resonance takes place is called the frequency of resonance $\omega_r$ (radians/sec) or $f_r$ (Hz).

Resonance may occur by varying frequency of the applied voltage to the complex network or by varying inductance, L or Capacitance, C, by keeping the frequency constant. Under resonance the following conditions occur in the circuit.

- The impedance of the inductance is equal to the impedance Capacitance.
- The phase of the current in the circuit is in phase with the applied voltage.
- Maximum current will flow in the circuit.
- The voltage across the capacitor or inductor is $I \times X_C$ or $I \times X_L$ where I is the current at resonance and $X_C$ or $X_L$ is the impedance of the circuit.
- The total power is dissipated in the resistor and the absorbed average power is maximum.

1.1 Series Resonance

Consider a series circuit consists of resistor, inductor and capacitor as shown in Figure 1.1.

![Series resonance circuit](image)

Figure 1.1: Series resonance circuit

The impedance of the circuit is

$$Z = R + j(X_L - X_C)$$

At resonance the imaginary part is zero

$$X_L - X_C = 0$$
$$\omega_r L - \frac{1}{\omega_r C} = 0$$

$$\omega_r L = \frac{1}{\omega_r C}$$
$$\omega_r^2 = \frac{1}{LC}$$
$$\omega_r = \frac{1}{\sqrt{LC}} \text{ radians/sec}$$

$$f_r = \frac{1}{2\pi \sqrt{LC}} \text{ Hz}$$
1.1. SERIES RESONANCE

Chapter 1. Resonance

The plot of the frequency response of series circuit is as shown in Figure 1.2. At resonant frequency \( \omega_r \), the current is maximum.

\[
\omega_r = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}
\]

Frequency is always positive

\[
\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}
\]

In terms of frequency \( f_1 \)

\[
f_1 = \frac{1}{2\pi} \left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]
\]

At frequency \( \omega_2 \) the circuit impedance \( X_L > X_C \)

\[
\omega_2 = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}
\]

Frequency is always positive

\[
\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}
\]

In terms of frequency \( f_2 \)

\[
f_2 = \frac{1}{2\pi} \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]
\]
1.1. SERIES RESONANCE

Chapter 1. Resonance

Relation between \( \omega_r \), \( \omega_1 \) and \( \omega_2 \)

\[
\omega_1 \times \omega_2 = \left[ \frac{R}{2L} + \sqrt{\frac{(R/2L)^2 + 1}{LC}} \right] \times \left[ \frac{-R}{2L} + \sqrt{\frac{(R/2L)^2 + 1}{LC}} \right] = \frac{1}{LC}
\]

\[
\omega_r = \sqrt{\frac{1}{LC}} \quad \omega_r^2 = \frac{1}{LC} = \omega_1 \omega_2
\]

\[
\omega_r = \sqrt{\omega_1 \omega_2}
\]

\[
f_r = \sqrt{f_1 f_2}
\]

Resonance by varying circuit inductance

Consider a series RLC circuit as shown in Figure 1.5 is become resonant by varying inductance of the circuit.

![Figure 1.5: Resonance by varying inductance](image)

Let \( L_1 \) is the inductance at \( \omega \)

\[
X_C - X_L = R
\]

\[
\frac{1}{\omega C} - \omega L_1 = R
\]

\[
L_1 = \frac{1}{\omega^2 C} - \frac{R}{\omega}
\]

Let \( L_2 \) is the inductance at \( \omega \)

\[
X_L - X_C = R
\]

\[
\omega L_2 - \frac{1}{\omega C} = R
\]

\[
L_2 = \frac{1}{\omega^2 C} + \frac{R}{\omega}
\]

Resonance by varying circuit capacitance

Consider a series RLC circuit as shown in Figure ?? is become resonant by varying capacitance of the circuit.

![Figure 1.6: Resonance by varying capacitance](image)

Let \( C_1 \) is the capacitance at \( \omega_1 \)

\[
X_C - X_L = R \Rightarrow \frac{1}{\omega_1 C_1} - \omega_1 L = R
\]

\[
\frac{1}{\omega_1 C_1} = R + \omega_1 L
\]

\[
C_1 = \frac{1}{\omega_1^2 L + \omega_1 R}
\]

Let \( C_2 \) is the capacitance at \( \omega_2 \)

\[
X_L - X_C = R \Rightarrow \omega_2 L - \frac{1}{\omega_2 C_2} = R
\]

\[
\frac{1}{\omega_2 C_2} = \omega_2 L - R
\]

\[
C_2 = \frac{1}{\omega_2^2 L - \omega_2 R}
\]
### Table 1.1: Important Formulae

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>At resonance</td>
<td>( Z = R ), ( X_L = X_C )  ( I_r = \frac{E}{R} )</td>
</tr>
<tr>
<td>Resonance</td>
<td>( \omega_r = \frac{1}{\sqrt{LC}} ), ( f_r = \frac{1}{2\pi\sqrt{LC}} )</td>
</tr>
<tr>
<td>Half power frequency</td>
<td>( \omega_1 = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}} ), ( f_1 = \frac{\omega_1}{2\pi} )</td>
</tr>
<tr>
<td>Half power frequency</td>
<td>( \omega_2 = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}} ), ( f_2 = \frac{\omega_2}{2\pi} )</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>( B = \omega_2 - \omega_1 = \frac{R}{L} ) Radians</td>
</tr>
<tr>
<td></td>
<td>( B = f_2 - f_1 = \frac{R}{2\pi L} ) Hz</td>
</tr>
<tr>
<td>Quality factor</td>
<td>( Q = \frac{\omega_r L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} )</td>
</tr>
<tr>
<td>( \omega_r \omega_1 \omega_2 )</td>
<td>( \omega_r = \sqrt{\omega_1\omega_2} ) OR ( f_r = \sqrt{f_1f_2} )</td>
</tr>
<tr>
<td>Voltage across capacitor/inductor</td>
<td>( V_{LR} = V_{CR} = IX_{LR} )</td>
</tr>
<tr>
<td>Value of inductor at ( f_1 ), ( f_2 )</td>
<td>( L_1 = \frac{1}{\omega^2 C} - \frac{R}{\omega}, \quad L_2 = \frac{1}{\omega^2 C} + \frac{R}{\omega} )</td>
</tr>
<tr>
<td>Value of capacitor at ( f_1 ), ( f_2 )</td>
<td>( C_1 = \frac{1}{\omega^2 L} - \frac{R}{\omega}, \quad C_2 = \frac{1}{\omega^2 L} + \frac{R}{\omega} )</td>
</tr>
</tbody>
</table>

