Spread Spectrum Signals for Digital Communication

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Spread Spectrum Signals for Digital Communication

1. Model of Spread Spectrum Digital Communication System
2. Direct Sequence Spread Spectrum Signals
3. Frequency-Hopped Spread Spectrum Signals
4. CDMA
5. Time-hopping SS
6. Synchronization of SS systems

Generation of Pseudo Noise codes

 Generation of Pseudo Noise codes

Spread Spectrum Modulation (SSM) Background

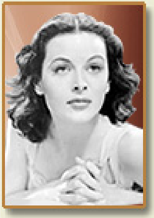
- Used in military for the past 50 years
- Its commercial use started in 1980
- Frequency Hopping: Invented in 1940 by Hedy Lamarr

Hedy Lamarr and George Antheil. Photo of Hedy Lamarr courtesy of the Academy of Motion Picture Arts & Sciences. Photo of George Antheil courtesy of the Estate of George Antheil.

Figure: Hedy Lamarr and George Antheil [1]

Hedy Lamarr

Invention of Spread Spectrum Technology



Although better known for her Silver Screen exploits, Austrian actress Hedy Lamarr (born Hedwig Eva Maria Kiesler) also became a pioneer in the field of wireless communications following her emigration to the United States. The international beauty icon, along with co-inventor George Anthiel, developed a "Secret Communications System" to help combat the Nazis in World War II. By manipulating radio frequencies at irregular intervals between transmission and reception, the invention formed an unbreakable code to prevent classified messages from being intercepted by enemy personnel.

Lamarr and Anthiel received a patent in 1941, but the enormous significance of their invention was not realized until decades later. It was first implemented on naval ships during the Cuban Missile Crisis and subsequently emerged in numerous military applications. But most importantly, the "spread spectrum" technology that Lamarr helped to invent would galvanize the digital communications boom, forming the technical backbone that makes cellular phones, fax machines and other wireless operations possible.

Figure: Hedy Lamarr invention details [2]

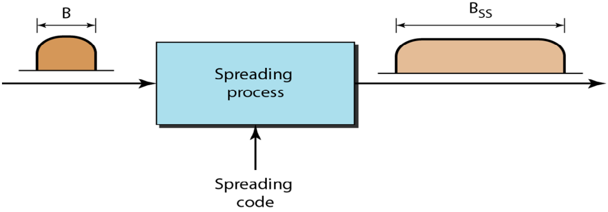
- Spread spectrum modulation was originally developed for military to be used in the battle ground and in the hostile territories where the enemy always tries to intrude into the communication system of the friendly forces to steal information and to jam the systems.

Definition: Spread spectrum is a means of transmission in which the signal occupies a bandwidth in excess of the minimum necessary to send the information: the band spread is accomplished by means of a code which is independent of the data, and synchronized reception with the code at the receiver is used for de-spreading and subsequent data recovery.

Applications and advantages of spread spectrum systems

- Low density power spectra for signal (To avoid being detected.)
- To prevent eavesdropping.
- To prevent the jamming of signals.
- Improved interference rejection
- CDMA applications
- Secure communication
- High resolution ranging
- Antijam capability
- Increased capacity and spectral efficiency in some mobile-cellular pesonal communication system applications
- Lower cost of implementation
- Readily available IC components

- A type of modulation in which the modulated signal bandwidth is much greater than the message signal bandwidth.
- The spreading of the message signal spectrum is done by a spreading code called Pseudo Noise Code (PN Code) which is independent of the message signal.



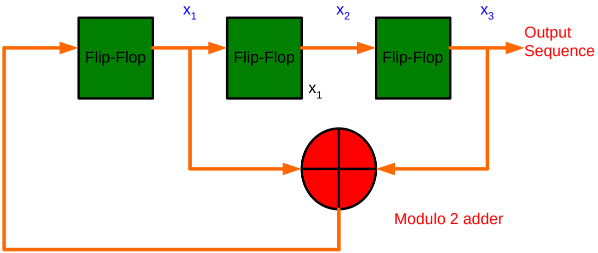
Types of Spread Spectrum

- Direct Sequence Spread Spectrum (DS/SS)
- Frequency Hopping Spread Spectrum (FH/SS)
  1. Slow Frequency Hopping Spread Spectrum
  2. Fast Frequency Hopping Spread Spectrum
- Time Hopping Spread Spectrum (TH/SS)
- Hybrid spread spectrum methods

Spreading Codes

- Maximal length sequences
  - Good auto and cross correlation, small code set
- Gold codes and Kasami sequences
  - Are derived from M-sequences with similar correlation properties, and a larger code set.
- Walsh and Hadamard sequences
  - zero correlation between codes when aligned cross-correlation non-zero when time shifted fixed spreading factor (codes of different length are not orthogonal)

