

Solutions to Chapter 10: Communication-Through-BLF-Channels:[1, 2, 3, 4, 5]

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Solutions to Chapter 10: Communication-Through-BLF-Channels

(John G Proakis)



10.2 In a binary PAM system, the clock that specifies the sampling of the correlator output is offset from the optimum sampling time by 10%. If the signal pulse used is rectangular

- Determine the loss in SNR due to the mistiming.
- Determine the amount of ISI introduced by mistiming and determine its effect on performance.

Solution:

- If the transmitted signal is :

$$r(t) = \sum_{n=-\infty}^{\infty} I_n h(t - nT) + n(t)$$

then the output of the receiving filter is :

$$y(t) = \sum_{n=-\infty}^{\infty} I_n x(t - nT) + v(t)$$

where $x(t) = h(t) * h(t)$ and $v(t) = n(t) * h(t)$

If the sampling time is off by 10%, then the samples at the output of the correlator are taken at

$$t = (m \pm \frac{1}{10})T.$$

Assuming that

$$t = (m - \frac{1}{10})T.$$

without loss of generality, then the sampled sequence is :

$$y(m) = \sum_{n=-\infty}^{\infty} I_n x((m - \frac{1}{10})T - nT) + v(m - \frac{1}{10})T$$



If the signal pulse is rectangular with amplitude A and duration T, then

$$\sum_{n=-\infty}^{\infty} I_n x\left(m - \frac{1}{10}T - nT\right)$$

is nonzero only for $n = m$ and $n = m-1$. Therefore, the sampled sequence is given by :

$$y_m = \sum_{n=-\infty}^{\infty} I_n x\left(-\frac{1}{10}T\right) + I_{m-1} x\left(-\frac{1}{10}T\right) + v\left(m - \frac{1}{10}T\right)$$

$$= \frac{9}{10} I_m A^2 T + I_{m-1} A^2 T + v\left(m - \frac{1}{10}T\right)$$

The variance of the noise is :

$$\sigma_v^2 = \frac{N_0}{2} A^2 T$$

and therefore, the SNR is :

$$\text{SNR} = \left(\frac{9}{10}\right)^2 \frac{2(A^2 T)^2}{N_0 A^2 T} = \frac{81}{100} \frac{2A^2 T}{N_0}$$

As it is observed, there is a loss of

$$10 \log_{10} \frac{81}{100} = -0.9151 \text{ db}$$

due to the mistiming.



(b) Recall from part (a) that the sampled sequence is

$$y_m = \frac{9}{10} I_m A^2 T + I_{m-1} \frac{1}{10} A^2 T + v_m$$

The term

$$I_{m-1} \frac{1}{10} A^2 T$$

expresses the ISI introduced to the system. If $I_m = 1$ is transmitted, then the probability of error is

$$\begin{aligned} P(e|I_m = 1) &= \frac{1}{2} P(e|I_m = 1, I_{m-1} = 1) + \frac{1}{2} P(e|I_m = 1, I_{m-1} = -1) \\ &= \frac{1}{2\sqrt{\pi N_0 A^2 T}} \int_{-\infty}^{-A^2 T} e^{-\frac{v^2}{N_0 A^2 T}} dv + \frac{1}{2\sqrt{\pi N_0 A^2 T}} \int_{-\infty}^{-\frac{8}{10} A^2 T} e^{-\frac{v^2}{N_0 A^2 T}} dv \\ &= \frac{1}{2} Q \left[\sqrt{\frac{2A^2 T}{N_0}} \right] + \frac{1}{2} Q \left[\sqrt{\left(\frac{8}{10}\right)^2 \frac{2A^2 T}{N_0}} \right] \end{aligned}$$

- Since the symbols of the binary PAM system are equiprobable the previous derived expression is the probability of error when a symbol by symbol detector is employed.
- Comparing this with the probability of error of a system with no ISI, we observe that there is an increase of the probability of error by

$$p_{diff}(e) = \frac{1}{2} Q \left[\sqrt{\left(\frac{8}{10}\right)^2 \frac{2A^2 T}{N_0}} \right] - \frac{1}{2} Q \left[\sqrt{\frac{2A^2 T}{N_0}} \right]$$



10.4: A wireline channel of length 1000km is used to transmit data by means of binary PAM. Regenerative repeaters are spaced 50km apart along the system. Each segment of the channel has an ideal(constant) frequency response over the frequency band $0 \leq f \leq 1200$ Hz and an attenuation of 1dB/km. The channel noise is AWGN.