Selectivity: is property of circuit in which the circuit is allowed to select a band of frequencies between \( f_1 \) and \( f_2 \).
1: For the circuit shown in Figure Find (a) The resonant and half power frequencies (b) Calculate the quality factor and bandwidth (c) Determine the amplitude of the current at \( \omega_o, \omega_1, \omega_2 \)

\[
\begin{align*}
\text{Figure 1.7: Example 1} \\
LC = 100 \times 10^{-3} \times 3.09 \times 10^{-6} = 3.09 \times 10^{-7} \\
\text{The resonant frequency } \omega_o \text{ is} \\
\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.09 \times 10^{-7}}} = 1800 \text{ rad/s} \\
\text{The half power frequency } \omega_1, \omega_2 \text{ is} \\
\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\
= -15 + \sqrt{225 + \frac{1}{3.09 \times 10^{-7}}} \\
= -15 + \sqrt{225 + 3.236 \times 10^6} \\
= -15 + 1798.96 = 1784 \text{ rad/s} \\
\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\
= 15 + 1798.96 = 1814 \text{ rad/s} \\
\text{Frequency in Hz is} \\
f_1 = \frac{\omega_1}{2\pi} = \frac{1784}{2\pi} = 284 \text{ Hz} \\
f_2 = \frac{\omega_2}{2\pi} = \frac{1814}{2\pi} = 289 \text{ Hz} \\
\text{Bandwidth B is} \\
B = \omega_2 - \omega_1 = 1814 - 1784 = 30 \text{ rad/s} \\
\end{align*}
\]

Also B is

\[
B = \frac{R}{L} = \frac{3}{100 \times 10^{-3}} = 30 \text{ rad/s} \\
\]

Quality factor Q is

\[
Q = \frac{\omega_o}{B} = \frac{1800}{30} = 60 \\
\]

The amplitude of the current at \( \omega_o \) is

\[
I = \frac{V}{R} = \frac{10}{3} = 3.33 \text{A} \\
\]

The amplitude of the current at \( \omega_1, \omega_2 \) is

\[
I = \frac{V}{\sqrt{2R}} = \frac{10}{\sqrt{2 \times 3}} = 2.36 \text{A} \\
\]

2: For the circuit shown in Figure find the resonant frequency, quality factor and bandwidth for the circuit. Determine the change in Q and the bandwidth if R is changed from \( R = 2 \Omega \) to \( R = 0.4 \Omega \)

\[
\begin{align*}
\text{Figure 1.8: Example} \\
LC = 2 \times 10^{-3} \times 5 \times 10^{-6} = 10 \times 10^{-9} \\
\text{The resonant frequency } \omega_o \text{ is} \\
\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 10^{-9}}} = 10 \times 10^3 \text{ rad/s} \\
\text{B is} \\
B = \frac{R}{L} = \frac{2}{2 \times 10^{-3}} = 1000 \text{ rad/s} \\
\text{Quality factor Q is} \\
Q = \frac{\omega_o}{B} = \frac{10 \times 10^3}{1000} = 10 \\
\text{When } R = 0.4 \Omega \text{ B is} \\
B = \frac{R}{L} = \frac{0.4}{2 \times 10^{-3}} = 200 \text{ rad/s} \\
\text{Quality factor Q is} \\
Q = \frac{\omega_o}{B} = \frac{10 \times 10^3}{200} = 50 \\
\end{align*}
\]

3: For the circuit shown in Figure find the following (a) The resonant frequency \( f_o \) (b) Quality factor Q (c) \( f_{c1}, f_{c2} \) (d) Bandwidth B

\[
\begin{align*}
\text{Figure} \\
12.5k\Omega 1.25pF 312mH \\
\text{Dr. Manjunatha P Professor Dept of ECE, JNN College of Engineering, Shivamogga} \\
\end{align*}
\]
Figure 1.9: Example

\[ LC = 312 \times 10^{-3} \times 1.25 \times 10^{-12} = 39 \times 10^{-12} \]

The resonant frequency \( \omega_0 \) is

\[ \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.39 \times 10^{-12}}} = 1.6 \times 10^6 \text{ rad/s} \]

The half power frequency \( f_o \) is

\[ f_o = \frac{\omega_0}{2\pi} = \frac{1.6 \times 10^6}{2\pi} = 254 \times 10^3 \text{ Hz} \]

\[ B = \frac{R}{L} = \frac{62.5 \times 10^3}{312 \times 10^{-3}} = 200 \times 10^3 \]

Quality factor \( Q \) is

\[ Q = \frac{\omega_0}{B} = \frac{1.6 \times 10^6}{200 \times 10^3} = 8 \]

The half power frequency \( \omega_1 \) is

\[ \omega_1 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \]

\[ \omega_1 = -1 \times 10^5 + \sqrt{(1 \times 10^5)^2 + \frac{1}{0.39 \times 10^{-12}}} \]

\[ \omega_1 = -1 \times 10^5 + \sqrt{1 \times 10^{10} + 2.5641 \times 10^{12}} \]

\[ \omega_1 = -1 \times 10^5 + 1.6 \times 10^6 = 1.5 \times 10^6 \text{ rad/s} \]

Bandwidth \( B \) in Hz is

\[ B = \omega_2 - \omega_1 = 1.7 \times 10^6 - 1.5 \times 10^6 = 200 \times 10^3 \text{ rad/s} \]

\[ B = \frac{200 \times 10^3}{2 \times \pi} = 31.83 \times 10^3 \text{ Hz} \]

4: A variable frequency voltage source drives the network shown in Figure. Find the resonant frequency \( f_o \), Quality factor \( Q \), Bandwidth

\[ B \] and the average power dissipated by the network at resonance

Figure 1.10: Example

\[ LC = 50 \times 10^{-3} \times 5 \times 10^{-6} = 2.5 \times 10^{-6} \]

The resonant frequency \( \omega_0 \) is

\[ \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.5 \times 10^{-6}}} = 632.45 \text{ rad/s} \]

The half power frequency \( f_o \) is

\[ f_o = \frac{\omega_0}{2\pi} = \frac{632.45}{2\pi} = 100 \text{ Hz} \]

Quality factor \( Q \) is

\[ Q = \frac{\omega_0}{B} = \frac{632.45}{80} = 7.9 \]

The average power dissipated at resonance

\[ P = \frac{1}{2} \times \frac{V^2}{R} \]

\[ P = \frac{1}{2} \times \frac{12^2}{4} = 18 \text{ Watts} \]

5: For the circuit shown in Figure the maximum amplitude of current is 10A, circuit quality factor \( Q \) is 100 and \( L=0.1H \). If the applied voltage is 100 V find the value of capacitance

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ \omega_0 = \frac{V}{I_m} \]

\[ Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \sqrt{\frac{L}{C}} = Q \times R \]

Solution:

The maximum value of current flows in a circuit when the circuit is in resonance and the impedance of the circuit is pure resistor and its value is

\[ R = \frac{V}{I_m} = \frac{100}{10} = 10 \text{ } \Omega \]

The relation between \( Q \) and Bandwidth \( B \) is

\[ Q = \frac{1}{R} \sqrt{\frac{L}{C}} \Rightarrow \sqrt{\frac{L}{C}} = Q \times R \]
1.1. SERIES RESONANCE

\[
\frac{L}{C} = (Q \times R)^2 = (100 \times 10)^2 = 1 \times 10^6
\]

\[
C = \frac{L}{1 \times 10^6} = \frac{0.1}{1 \times 10^6} = 0.1 \mu F
\]

