Solutions to Chapte 7: Channel Coding

(Bernard Sklar)

Note: State diagram, tree diagram and trellis diagram for K=3 are same only changes will occur in the output that depends upon the connection vector.

7.1 Draw the state diagram, tree diagram and trellis diagram for the K=3, rate=1/3 code generated by
$g_1(X) = X + X^2, \quad g_2(X) = 1 + X + X^2, \quad g_3(X) = 1 + X$

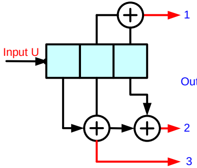


Table: State transition table (rate=1/2, K=3)

Input bit	State at time t_i	State at time $t_i + 1$	Output
0	00	00	000
1	00	10	011
0	01	00	110
1	01	10	101
0	10	01	111
1	10	11	100
0	11	01	001
1	11	11	010

- States represent possible contents of the rightmost K-1 register content.
- For this example there are only two transitions from each state corresponding to two possible input bits.
- Solid line denotes for input bit zero, and dashed line denotes for input bit one.

Tuple	State
00	a
10	b
01	c
11	d

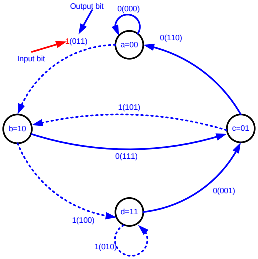


Figure: State diagram for rate=1/3 and K=3

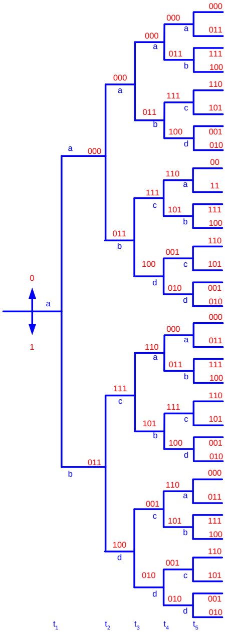


Figure: Tree diagram for rate=1/3 and

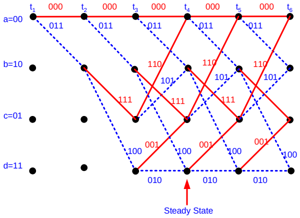


Figure: Trellis diagram for rate=1/3 and K=3

7.2 Given a K=3 and rate 1/2 binary convolutional code with the partially completed state diagram shown in Figure 7.1 find the complete state diagram, and sketch a diagram for encoder

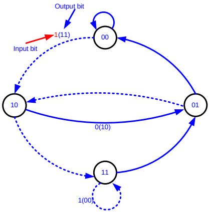


Figure: State diagram for rate=1/2, K=3

Table: State transition table (rate=1/2, K=3)

Input bit	State at time t_i	State at time $t_i + 1$	Output
0	00	00	00
1	00	10	11
0	01	00	01
1	01	10	10
0	10	01	10
1	10	11	01
0	11	01	11
1	11	11	00

- States represent possible contents of the rightmost K-1 register content.
- For this example there are only two transitions from each state corresponding to two possible input bits.

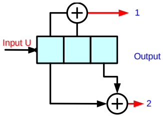


Figure: Encoder diagram

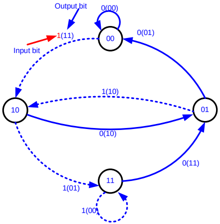


Figure: Modified State diagram

7.3 Draw the state diagram, tree diagram and trellis diagram for the convolutional encoder characterized by the block diagram in Figure p7.2

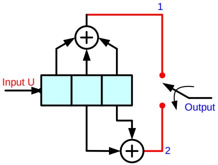


Table: State transition table (rate=1/2, K=3)

Input bit	State at time t_i	State at time t_i+1	Output
0	00	00	00
1	00	10	10
0	01	00	11
1	01	10	01
0	10	01	11
1	10	11	01
0	11	01	00
1	11	11	10

- States represent possible contents of the rightmost K-1 register content.
- For this example there are only two transitions from each state corresponding to two possible input bits.
- Solid line denotes for input bit zero, and dashed line denotes for input bit one.