- (a) What is the highest bit rate that can be transmitted without ISI?
- (b) Determine the required $\frac{\epsilon_b}{N_0}$ to achieve a bit error of $P_2 = 10^{-7}$ for each repeater.
- (c) Determine the transmitted power at each repeater to achieve the desired $\frac{\epsilon_b}{N_0}$ where $N_0 = 4.1 \times 10^{-21}$ W/Hz

Solution

- (a) Each segment of the wire-line can be considered as a bandpass filter with bandwidth $W = 1200$ Hz. Thus, the highest bit rate that can be transmitted without ISI by means of binary PAM is :
 $R = 2W = 2400$ bps



(b) The probability of error for binary PAM transmission is :

$$P_2 = Q \left[\sqrt{\frac{2\varepsilon_b}{N_0}} \right]$$

Hence, using mathematical tables for the function Q we find that $P_2 = 10^{-7}$ is obtained for :

$$\sqrt{\frac{2\varepsilon_b}{N_0}} = 5.2 \Rightarrow \frac{\varepsilon_b}{N_0} = 13.52 = 11.30dB$$

(c) The received power P_R is related to the desired SNR per bit through the relation :

$$\frac{P_R}{N_0} = \frac{1}{T} \frac{\varepsilon_b}{N_0} = R \frac{\varepsilon_b}{N_0}$$

Hence, with $N_0 = 4.1 \times 10^{-21}$ W/Hz we obtain :

$$P_R = 4.1 \times 10^{-21} \times 1200 \times 13.52 = 6.6518 \times 10^{-17} = -161.77dBW$$

Since the power loss of each segment is :

$$L_s = 50Km \times 1dB/Km = 50dB$$

the transmitted power at each repeater should be :

$$P_T = P_R + L_s = -161.77 + 50 = -111.77dBW$$



E&CE 411, Spring 2009, Table of Q Function

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Table 1: Values of $Q(x)$ for $0 \leq x \leq 9$