6: The Q of a series circuit network is 10. The maximum amplitude of current at resonance is 1 A when applied voltage is 10 V. If \( L = 0.1 \) H find the value of capacitance

Solution:
The maximum value of current flows in a circuit when the circuit is in resonance and the impedance of the circuit is pure resistor and its value is

\[
R = \frac{V}{I_m} = \frac{10}{1} = 10 \Omega
\]

The relation between Q and Bandwidth B is

\[
Q = \frac{1}{R} \sqrt{\frac{L}{C}} \Rightarrow \sqrt{\frac{L}{C}} = Q \times R
\]

\[
\frac{L}{C} = (Q \times R)^2 = (10 \times 10)^2 = 10 \times 10^3
\]

\[
C = \frac{L}{10 \times 10^3} = \frac{0.1}{10 \times 10^3} = 10 \mu F
\]

7: A coil of inductance 9H and resistance 50 Ω in series with a capacitor is supplied at constant voltage from a variable frequency source. If the maximum current is 1A at 75 Hz, find the frequency when the current is 0.5A

Solution:

8: Design a series resonant circuit to have \( \omega_r = 2500 \) rad/sec \( Z(\omega_r) = 100 \) Ω and \( B = 500 \) rad/sec

Solution:

\[
B = \omega_r - \omega_1 = \frac{R}{L}
\]

\[
L = \frac{R}{B} = \frac{100}{500} = 0.2H
\]

The resonant frequency \( \omega_r \) is

\[
\omega_r = \frac{1}{\sqrt{LC}}
\]

\[
\sqrt{LC} = \frac{1}{\omega_r} = \frac{1}{2500} = 4 \times 10^{-4}
\]

\[
LC = 1.6 \times 10^{-7}
\]

\[
C = \frac{1.6 \times 10^{-7} \times 0.8 \times 10^{-6}}{0.2 \times 10^{-3}} = 0.8 \mu F
\]

\[
R = 100 \Omega \quad L = 0.2H \quad C = 0.8 \mu F
\]

9: A series resonant RLC circuit has resonant frequency of 80 K rad/sec and a quality factor of 8. Find the bandwidth, the upper cutoff frequency and lower cutoff frequency

Solution:

\[
Q = \frac{\omega_r}{B}
\]

\[
B = \frac{\omega_r}{Q} = \frac{80 \times 10^3}{8} = 10 \times 10^3 \text{ rad/sec}
\]

Bandwidth in Hertz

\[
B = \frac{\omega_r - \omega_1}{2\pi} = \frac{10 \times 10^3}{2 \times \pi} = 1.59 \times 10^3 \text{ Hz}
\]

\[
B = \omega_2 - \omega_1 = \frac{R}{L}
\]

\[
\omega_1 = \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}
\]

\[
= -5 \times 10^3 + \sqrt{\left(\frac{10 \times 10^3}{2}\right)^2 + (80 \times 10^3)^2}
\]

\[
= -5 \times 10^3 + \sqrt{25 \times 10^6 + 6.4 \times 10^9}
\]

\[
= -5 \times 10^3 + 80.156 \times 10^3 = 75156 \text{ rad/sec}
\]

\[
\omega_2 = \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}
\]

\[
= 5 \times 10^3 + 80.156 \times 10^3 = 85156 \text{ rad/sec}
\]

Frequency in Hertz

\[
f_1 = \frac{\omega_1}{2\pi} = \frac{75156}{2 \times \pi} = 11.96 \times 10^3 \text{ Hz}
\]

\[
f_2 = \frac{\omega_2}{2\pi} = \frac{85156}{2 \pi} = 13.55 \times 10^3 \text{ Hz}
\]

10: A series RLC circuit has R=10 Ω, L=0.1H, and C=100 μF and is connected across 200 V, variable frequency source. Find (a) the resonant frequency (b)impedance at this frequency (c) the voltage across drop across
inductance and capacitance at this frequency (d) quality factor and (e) bandwidth.

Solution: (a) Resonant frequency

\[ f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 100 \times 10^{-6}}} = 50.33 \text{Hz} \]

(b) Impedance at resonance \( Z = R = 10 \Omega \)

\[ I = \frac{V}{R} = \frac{200}{10} = 20 \text{A} \]

(c) Voltage across inductance/capacitance

\[ X_L = X_C = 2\pi f_r \times L = 2\pi \times 50.33 \times 0.1 = 31.62 \Omega \]

\[ V_L = IX_L = 20 \times 31.62 = 632.46 \text{V} \]

(d) Quality factor

\[ Q = \frac{X_L}{R} = \frac{31.62}{10} = 3.162 \]

(e) Bandwidth

\[ B = \frac{R}{2\pi L} = \frac{10}{2 \times \pi \times 0.1} = 15.9 \text{Hz} \]

11: A coil of resistance \( R=20 \Omega \) and inductance \( L=0.2 \text{ H} \) is connected in series with a capacitance across 230 V supply. Find (a) the value of the capacitance for which resonance occurs at 100 Hz frequency (b) the current through and voltage across the capacitor (c) Q factor of the coil.

Solution: (a) Resonant frequency

\[ f_r = \frac{1}{2\pi\sqrt{LC}} \]

\[ \sqrt{LC} = \frac{1}{2\pi f_r} = \frac{1}{2\pi \times 50} = 3.18 \times 10^{-3} \]

\[ LC = 1 \times 10^{-5} \]

\[ L = \frac{1}{100 \times 10^{-6}} = 101 \text{ mH} \]

(b) Quality factor

\[ Q = \frac{X_L}{R} = \frac{2\pi f_r \times L}{R} = \frac{2\pi \times 100 \times 10^{-3}}{10} = 3.14 \]

Impedance at resonance is \( Z = R = 10 \Omega \)

(c) Current through resistance is

\[ I = \frac{V}{R} = \frac{230}{10} = 23 \text{A} \]

Voltage drop across resistor is

\[ V = IR = 23 \times 10 = 230 \text{V} \]

Voltage drop across Inductor/capacitance is

\[ X_L = 2\pi f_r \times L = 2\pi \times 50 \times 101 \times 10^{-3} = 31.714 \Omega \]

\[ V_L = V_C = IX_L = 23 \times 31.714 = 729.2 \text{V} \]
1.2 Parallel Resonance

Consider a parallel circuit consists of resistor and inductor in one branch and capacitor in another branch which is as shown in Figure 1.27.

![Figure 1.27: General parallel resonant circuit](image)

The total admittance of the circuit is

\[ Y = G + j \left( \omega C - \frac{1}{\omega L} \right) \]

When the circuit is at resonance the imaginary part is zero

\[ \omega_r C - \frac{1}{\omega_r L} = 0 \]

\[ \omega_r^2 = \frac{1}{LC} \]

\[ \omega_r = \sqrt{\frac{1}{LC}} \]

\[ f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \]

Consider a parallel circuit consists of resistor and inductor in one branch and capacitor in another branch which is as shown in Figure 1.13.