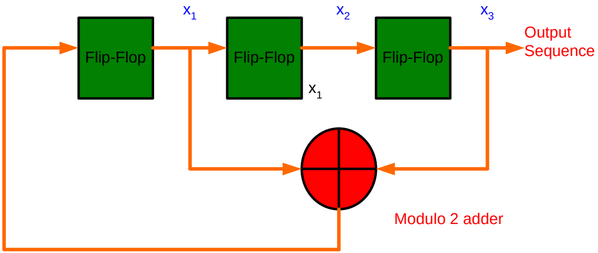
Pseudo Noise (PN) Sequence



- 100, 110, 111, 011 101, 010, 001, 100,
- 0011101

$N = 2^m - 1$

- A pseudo noise (PN) sequence is defined as a coded sequence of 1s and 0s with a certain autocorrelation properties.
- The class of sequence used in spread spectrum communication is usually periodic.
- The major tasks of PN sequences are:
  1. Spreading the bandwidth of the modulated signal to the larger bandwidth.
  2. Distinguishing between the different user signals utilizing the same transmission bandwidth.
- The maximum length sequence is a type of cyclic code represents a commonly used PN sequence.



- Consider the initial state of the shift register is 100 (i.e, x1=1, x2=0 and x3=0).
- Then, the succession of state will be as follows: 100, 110, 111, 011,101, 010, 001, 100
- The output of the sequence (the last position of each of the shift register) is therefore 0011101. The choice of initial state 100 is an arbitrary one.
- Any of the other six states could serve equally well as an initial state.

1. Property 1: Balance Property
   In each period of a maximum-length sequence, the number of 1s is always one more than the number of 0s.
2. Property2: Run Property
   Among the runs of 1s and of 0s in each period of maximum-length sequence, one-half the runs of each kind are of length one, one-fourth are of length two, one-eighth are of length three, and so on as long as these fractions represent meaningful numbers of runs
3. Property3: Autocorrelation property
   The autocorrelation function of a maximum-length sequence is periodic and binary valued.

Property3: Autocorrelation property

- It is a measure of similarity between a signal $f(t)$ and τ second time-shifted replica of itself.
- The autocorrelation function is plot of autocorrelation over all shifts $(-t)$ of the signal.
- Autocorrelation $R_a(\tau)$ in general is defined as:

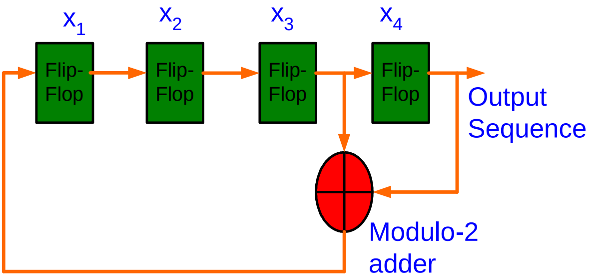
$$R_a(\tau) = \int_{-\infty}^{\infty} f(t)f(t-\tau)dt$$

The autocorrelation function of a maximum length sequence is periodic and binary valued. This property is called as correlation property.

$$R_c(\tau) = \frac{1}{L}\sum_{i=0}^{L-1} C_i * C_{(i+\tau)} \qquad mod\ L \qquad \tau = 0, 1, \ldots L-1$$

If symbol 1 and 0 are represented by +1 volt and -1 volt then autocorrelation has only two values.

$$R_c(\tau) = \begin{cases} 1 & for\ \tau = kL \quad k = 0, 1, 2, \ldots \\ -\frac{1}{L} & for\ \tau \neq kL \quad k = 0, 1, 2, \ldots \end{cases}$$



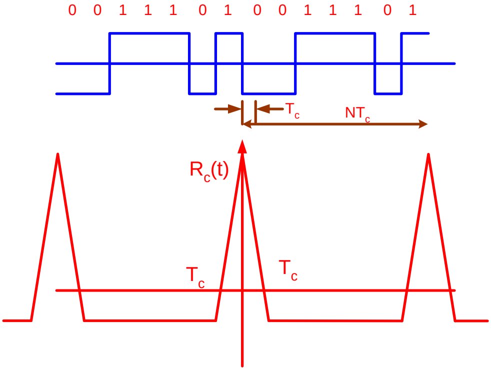
- 1000, 0100, 0010, 1001, 1100, 0110, 1011, 0101, 1010, 1101, 1110, 1111, 0111, 0011, 0001, 1000,
- 000100110101111
- $N = 2^m - 1$
- 000100110101111

$0011101 \Longrightarrow C_0 C_1 C_2 C_3 C_4 C_5 C_6 \;\; C_0 C_1 C_5 = -1 \;\; C_2 C_3 C_4 C_6 = 1$

$$R_c(0) = \frac{1}{7}\left(C_0^2 + C_1^2 + C_2^2 + C_3^2 + C_4^2 + C_5^2 + C_6^2\right) = 1$$

