Solutions to problems from Chapter 3

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- The information sequences $u = [u_0\ u_1\ u_2\ u_3]$ are encoded into codewords v of length n = 8, using a systematic encoder. If we assume that the systematic positions in a codeword are the last k = 4 positions, then codewords have the form

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  $v = [v_0\ v_1\ v_2\ v_3\ u_0\ u_1\ u_2\ u_3]$
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$$H = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right]$$

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$$G = \begin{bmatrix} 0 & 1 & 1 & 1 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & | & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & | & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & | & 0 & 0 & 0 & 1 \end{bmatrix}$$

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- there are no two identical columns in H $d_{min} > 2$
- there are no groups of 3 columns that sum up to 0 $d_{min} > 3$
- there exists a group of 4 columns (for example, columns 1,2,3,6) that sum up to 0 $d_{min} = 42$.

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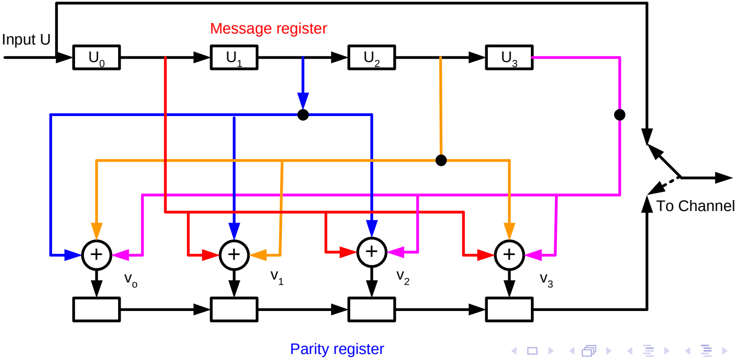

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- Syndrome vector is obtained from the received sequence $r = [r_0 \;\; r_1 \ldots r_7]$ using the parity check matrix of the code:

$$s = [s_0 \; s_1 \; s_2 \; s_3] = rH^T :$$

$$= (r_0, \; r_1, \; r_2, \; r_3, \; r_4, \; r_5, \; r_6, \; r_7) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

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- By parity check matrix the syndrome digits are:

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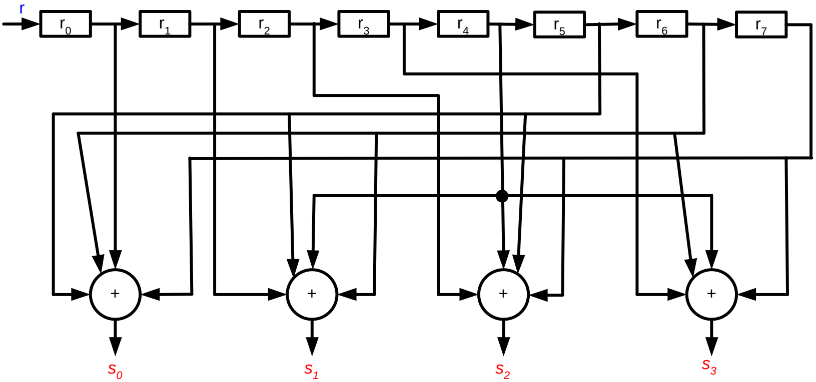

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- Let A_i denote the number of codewords of weight i in an (n; k) code C. Then the numbers $A_0, A_1, \ldots An$ are called the weight distribution of the code. For any linear code A0 = 1 (every linear code must contain the all-zero codeword). The first next non-zero element of the weight distribution of a linear code is $A_{d_{min}}$, corresponding to the number of minimal-weight codewords.
- For the (8; 4) code with the minimum distance $d_{min} = 4$, all the codewords except the all-zero and the all-one codeword have minimum weight, that is, the weight distribution is
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- If $p = 10^{-2}$ then $P_u(E) = 1.3448x10^{-7}$.

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