![Figure 1.13: General parallel resonant circuit](image)

The impedance of the inductor branch is

\[ Z_L = R + j\omega L \]

The admittance of the inductor branch is

\[ Y_L = \frac{1}{Z_L} = \frac{1}{R + j\omega L} \]

\[ Y_L = \frac{1}{Z_L} = \frac{1}{R + j\omega L} \]

\[ = \frac{1}{R + j\omega L} \times \frac{R - j\omega L}{R - j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} \]

Similarly the impedance of the capacitor branch is

\[ Z_C = \frac{1}{j\omega C} \]

The admittance of the capacitor branch is

\[ Y_C = \frac{1}{Z_C} = j\omega C \]

Total admittance of the circuit is

\[ Y = Y_L + Y_C = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C \]

Separating real and imaginary parts

\[ Y = \frac{R}{R^2 + \omega^2 L^2} + j \left[ \omega C - \frac{\omega L}{R^2 + \omega^2 L^2} \right] \]

\[ \omega_r C - \frac{\omega_r L}{R^2 + \omega^2 L^2} = 0 \]

\[ \omega_r C = \frac{\omega_r L}{R^2 + \omega^2 L^2} \]

\[ R^2 + \omega_r^2 L^2 = \frac{\omega_r L}{\omega_r C} = \frac{L}{C} \]

\[ \omega_r^2 L^2 = \frac{L}{C} - R^2 \]

\[ \omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \]

\[ f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \]

Consider a parallel circuit consists of resistor and inductor in one branch and capacitor in another branch which is as shown in Figure 1.14.

![Figure 1.14: General parallel resonant circuit](image)

The impedance of the inductor branch is

\[ Z_L = R_L + j\omega L \]
1.2. PARALLEL RESONANCE

The admittance of the inductor branch is

\[ Y_L = \frac{1}{Z_L} = \frac{1}{R_L + j\omega L} \]

\[ Y_L = \frac{1}{Z_L} = \frac{1}{R_L + j\omega L} = \frac{1}{R_L + j\omega L} \times \frac{R_L - j\omega L}{R_L - j\omega L} = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} \]

Similarly the impedance of the capacitor branch is

\[ Z_C = R_C - j \frac{1}{\omega C} \]

The admittance of the inductor branch is

\[ Y_C = \frac{1}{Z_C} = \frac{1}{R_C - j \frac{1}{\omega C}} \]

\[ Y_C = \frac{1}{Z_C} = \frac{1}{R_C - j \frac{1}{\omega C}} = \frac{R_C + j \frac{1}{\omega C}}{R_C + j \frac{1}{\omega C}} \]

Total admittance of the circuit is

\[ Y = Y_L + Y_C = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + j \frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} \]

Separating real and imaginary parts

Real part is

\[ \frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + \frac{1}{\omega^2 C^2}} \]

Imaginary part is

\[ \frac{\omega L}{R_C^2 + \frac{1}{\omega^2 C^2}} - \frac{\omega L}{R_L^2 + \omega^2 L^2} = 0 \]

\[ \frac{\omega L}{R_C^2 + \frac{1}{\omega^2 C^2}} = \frac{\omega L}{R_L^2 + \omega^2 L^2} \]

\[ \frac{1}{\omega_r C} \left( R_L^2 + \omega_r^2 L^2 \right) = \omega_r L \left( R_C^2 + \frac{1}{\omega_r^2 C^2} \right) \]

\[ \frac{1}{LC} \left( R_L^2 + \omega_r^2 L^2 \right) = \omega_r^2 \left( R_C^2 + \frac{1}{\omega_r^2 C^2} \right) \]

\[ \frac{R_L^2}{LC} + \omega_r^2 \frac{L}{C} = \omega_r^2 \frac{R_C^2}{C} + \frac{1}{C^2} \]

\[ \omega_r^2 \left( \frac{R_C^2 - \frac{L}{C}}{LC} \right) = \frac{R_L^2}{LC} - \frac{1}{C^2} \]

\[ \omega_r^2 \left( \frac{R_C^2 - \frac{L}{C}}{LC} \right) = \frac{1}{LC} \left( \frac{R_L^2 - \frac{L}{C}}{C} \right) \]

\[ \omega_r^2 = \frac{1}{LC} \times \left( \frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}} \right) \]

\[ \omega_r = \sqrt{\frac{1}{LC} \sqrt{ \frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}} } } \]

\[ f_r = \frac{1}{2\pi \sqrt{LC}} \sqrt{ \frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}} } \]

The frequency response of parallel graph is as shown in Figure 1.15. From the figure it is observed that the current is minimum at resonance. The parallel circuit is called as a rejector circuit. The circuit impedance is maximum at the resonance. The half power frequencies are at $\sqrt{2}f_r$.

![Figure 1.15: plot of parallel resonant circuit](image)

Calculate the resonant frequency of the circuit shown in Figure 1.16

\[ Z_{LC} = jXL + jXC = j\omega L + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C} \]

\[ Z_{LC} = \frac{j(\omega^2 LC - 1)}{\omega C} \]

\[ \omega^2 LC = 1 \]

\[ \omega^2 = \frac{1}{\sqrt{LC}} \]
1.2. PARALLEL RESONANCE

Chapter 1. Resonance

JAN-2018-CBCS 8 c) Show that a two-branch parallel circuit is resonant at all frequencies if \( R_L = R_C = \sqrt{\frac{L}{C}} \) where \( R_L \) is resistor in the inductor branch and \( R_C \) is resistor in the capacitor branch.

Solution

Figure 1.17: 2018-CBCS-Question Paper

\[
Z_L = R_L + jX_L
\]
\[
Y_L = \frac{1}{R_L + jX_L}
\]
\[
Y_L = \frac{1}{R_L + j\omega L}
\]
\[
Y_L = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2}
\]

\[
Z_C = R_C - jX_C
\]
\[
Y_C = \frac{1}{R_C - jX_C}
\]
\[
Y_C = \frac{1}{R_C + \frac{1}{j\omega C}}
\]
\[
Y_C = \frac{R_C + j\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}}
\]

\[ Y = Y_L + Y_C = \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C + j\frac{1}{\omega C}}{R_C^2 + \frac{1}{\omega^2 C^2}} \]

Separating imaginary parts and equated to zero

\[
\frac{\omega L}{R_L^2 + \omega^2 L^2} + \frac{1}{\omega C} = 0
\]
\[
\omega L(1 + R_C^2 \omega^2 C^2) = \omega C(R_L^2 + \omega^2 L^2)
\]
\[
L(1 + R_C^2 \omega^2 C^2) = C(R_L^2 + \omega^2 L^2)
\]
\[
L + \omega^2 LR_C C^2 = CR_L^2 + \omega^2 CL^2
\]

\[ \frac{1}{C} + \omega^2 R_C C = \frac{R_L^2}{L} + \omega^2 L \]

\[ \frac{1}{C} + \omega^2 C = \frac{L}{C} + \omega^2 L \]

From the above equation it is observed that on both sides it contains \( \omega \). As we vary \( \omega \) it varies on both sides and hence the circuit is resonant for all frequencies.