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0.00	0.5	2.30	0.010724	4.55	2.6823×10^{-6}	6.80	5.231×10^{-12}
0.05	0.48046	2.35	0.0093867	4.60	2.1125×10^{-6}	6.85	3.6925×10^{-12}
0.10	0.46017	2.40	0.0081975	4.65	1.6597×10^{-6}	6.90	2.6001×10^{-12}
0.15	0.44038	2.45	0.0071428	4.70	1.3008×10^{-6}	6.95	1.8264×10^{-12}
0.20	0.42074	2.50	0.0062097	4.75	1.0171×10^{-6}	7.00	1.2798×10^{-12}
0.25	0.40129	2.55	0.0053861	4.80	7.9333×10^{-7}	7.05	8.9459×10^{-13}
0.30	0.38209	2.60	0.0046612	4.85	6.1731×10^{-7}	7.10	6.2378×10^{-13}
0.35	0.36317	2.65	0.0040246	4.90	4.7918×10^{-7}	7.15	4.3389×10^{-13}
0.40	0.34458	2.70	0.003467	4.95	3.7107×10^{-7}	7.20	3.0106×10^{-13}
0.45	0.32636	2.75	0.0029798	5.00	2.8665×10^{-7}	7.25	2.0839×10^{-13}
0.50	0.30854	2.80	0.0025531	5.05	2.2091×10^{-7}	7.30	1.4388×10^{-13}
0.55	0.29116	2.85	0.002186	5.10	1.6983×10^{-7}	7.35	9.9103×10^{-14}
0.60	0.27425	2.90	0.0018658	5.15	1.3024×10^{-7}	7.40	6.8092×10^{-14}
0.65	0.25785	2.95	0.0015889	5.20	9.9644×10^{-8}	7.45	4.667×10^{-14}
0.70	0.24196	3.00	0.0013499	5.25	7.605×10^{-8}	7.50	3.1909×10^{-14}
0.75	0.22663	3.05	0.0011442	5.30	5.7901×10^{-8}	7.55	2.1763×10^{-14}
0.80	0.21186	3.10	0.0009676	5.35	4.3977×10^{-8}	7.60	1.4807×10^{-14}
0.85	0.19766	3.15	0.00081635	5.40	3.332×10^{-8}	7.65	1.0049×10^{-14}
0.90	0.18406	3.20	0.00068714	5.45	2.5185×10^{-8}	7.70	6.8033×10^{-15}
0.95	0.17106	3.25	0.00057703	5.50	1.899×10^{-8}	7.75	4.5946×10^{-15}
1.00	0.15866	3.30	0.00048342	5.55	1.4283×10^{-8}	7.80	3.0954×10^{-15}
1.05	0.14686	3.35	0.00040406	5.60	1.0718×10^{-8}	7.85	2.0802×10^{-15}
1.10	0.13567	3.40	0.00033693	5.65	8.0224×10^{-9}	7.90	1.3945×10^{-15}
1.15	0.12507	3.45	0.00028029	5.70	5.9904×10^{-9}	7.95	9.3256×10^{-16}
1.20	0.11507	3.50	0.00023263	5.75	4.4622×10^{-9}	8.00	6.221×10^{-16}
1.25	0.10565	3.55	0.00019262	5.80	3.3157×10^{-9}	8.05	4.1397×10^{-16}
1.30	0.0968	3.60	0.00015911	5.85	2.4579×10^{-9}	8.10	2.748×10^{-16}
1.35	0.088598	3.65	0.00013112	5.90	1.8175×10^{-9}	8.15	1.8196×10^{-16}
1.40	0.080757	3.70	0.0001079	5.95	1.3407×10^{-9}	8.20	1.2019×10^{-16}
1.45	0.073529	3.75	8.8417×10^{-5}	6.00	9.8650×10^{-10}	8.25	7.9197×10^{-17}
1.50	0.066807	3.80	7.2348×10^{-5}	6.05	7.2423×10^{-10}	8.30	5.2056×10^{-17}
1.55	0.060571	3.85	5.9059×10^{-5}	6.10	5.3034×10^{-10}	8.35	3.4131×10^{-17}
1.60	0.054799	3.90	4.8096×10^{-5}	6.15	3.8741×10^{-10}	8.40	2.2324×10^{-17}
1.65	0.049471	3.95	3.9076×10^{-5}	6.20	2.8232×10^{-10}	8.45	1.4565×10^{-17}
1.70	0.044565	4.00	3.1671×10^{-5}	6.25	2.0523×10^{-10}	8.50	9.4795×10^{-18}
1.75	0.040069	4.05	2.5609×10^{-5}	6.30	1.4882×10^{-10}	8.55	6.1544×10^{-18}
1.80	0.03593	4.10	2.0658×10^{-5}	6.35	1.0766×10^{-10}	8.60	2.9858×10^{-18}
1.85	0.032157	4.15	1.6624×10^{-5}	6.40	7.7688×10^{-11}	8.65	2.375×10^{-18}
1.90	0.028717	4.20	1.3346×10^{-5}	6.45	5.5925×10^{-11}	8.70	1.6294×10^{-18}
1.95	0.025588	4.25	1.0689×10^{-5}	6.50	4.016×10^{-11}	8.75	1.0668×10^{-18}
2.00	0.02275	4.30	8.5399×10^{-6}	6.55	2.8769×10^{-11}	8.80	6.8408×10^{-19}
2.05	0.020182	4.35	6.8069×10^{-6}	6.60	2.0558×10^{-11}	8.85	4.376×10^{-19}
2.10	0.017864	4.40	5.4125×10^{-6}	6.65	1.4655×10^{-11}	8.90	2.7923×10^{-19}
2.15	0.015778	4.45	4.2935×10^{-6}	6.70	1.0421×10^{-11}	8.95	1.7774×10^{-19}
2.20	0.013903	4.50	3.3977×10^{-6}	6.75	7.3923×10^{-12}	9.00	1.1286×10^{-19}
2.25	0.012224						

Figure: Q table



10.10 Binary PAM is used to transmit information over an unequalized linear filter channel. When $a=1$ is transmitted, the noise-free output of the demodulator is

$$x_m = \begin{cases} 0.3 & (m = 1) \\ 0.9 & (m = 0) \\ 0.3 & (m = -1) \\ 0 & (\text{otherwise}) \end{cases}$$

- (a) Design a three tap zero forcing linear equalizer so that the output is $q_m = \begin{cases} 1 & (m = 0) \\ 0 & (m \neq \pm 1) \end{cases}$
- (b) Determine q_m for $m = \pm 2, \pm 3$, by convolving the impulse response of the equalizer with the channel response.