Tuple	State
00	a
10	b
01	c
11	d

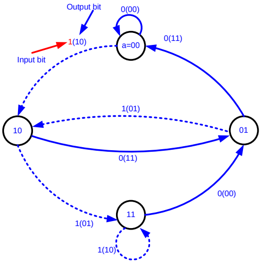
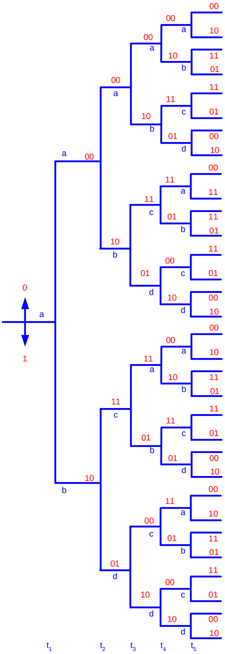


Figure: State diagram for rate=1/3 and K=3



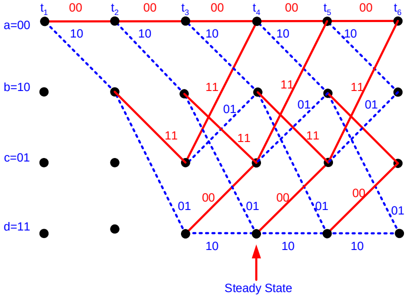
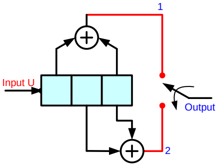


Figure: Trellis diagram for rate=1/3 and K=3

7.5 Consider the convolutional encoder shown in Fiugre a)Write the connection vectors and polynomials for this encoder. b)Draw the state diagram, tree diagram and trellis diagram

Solution: $g_1 = [1\ 0\ 1]$, $g_2 = [0\ 1\ 1]$
$g_1(X) = 1 + X^2$, $g_2(X) = X + X^2$



- States represent possible contents of the rightmost K-1 register content.
- For this example there are only two transitions from each state corresponding to two possible input bits.

Tuple	State
00	a
10	b
01	c
11	d

Table: State transition table (rate=1/2, K=3)

Input bit	State at time t_i	State at time $t_i + 1$	Output
0	00	00	00
1	00	10	10
0	01	00	11
1	01	10	01
0	10	01	01
1	10	11	11
0	11	01	10
1	11	11	00

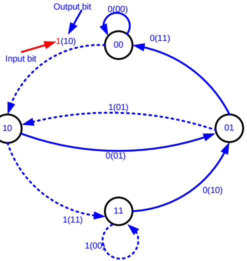


Figure: State diagram for rate=1/3 and K=3

 Solutions

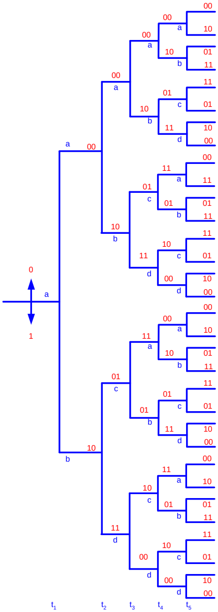


Figure: Tree diagram for rate=1/3 and K=3

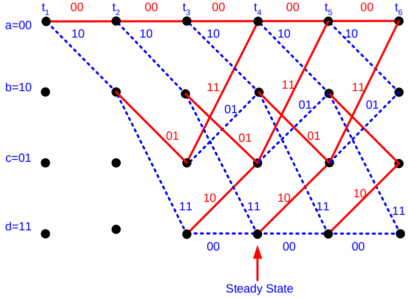


Figure: Trellis diagram for rate=1/3 and K=3

7.6 An encoder diagram is shown in Figure. Find the encoder output for an input sequence **1 0 0 1 0 1 0**

Solution: $g_1 = [1\ 0\ 1]$, $g_2 = [1\ 1\ 1]$
$g_1(X) = 1 + X^2$, $g_2(X) = 1 + X + X^2$

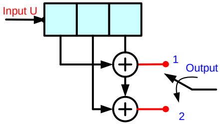
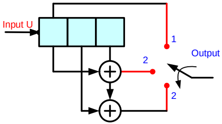


Table: State transition table (rate=1/2, K=3)

Input bit	Register Contents	State at time t_i	State at time $t_i + 1$	Output
-	000	00	00	00
1	100	00	10	11
0	010	10	01	01
0	001	01	00	11
1	100	00	10	11
0	010	10	01	01
1	101	01	10	00
0	010	10	01	01

7.6 Figure shows an encoder for a (3,2) convolutional code. Find the transfer function $T(D)$ and minimum free distance for this code. Also, draw the state diagram for the code.



References

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