Selectivity: It is defined as the ratio of bandwidth to resonant frequency of a series resonant circuit.

\[ Selectivity = \frac{B}{f_r} = \frac{f_2 - f_1}{f_r} \]

1: For the circuit as shown in Figure 1.18 find the resonant frequency and the corresponding current in each branch.

Figure 1.18: General parallel resonant circuit

Solution:

\[ \frac{L}{C} = \frac{1 \times 10^{-3}}{20 \times 10^{-6}} = 50 \]
1.2. PARALLEL RESONANCE

\[ f_r = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}} \]

\[ = \frac{1}{2\pi\sqrt{2 \times 10^{-8}}} \sqrt{\frac{6^2 - 50}{4^2 - 50}} \]

\[ = 1125.4 \times 0.641 = 721 \text{ Hz} \]

\[ X_L = 2\pi \times f \times L = 2\pi \times 721 \times 1 \times 10^{-3} = 4.53 \Omega \]

\[ X_C = \frac{1}{2\pi \times f \times C} = \frac{1}{2\pi \times 721 \times 20 \times 10^{-6}} = 11.03 \Omega \]

\[ I_L = \frac{V}{Z_L} = \frac{200}{R + jX_L} = \frac{200}{6 + j4.53} \]

\[ = \frac{200}{7.51, \angle 37} = 26.63 \angle -37 \]

\[ I_C = \frac{V}{Z_C} = \frac{200}{R + jX_C} = \frac{200}{4 - j11.03} \]

\[ = \frac{200}{11.73, \angle 70} = 17.05 \angle 70 \]

2: Find the value of L for which the circuit as shown in Figure 1.19 is resonance at \( \omega = 5000 \text{ rad/sec} \).

Solution: The admittance of circuit is given by

\[ Y = \frac{1}{4 + jX_L} + \frac{1}{8 - j12} \]

\[ = \frac{4 - jX_L}{4^2 + X_L^2} + \frac{8 + 12}{8^2 + 12^2} \]

At resonance

\[ \frac{X_L}{4^2 + X_L^2} = \frac{12}{8^2 + 12^2} \]

\[ 12(4^2 + X_L^2) = X_L(8^2 + 12^2) \]

\[ 192 + 12X_L^2 = 208X_L \]

\[ 12X_L^2 - 208X_L + 192 = 0 \]

\[ 3X_L^2 - 52X_L + 48 = 0 \]

\[ X_{L1} = 17.57 \Omega \]

\[ X_{L2} = 5.69 \Omega \]

\[ L1 = \frac{X_{L1}}{\omega} = \frac{17.57}{2\pi \times 1000} = 2.797mH \]

\[ L2 = \frac{X_{L2}}{\omega} = \frac{5.69}{2\pi \times 1000} = 0.9mH \]

3: Find the value of L for which the circuit as shown in Figure 1.20 is resonance at 1000 Hz.

Solution: The admittance of circuit is given by

\[ X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 50 \times 10^{-6}} = 3.18 \Omega \]

\[ Y = \frac{1}{10 + jX_L} + \frac{1}{10 - jX_L} \]

\[ = \frac{10 - jX_L}{10^2 + X_L^2} + \frac{8 + j3.18}{8^2 + 3.18^2} \]

At resonance

\[ \frac{X_L}{10^2 + X_L^2} = \frac{3.18}{8^2 + 3.18^2} \]

\[ 3.18(100 + X_L^2) = X_L(64 + 10.11) \]

\[ 318 + 3.18X_L^2 = 74X_L \]

\[ 3.18X_L^2 - 74X_L + 318 = 0 \]

\[ X_L = \frac{74 \pm \sqrt{74^2 - (4 \times 3.18 \times 318)}}{2 \times 3} \]

\[ = \frac{74 \pm \sqrt{5476 - (4045)}}{6.36} \]

\[ = \frac{74 \pm 37.8}{6.36} \]

\[ X_{L1} = 17.57 \Omega \]

\[ X_{L2} = 5.69 \Omega \]

\[ L1 = \frac{X_{L1}}{\omega} = \frac{17.57}{2\pi \times 1000} = 2.797mH \]

\[ L2 = \frac{X_{L2}}{\omega} = \frac{5.69}{2\pi \times 1000} = 0.9mH \]
4: Find the value of C for which the circuit as shown in Figure 1.28 is resonance at 750 Hz.

![General parallel resonant circuit](image1.png)

**Figure 1.21: General parallel resonant circuit**

**Solution:** The admittance of circuit is given by

\[
Y = \frac{1}{10 + j8} + \frac{1}{6 - jX_C}
\]

\[
= \frac{10 + j8}{10^2 + 8^2} + \frac{6 - jX_C}{6^2 + X_C^2}
\]

\[
= \left( \frac{10}{164} + \frac{6}{36 + X_C^2} \right) + j \left( \frac{X_C}{36 + X_C^2} - \frac{8}{164} \right)
\]

At resonance

\[
\frac{X_C}{36 + X_C^2} - \frac{8}{164} = 0
\]

\[
164X_C - 8X_C^2 - 288 = 0
\]

\[
2X_C^2 - 41X_C + 78 = 0
\]

\[
X_{C1} = 41 \pm \sqrt{41^2 - (4 \times 2 \times 72)} \times 2 = 38.98 \Omega
\]

\[
X_{C2} = 1.94 \Omega
\]

\[
C_1 = \frac{1}{\omega X_{C1}} = \frac{1}{2 \times \pi \times 750 \times 18.56} = 11.44 \mu F
\]

\[
C_2 = \frac{1}{\omega X_{C2}} = \frac{1}{2 \times \pi \times 750 \times 1.94} = 109.45 \mu F
\]

5: Find the value of C for which the circuit as shown in Figure 1.22 is resonance at 3000 rad/sec.

![General parallel resonant circuit](image2.png)

**Figure 1.22: General parallel resonant circuit**

**Solution:**

\[
X_L = \omega \times L = 3000 \times 0.005 = 15 \Omega
\]

6: Find the value of \( R_L \) for which the circuit as shown in Figure 1.23 is resonant.