By solving the previous matrix equation the coefficients:

$$c_{-1} = \frac{\begin{pmatrix} 0 & 0.3 & 0 \\ 1 & 0.9 & 0.3 \\ 0 & 0.3 & 0.9 \end{pmatrix}}{\Delta} = -0.4762$$

where

$$\Delta = \begin{vmatrix} 0.9 & 0.3 & 0 \\ 0.3 & 0.9 & 0.3 \\ 0 & 0.3 & 0.9 \end{vmatrix} = 0.567$$

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} -0.4762 \\ 1.4286 \\ -0.4762 \end{pmatrix}$$



By solving the previous matrix equation the coefficients:

$$c_{-1} = \frac{\begin{pmatrix} 0 & 0.3 & 0 \\ 1 & 0.9 & 0.3 \\ 0 & 0.3 & 0.9 \end{pmatrix}}{\Delta} = -0.4762$$

where

$$\Delta = \begin{vmatrix} 0.9 & 0.3 & 0 \\ 0.3 & 0.9 & 0.3 \\ 0 & 0.3 & 0.9 \end{vmatrix} = 0.567$$

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} -0.4762 \\ 1.4286 \\ -0.4762 \end{pmatrix}$$

(b) The values of q_m at $m = \pm 2, \pm 3$ are given by

$$q_2 = \sum_{n=-1}^1 c_n h_{2-n} = c_1 h_1 = -0.1429$$

$$q_{-2} = \sum_{n=-1}^1 c_n h_{-2-n} = c_{-1} h_{-1} = -0.1429$$

$$q_3 = \sum_{n=-1}^1 c_n h_{3-n} = 0 \quad \text{and} \quad q_{-3} = \sum_{n=-1}^1 c_n h_{-3-n} = 0$$



10.11 The transmission of a signal pulse with a raised cosine spectrum through a channel results in the following (noise-free) sampled output from the demodulator:

$$x_k = \begin{cases} -0.5 & (k = -2) \\ 0.1 & (k = -1) \\ 1 & (k = 0) \\ -0.2 & (k = 1) \\ -0.05 & (k = 2) \\ 0 & (\text{otherwise}) \end{cases}$$

- (a) Determine the tap coefficients of a three tap linear equalizer based on the zero-forcing criterion.
 (b) For the coefficients determined in (a), determine the output of the equalizer for the case of the isolated pulse. Thus determine the residual ISI and its span in time.

Solution:

(a) The discrete-time impulse response of the output is :

Therefore $x_{-2} = -0.5$, $x_{-1} = 0.1$, $x_0 = 1$, $x_1 = -0.2$, $x_2 = -0.05$ with $q_0 = 1$ and $q_m = 0$ for $m \neq 0$
 If $\{c_n\}$ denote the coefficients of the equalizer, then the output of the three tap zero-force equalizer is:

$$q_m = \sum_{n=-1}^1 c_n x_{m-n}$$

$$q_{-1} = c_{-1}x_0 + x_0x_{-1} + c_1x_{-2}$$

$$q_0 = c_{-1}x_1 + c_0x_0 + c_1x_{-1}$$

$$q_1 = c_{-1}x_2 + c_0x_1 + c_1x_0$$

$$\begin{pmatrix} x_0 & x_{-1} & x_{-2} \\ x_1 & x_0 & x_{-1} \\ x_2 & x_1 & x_0 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0.1 & -0.5 \\ -0.2 & 1 & 0.1 \\ -0.05 & -0.2 & 1 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$



By solving the previous matrix equation the coefficients:

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0.000 \\ 0.980 \\ 0.196 \end{pmatrix}$$

(b) The output of the equalizer is :

$$q_m = \begin{cases} 0(m \leq -4) \\ c_{-1}x_{-2} = 0(m = -3) \\ c_{-1}x_{-1} + c_0x_{-2} = -0.49(m = -2) \\ 0(m = -1) \\ 1(m = 0) \\ 0(m = 1) \\ c_0x_2 + c_1x_1 = 0.0098(m = 2) \\ c_1x_2 = 0.0098(m = 3) \\ 0(m \geq 4) \end{cases}$$

Hence, the residual ISI sequence is =

$$\{\dots, 0, -0.49, 0, 0, 0, 0.0098, 0.0098, 0\dots\}$$

and its span is 6 symbols.