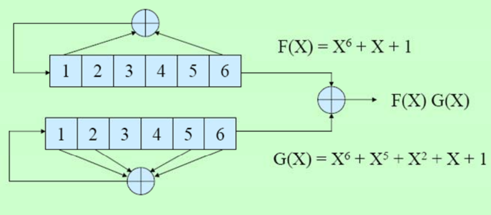
$$R_c(1) = \frac{1}{7}\left(C_0.C_6 + C_1.C_0 + C_2.C_1 + C_3.C_2 + C_4.C_3 + C_5.C_4 + C_6.C_5\right) = -\frac{1}{7}$$

$$= \frac{1}{7}\left(-1.1 + -1. - 1 + 1. - 1 + 1.1 + 1.1 + -1.1 + 1. - 1\right) = -\frac{1}{7}$$



Gold codes

- Combining two m-sequences creates Gold codes.
- Gold sequences are an important class of sequences that allow construction of long sequences with three valued Auto Correlation Function ACFs.
- Gold sequences are constructed from pairs of preferred m-sequences by modulo-2 addition of two maximal sequences of the same length.
- Gold sequences are in useful in non-orthogonal (asynchronous) CDMA.
- The use of Gold sequences permits the transmission to be asynchronous.
- The receiver can synchronize using the auto-correlation property of the Gold sequence.

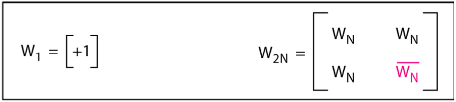


Walsh Codes

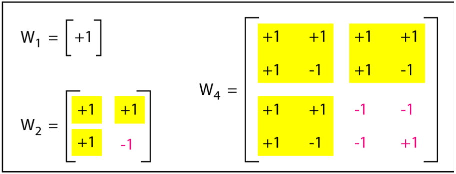
- There is a simple method on how to generate a form of orthogonal Walsh codes.

$$H_0 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_k = \begin{bmatrix} H_{k-1} & H_{k-1} \\ H_{k-1} & -H_{k-1} \end{bmatrix}$$

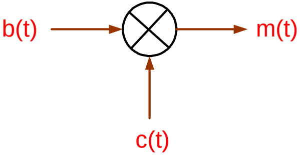


a. Two basic rules

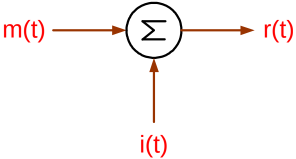


b. Generation of W_1, W_2, and W_4

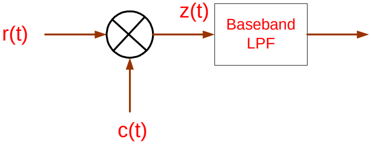
Notion of spread spectrum



$$m(t) = c(t)b(t)$$



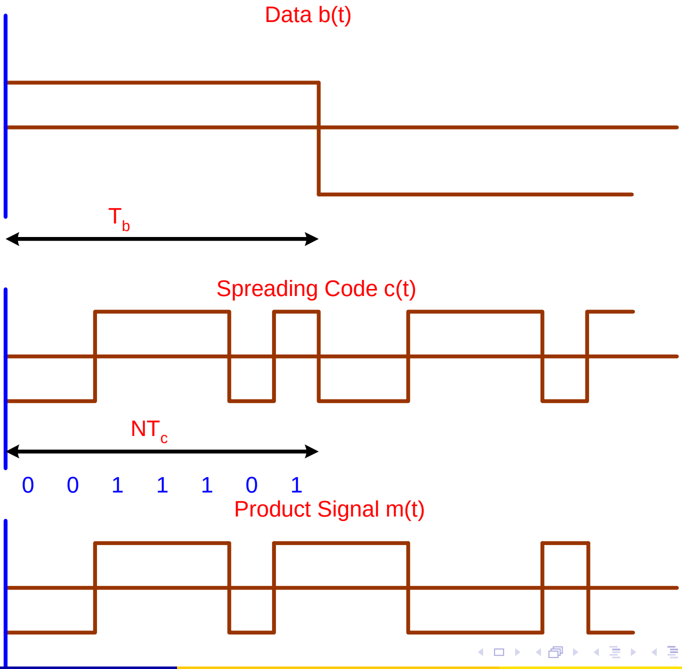
$$r(t) = m(t) + i(t) = c(t)b(t) + i(t)$$



$$z(t) = c(t)r(t) = c^2(t)b(t) + c(t)i(t)$$

$$c^2(t) = 1$$

$$z(t) = b(t) + c(t)i(t)$$


Data b(t)
Tb
Spreading Code c(t)
NTc
0 0 1 1 1 0 1
Product Signal m(t)