![General parallel resonant circuit](image3.png)

**Figure 1.23: General parallel resonant circuit**

**Solution:** The admittance of circuit is given by

\[
Y = \frac{1}{R_L + j10} + \frac{1}{10 - j15}
\]

\[
= \frac{R_L - j10}{R_L^2 + 10^2} + \frac{10 + j15}{100 + 225}
\]

\[
= \left( \frac{R_L}{R_L^2 + 100} + \frac{10}{325} \right) + j \left( \frac{15}{325} - \frac{10}{R_L^2 + 100} \right)
\]

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At resonance

\[
\frac{15}{325} - \frac{10}{R_L^2 + 100} = 0
\]

\[
15(R_L^2 + 100) = 3250
\]

\[
R_L^2 = 116.6
\]

\[
R_L = 10.8
\]

7: Two impedances \((10 + j12) \Omega\) and \((20 - j15)\Omega\) are connected in parallel and this combination is connected in series with an impedance \((5 - jX_C) \Omega\). Find the value of \(X_C\), for which resonance occurs.

\[
Z = (5 - jX_C) + \left[\frac{(10 + j12)(20 - j15)}{(10 + j12) + (20 - j15)}\right]
\]

\[
= (5 - jX_C) + \frac{(10 + j12)(20 - j15)}{30 - j3}
\]

\[
= (5 - jX_C) + \frac{380 + j90}{30 - j3}
\]

\[
= (5 - jX_C) + \frac{390.51j}{13.325}
\]

\[
= (5 - jX_C) + 12.78 + j2.33
\]

\[
= (5 - jX_C) + 17.78 + j(2.33 - jX_C)
\]

Figure 1.24: General parallel resonant circuit

**Solution:** The impedance of circuit is given by

\[
Y = \frac{1}{20 + j37.7} + \frac{1}{20 - j37.7} + \frac{10 - jX_C}{20^2 + j37.7^2 + 10^2 + X_C^2}
\]

\[
= \frac{1}{20 - j37.7} + \frac{10 - jX_C}{1821.29 + 10^2 + X_C^2}
\]

At resonance imaginary part is zero

\[
X_C = 2.33
\]

8: (2015-JAN) Find the value of C for which the circuit as shown in Figure 1.25 is resonance at 50 Hz.

Figure 1.25: General parallel resonant circuit

**Solution:**

The admittance of circuit is given by

\[
Y = \frac{1}{20 + j37.7} + \frac{1}{20 - j37.7} + \frac{10 - jX_C}{20^2 + j37.7^2 + 10^2 + X_C^2}
\]

\[
= \frac{1}{20 - j37.7} + \frac{10 - jX_C}{1821.29 + 10^2 + X_C^2}
\]
At resonance imaginary part is

\[
\frac{X_C}{100 + X_C^2} = \frac{37.7}{1821.29} = 0
\]

\[
X_C = \frac{37.7}{100 + X_C^2} = 1821.29
\]

\[
1821.29X_C = 37.7X_C^2 + 3770
\]

\[37.7X_C^2 - 1821.29X_C + 3770 = 0\]

\[X_C = \frac{1821.29 \pm \sqrt{1821.29^2 - (4 \times 37.7 \times 3770)}}{2 \times 37.7} = \frac{1821.29 \pm \sqrt{3317097 - 568516}}{75.4}
\]

\[X_C = 46.14 \Omega
\]

\[X_C = 2.16 \Omega
\]

\[\omega = 2 \times \pi \times f = 2 \times \pi \times 50 = 314.16
\]

\[C_1 = \frac{1}{\omega X_{C1}} = \frac{1}{314.16 \times 46.14} = 68.98 \mu F
\]

\[C_2 = \frac{1}{\omega X_{C2}} = \frac{1}{314.16 \times 2.16} = 1.47 \times 10^{-3} F
\]

9: (2012-DEC) Determine the value of \(R_L\) \(R_C\) for which the circuit as shown in Figure 1.26 is resonates at all frequencies.

Figure 1.26: General parallel resonant circuit

Solution:

The admittance of circuit is given by

\[f_r = \frac{1}{2\pi \sqrt{LC}} \sqrt{\frac{R_C^2}{R_L^2} - \frac{L}{C}}
\]

The circuit will resonate at any frequency if

\[R_L^2 = R_C^2 = \frac{L}{C}
\]

\[R_L = R_C = \sqrt{\frac{L}{C}} = \sqrt{\frac{40 \times 10^{-3}}{40 \times 10^{-6}}} = 31.6 \Omega
\]
JAN 2018 CBCS Q 7 a) What is resonance? Derive an expression for half power frequencies 8 Marks JAN 2018 CBCS Q 7 b) Define the Q factor Selectivity and 3 Marks JAN 2018 CBCS Q 7 c) A series RLC circuit of \( R = 4 \Omega \), \( L=1\text{mH} \), and \( C=10\mu\text{F} \). Calculate the resonant frequency Q factor half power frequencies and bandwidth 8 Marks Solution:

i) \( LC = 1 \times 10^{-3} \times 10 \times 10^{-6} = 10 \times 10^{-9} \)

\[
f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10 \times 10^{-9}}} = 1.59 \times 10^9 \text{Hz}
\]

ii) \( Q = \frac{1}{R\sqrt{LC}} = \frac{2\pi f_o L}{R} \)

\[
Q = \frac{2\pi f_o L}{R} = \frac{2\pi \times 1.59 \times 10^9 \times 1 \times 10^{-3}}{4} = 2.49
\]

iii) Bandwidth BW

\[
B = \frac{R}{L} = \frac{4}{1 \times 10^{-3}} = 4 \times 10^3 \text{rad/sec}
\]

Bandwidth in Hz

\[
B = \frac{B}{2\pi} = \frac{4 \times 10^3}{2\pi} = 636 \text{Hz}
\]

iv) \( f_1 \) and \( f_2 \)

\[
\frac{R}{2L} = \frac{4}{2 \times 1 \times 10^{-3}} = 2 \times 10^3
\]

\[
f_1 = \frac{1}{2\pi} \left[ -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]
\]

\[
= \frac{1}{2\pi} \left[ -2 \times 10^3 + \sqrt{(2 \times 10^3)^2 + \frac{1}{10 \times 10^{-9}}} \right]
\]

\[
= \frac{1}{2\pi} \left[ -2 \times 10^3 + \sqrt{4 \times 10^6 + 0.1 \times 10^9} \right]
\]

\[
= \frac{1}{2\pi} \left[ -2 \times 10^3 + 10.19 \times 10^3 \right] = 8.198 \times 10^3 \text{Hz}
\]

\[
f_2 = \frac{1}{2\pi} \left[ \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]
\]

\[
= \frac{1}{2\pi} \left[ 2 \times 10^3 + 10.19 \times 10^3 \right] = 12.198 \times 10^3 \text{Hz}
\]

JAN 2018 CBCS Q 8 a) Obtain an expression for resonant frequency in a parallel resonant circuit 6 Marks JAN 2018 CBCS Q 8 b) Obtain an expression for resonant frequency in a parallel resonant circuit 6 Marks