10.12 A non ideal band limited channel introduces ISI over three successive symbols. The (noise-free) response of the matched filter demodulator sampled at the sampling time kT is:

$$\int_{-\infty}^{\infty} s(t)s(t - kT)dt = \begin{cases} \epsilon_b(k = 0) \\ 0.9\epsilon_b(k = \pm 1) \\ 0.1\epsilon_b(k = \pm 2) \\ 0(\text{otherwise}) \end{cases}$$

- (a) Determine the tap coefficients of a three tap linear equalizer that equalizes the channel (received signal) response to an equivalent partial response (duobinary) signal $y_k = \begin{cases} \epsilon_b(k = 0, 1) \\ 0(\text{otherwise}) \end{cases}$
- (b) Suppose that the linear equalizer in (a) is followed by a viterbi sequence detector for the partial signal. Give an estimate of the error probability if the additive noise is white and gaussian, with power spectral density $\frac{1}{2} N_0$ W/Hz

Solution:

- (a) If c_n denote the coefficients of the zero-force equalizer and q_m is the 'sequence of the equalizers output samples, then :

$$q_m = \sum_{n=-1}^1 c_n x_{m-n}$$

where x_k is the noise free response of the matched filter demodulator sampled at $t = kT$. With $q_{-1} = 0, q_0 = q_1 = \epsilon_b$, we obtain the system :

$$\begin{aligned} q_{-1} &= c_{-1}x_0 + x_0x_{-1} + c_1x_{-2} \\ q_0 &= c_{-1}x_1 + c_0x_0 + c_1x_{-1} \\ q_1 &= c_{-1}x_2 + c_0x_1 + c_1x_0 \end{aligned} \quad \begin{pmatrix} x_0 & x_{-1} & x_{-2} \\ x_1 & x_0 & x_{-1} \\ x_2 & x_1 & x_0 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \epsilon_b & 0.9\epsilon_b & 0.1\epsilon_b \\ 0.9\epsilon_b & \epsilon_b & 0.9\epsilon_b \\ 0.1\epsilon_b & 0.9\epsilon_b & \epsilon_b \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ \epsilon_b \\ \epsilon_b \end{pmatrix}$$

The solution to the system is : $(c_{-1} \ c_0 \ c_1) = (0.2137 \ -0.3846 \ 1.3248)$



10.13 Determine the tap weight coefficients of a three-tap zero-forcing equalizer if the ISI spans three symbols and is characterized by the values $x(0)=1$, $x(-1)=0.3$, $x(1)=0.2$. Also determine the residual ISI at the output of the equalizer for the optimum tap coefficients.

Solution:

$$q_m = \sum_{n=-1}^1 c_n x_{m-n}$$

where x_k is the noise free response of the matched filter demodulator sampled at $t = kT$. With $q_{-1} = 0$, $q_0 = q_1 = \varepsilon_b$, we obtain the system :

$$\begin{aligned} q_{-1} &= c_{-1}x_0 + x_0x_{-1} + c_1x_{-2} \\ q_0 &= c_{-1}x_1 + c_0x_0 + c_1x_{-1} \\ q_1 &= c_{-1}x_2 + c_0x_1 + c_1x_0 \end{aligned} \quad \begin{pmatrix} x_0 & x_{-1} & x_{-2} \\ x_1 & x_0 & x_{-1} \\ x_2 & x_1 & x_0 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1.0 & 0.3 & 0.0 \\ 0.2 & 1.0 & 0.3 \\ 0.0 & 0.2 & 0.3 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

The solution to the system is : $(c_{-1} \ c_0 \ c_1) = (-0.3409 \ 1.1364 \ -0.2273)$ The output of the equalizer is :

$$q_m = \begin{cases} 0 & (m \leq -3) \\ c_{-1}x_{-1} = -0.1023 & (m = -1) \\ 0 & (m = -1) \\ 1 & (m = 0) \\ 0 & (m = 1) \\ c_1x_1 = -0.0455 & (m = 2) \\ 0 & (m \geq 3) \end{cases}$$






Hence, the residual ISI sequence is: $\dots, 0, -0.1023, 0, 0, 0, -0.0455, 0, \dots$



Thank You



References

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