Solution:

i) \( LC = 0.01 \times 0.01 \times 10^{-6} = 1 \times 10^{-10} \)

\[
f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1 \times 10^{-10}}} = 15.9 \times 10^3 \text{Hz}
\]

ii) \( Q = \frac{1}{R\sqrt{LC}} = \frac{2\pi f_o L}{R} \)

\[
Q = \frac{2\pi f_o L}{R} = \frac{2\pi \times 15.9 \times 10^3 \times 0.01}{100} = 10
\]

iii) Bandwidth BW

\[
B = \frac{R}{L} = \frac{100}{0.01} = 10 \times 10^3 \text{rad/sec}
\]

Bandwidth in Hz

\[
B = \frac{B}{2\pi} = \frac{10 \times 10^3}{2\pi} = 1.59 \times 10^3 \text{Hz}
\]

iv) \( f_1 \) and \( f_2 \)

\[
\frac{R}{2L} = \frac{100}{2 \times 0.01} = 5 \times 10^3
\]
f₁ = \frac{1}{2\pi} \left[ \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]

= \frac{1}{2\pi} \left[ -5 \times 10^3 + \sqrt{(5 \times 10^3)^2 + \frac{1}{1 \times 10^{-10}}} \right]

= \frac{1}{2\pi} \left[ -5 \times 10^3 + 25 \times 10^6 + 1 \times 10^{10} \right]

= \frac{1}{2\pi} \left[ 5 \times 10^3 + 100 \times 10^3 \right] = 15.12 \times 10^3 \text{ Hz}

f₂ = \frac{1}{2\pi} \left[ \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]

= \frac{1}{2\pi} \left[ 5 \times 10^3 + 100 \times 10^3 \right] = 16.71 \times 10^3 \text{ Hz}

iv) I₀

I₀ = \frac{V}{R} = \frac{10 \times 10^{-3}}{100} = 0.1 \times 10^{-3} \text{ Ampere}

2017 JAN 5 a) A series R-L-C circuit is fed with 50 V rms supply. At resonance the current through circuit is 25 A and the voltage across inductor is 1250 volts. If \( C = 4\mu F \) determine the values of R, L, Q, resonant frequency, bandwidth and half power frequencies. (12 M)

Solution:

\[ R = \frac{V}{I} = \frac{50}{25} = 2 \Omega \]

At resonance

\[ X_L = X_C \]

Quality factor Q is

\[ Q = \frac{V_L \text{ or } V_C}{E} = \frac{1250}{50} = 25 \]

\[ Q = \frac{R}{2\pi I} = \frac{\sqrt{L}}{C} \Rightarrow \frac{\sqrt{L}}{C} = QR \]

\[ L = (QR)^2C = (25 \times 2)^2 \times 4 \times 10^{-6} = 0.01 \text{ H} \]

\[ f_0 = \frac{1}{2\pi \sqrt{LC}} \]

\[ f_0 = \frac{1}{2\pi \sqrt{0.01 \times 4 \times 10^{-6}}} = 795.7 \text{ Hz} \]

iii) Bandwidth BW

\[ B = \frac{R}{L} = \frac{2}{0.01} = 200 \text{ rad/sec} \]

JULY 2016 Q 5 a) With respect to series resonant circuit define resonant frequency \( f_r \) and half power frequencies \( f_1 \) and \( f_2 \). Also show that the resonant frequency is equal to the geometric mean of half power frequencies 10 Marks

JULY 2016 Q 5 B) A series circuit is energized by constant voltage and constant frequency supply. Resonance takes place due to variation of inductance and the supply frequency is 300 Hz. The capacitance in the circuit is 10\( \mu F \). Determine the value of resistance in the circuit if the quality factor is 5. Also find the value of the inductance at half power frequencies. 10 Marks

Solution: \( LC = 0.01 \times 0.01 \times 10^{-6} = 1 \times 10^{-10} \)

The resonant frequency \( f_r \) is

\[ f_r = \frac{1}{2\pi \sqrt{LC}} \]

\[ \sqrt{LC} = \frac{1}{2\pi f_r} = \frac{1}{2\pi 300} = 5.3 \times 10^{-4} \]

\[ LC = 2.8 \times 10^{-7} \]

\[ L = \frac{2.8 \times 10^{-7}}{10 \times 10^{-6}} = 28.1 \times 10^{-3} \text{ Henry} \]

\[ Q = \frac{2\pi f_r L}{R} \]
1.2. PARALLEL RESONANCE

Chapter 1. Resonance

\[ R = \frac{2\pi f_r L}{Q} = \frac{2\pi \times 300 \times 0.0281}{5} = 10.6 \, \Omega \]

The value of the inductance at half power frequencies is

\[ \omega_r = 2\pi f_r = 2 \times \pi \times 300 = 1.88 \times 10^3 \, \text{radians} \]

\[ L_1 = \frac{1}{\omega^2 C} - \frac{R}{\omega} = \frac{1}{(1.88 \times 10^3)^2 \times 10^{-6} - \frac{10.6}{1.88 \times 10^3}} \approx 0.02829 - 5.638 \times 10^{-3} = 22.65 \times 10^{-3} \, \text{H} \]

\[ L_2 = \frac{1}{\omega^2 C} + \frac{R}{\omega} = \frac{1}{(1.88 \times 10^3)^2 \times 10^{-6} + \frac{10.6}{1.88 \times 10^3}} \approx 0.02829 + 5.638 \times 10^{-3} = 33.93 \times 10^{-3} \, \text{H} \]

DEC/JAN 2015 5 a) What is resonance? Derive an expression for cut-off frequencies 8 Marks

DEC/JAN 2015 5 b) Calculate the half power frequencies of series resonant circuit where the resonance frequency is 150 KHz and bandwidth is 75 KHz 4 Marks

Solution:

Frequency in radians is

\[ \omega_r = 2\pi f_r = 2 \times \pi \times 150 \times 10^3 = 942.5 \times 10^3 \, \text{radians} \]

Bandwidth in radians is

\[ B = 2\pi f_0 = 2 \times \pi \times 75 \times 10^3 = 471.2 \times 10^3 \, \text{radians} \]

\[ \omega_1 = \frac{-B + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2}}{2} = \frac{-235.6 \times 10^3 + \sqrt{55.5 \times 10^9 + 888.3 \times 10^9}}{2} = 736 \times 10^3 \, \text{Radians} \]

\[ \omega_2 = \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_r^2} = \frac{235.6 \times 10^3 + \sqrt{971.5 \times 10^3}}{2} = 1.2 \times 10^6 \, \text{Radians} \]

At resonance current I is

\[ I = \frac{V}{R} = \frac{75}{100} = 0.75 A \]

\[ X_C = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi \times 500 \times 10^3 \times 62.83 \times 10^{-12}} = 5 \times 10^3 \, \Omega \]

\[ V_C = V_L = I X_C = 0.75 \times 5 \times 10^3 = 3750 \, \text{V} \]

JULY 2014 5 a) Define the following terms i) Resonance ii) Q factor iii) Selectivity of series RLC circuit iv) Bandwidth 4 Marks
1.2. PARALLEL RESONANCE

Chapter 1. Resonance

JULY 2014 5 b) Prove that \( f_0 = \sqrt{f_1 f_2} \) where \( f_1 \) and \( f_2 \) are the two half power frequencies of resonant circuit 8 Marks

JULY 2014 5 C) A series RLC circuit has \( R=4 \ \Omega \) \( L=1 \ mH \) and \( C= 10 \ \mu F \). Calculate the Q factor, band width resonant frequency and the half power frequencies \( f_1 \) and \( f_2 \) 8 Marks

Solution:

\[
\begin{align*}
B &= \frac{R}{2\pi L} = \frac{4}{2 \pi \times 1 \times 10^{-3}} = 636.62 \text{Hz} \\
R &= \frac{4}{2 \times 1 \times 10^{-3}} = 2 \times 10^3 \\
\end{align*}
\]

\[
\begin{align*}
f_1 &= \frac{1}{2\pi} \left[ \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right] \\
&= \frac{1}{2\pi} \left[ -2 \times 10^3 + \sqrt{4 \times 10^6 + 0.1 \times 10^9} \right] \\
&= \frac{1}{2\pi} \left[ -2 \times 10^3 + 10198 \right] = 1305 \text{ Hz} \\
f_2 &= \frac{1}{2\pi} \left[ \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right] \\
&= \frac{1}{2\pi} \left[ 2 \times 10^3 + 10198 \right] = 1941 \text{ Hz}
\end{align*}
\]

JULY 2013 5 a) For the series RLC circuit shown in Figure find the resonant frequency, half power frequencies, band width Quality factor and 10 Marks

Solution:

\[
\begin{align*}
f_r &= \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{0.4 \times 0.5}} \\
&= 0.356 \text{ Hz}
\end{align*}
\]

Given data may wrongly printed because frequency is always more than 1 Hz

JULY 2012 5 a) Define the following terms i) Resonance ii) Q factor iii) Selectivity of series RLC circuit iv) Bandwidth 6 Marks

DEC 2012 5 b) A series RLC circuit has \( R=10 \ \Omega \) \( L=0.01 \ mH \) and \( C= 0.01 \ \mu F \) and it is connected across 10 mV supply. Calculate i)\( f_0 \), ii)\( Q_0 \) iii)band width iv) \( f_1 \) and \( f_2 \) v) \( I_0 \) 10 Marks

Solution:

\[
\begin{align*}
f_o &= \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{0.01 \times 0.01 \times 10^{-6}}} \\
&= 15915.5 \text{ Hz} \\
Q_o &= \frac{2\pi f_0 L}{R} = \frac{2\pi \times 15915.5 \times 0.01}{10} = 100 \\
B &= \frac{R}{2\pi L} = \frac{10}{2\pi \times 10^{-3}} = 160 \text{Hz} \\
R &= \frac{R}{2L} = \frac{2 \times 10 \times 10^{-3}} = 500 \\
f_1 &= \frac{1}{2\pi} \left[ \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right] \\
&= \frac{1}{2\pi} \left[ -500 + \sqrt{250 \times 10^3 + 1 \times 10^{10}} \right] \\
&= \frac{1}{2\pi} \left[ -500 + 100 \times 10^3 \right] \\
&= 15836 \text{ Hz} \\
f_2 &= \frac{1}{2\pi} \left[ \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right] \\
&= \frac{1}{2\pi} \left[ 500 + 100 \times 10^3 \right] \\
&= 15995 \text{ Hz} \\
I_o &= \frac{V}{R} = \frac{10 \times 10^{-3}}{10} = 1 \times 10^{-3} \text{A}
\end{align*}
\]

DEC 2011 5 a) Define the following terms i) Resonance ii) Q factor iii) Selectivity of series RLC circuit iv) Bandwidth 6 Marks

JULY 2010 5 b) Derive for resonant circuit \( f_0 = \sqrt{f_1 f_2} \) where \( f_1 \) and \( f_2 \) are the two half power frequencies 8 Marks

Solution:

\[
\begin{align*}
f_r &= \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{100 \times 10^{-3} \times 10 \times 10^{-6}}} \\
&= 0.356 \text{ Hz}
\end{align*}
\]

DEC 2010 5 C) An RLC series circuit has \( R = 1K \ \Omega \) \( L=100 \ mH \) and \( C= 10 \ \mu F \). If a voltage

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of 100 V is applied across series combination determine i) Resonant frequency ii) Q factor, and iii) Half power frequencies 4 Marks

Solution:

\[ f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 10 \times 10^{-6}}} = 159.15 \text{ Hz} \]

ii) Q factor

\[ Q = \frac{2\pi f_r L}{R} = \frac{2\pi \times 159.15 \times 100 \times 10^{-3}}{1 \times 10^3} = 0.1 \]

\[ B = \frac{R}{2\pi L} = \frac{1 \times 10^3}{2\pi \times 100 \times 10^{-3}} = 1592.35 \text{ Hz} \]

iii) Half power frequencies

\[ f_1 = \frac{1}{2\pi} \left[ \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right] \]

\[ = \frac{1}{2\pi} \left[ -5 \times 10^3 + \sqrt{25 \times 10^6 + 1 \times 10^6} \right] = 15.75 \text{ Hz} \]

\[ f_2 = \frac{1}{2\pi} \left[ \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right] \]

\[ = \frac{1}{2\pi} \left[ 5 \times 10^3 + 5099 \right] = 1607 \text{ Hz} \]

June/July 2009 5 C) A series circuit RLC consists of \( R = 1 \text{ KΩ} \) and an inductance of 100 mH in series with capacitance of 10 nF. If a voltage of 100 V is applied across series combination determine i) Resonant frequency ii) maximum current in the circuit iii) Q factor, and iii) Half power frequencies 8 Marks

Solution:

\[ f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 10 \times 10^{-9}}} = 5 \text{ KHz} \]

ii) Maximum current in the circuit

\[ I = \frac{V}{R} = \frac{100}{1 \times 10^3} = 0.1 \text{ A} \]

iii) Q factor

\[ Q = \frac{2\pi f_r L}{R} = \frac{2\pi \times 5 \times 10^3 \times 100 \times 10^{-3}}{1 \times 10^4} = 3.14 \]
iv) Half power frequencies

\[
\frac{R}{2L} = \frac{1 \times 10^3}{2 \times 100 \times 10^{-3}} = 5 \times 10^3
\]

\[
f_1 = \frac{1}{2\pi} \left[ \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]
\]

\[
= \frac{1}{2\pi} \left[ -5 \times 10^3 + \sqrt{25 \times 10^6 + 1 \times 10^9} \right]
\]

\[
= \frac{1}{2\pi} \left[ -5 \times 10^3 + 32 \times 10^3 \right] = 27 \text{ KHz}
\]

\[
f_2 = \frac{1}{2\pi} \left[ \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \right]
\]

\[
= \frac{1}{2\pi} \left[ 5 \times 10^3 + 32 \times 10^3 \right] = 37 \text{ KHz}
\]