Direct Sequence Spread Binary PSK

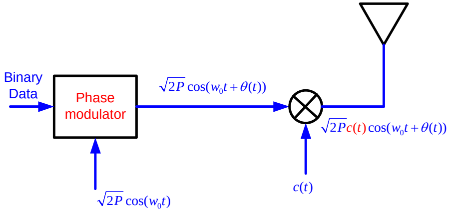


Figure: DSS Transmitter

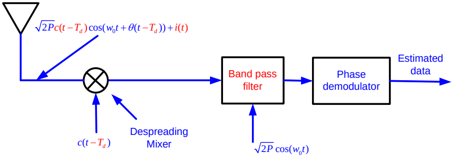


Figure: DSS Receiver

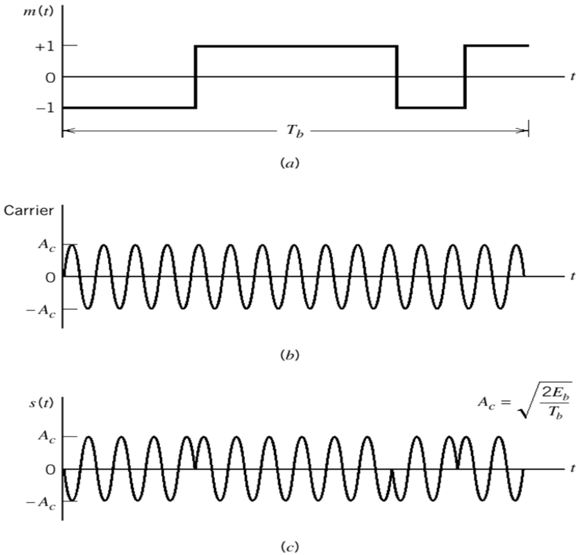


Figure: (a) m(t)=c(t).b(t) (b)Carrier (c) DS-BPSK signal

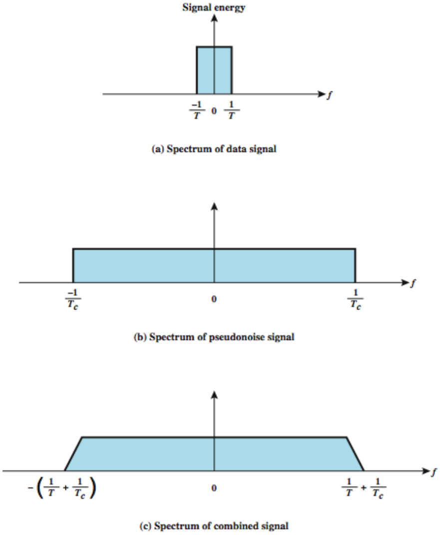


Figure: Bandwidth

Model of Spread Spectrum Digital Communication

- The channel encoder and decoder and the modulator and demodulator are the basic elements of the system.
- There are two identical pseudorandom pattern generator one with modulator at the transmitting end and a second with the demodulator at the receiving end.
- The synchronization of the PN sequence generated at the receiver with the PN sequence contained in the incoming received signal is required in order to demodulate the received signal

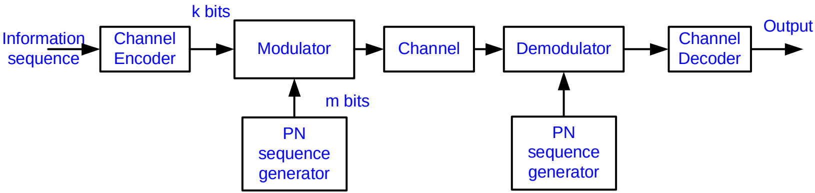


Figure: Model of spread spectrum communication system

Direct sequence spread spectrum signals

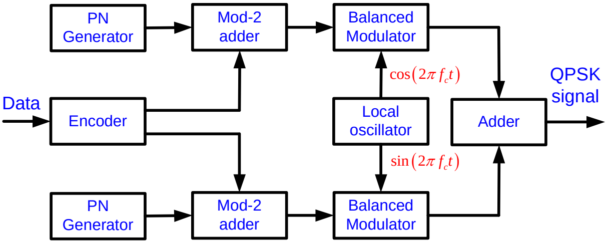


Figure: Direct sequence spread spectrum QPSK modulator

The message bits are modulo-2 added by PN sequence and if b_i is the ith bit of the PN sequence and c_i is the corresponding bit from the encoder, the modulo-2 sum is

$$a_i = b_i \oplus c_i$$

From this, $a_i = 0$ when $b_i = c_i$ and $a_i = 1$ when $b_i \neq c_i$. The sequence a_i is mapped into a binary PSK signal of the form $s(t) = \pm Re[g(t)e^{j2\pi f_c t}]$ as follows:

$$g_i(t) = \begin{cases} g(t - iT_c) & (a_i = 0) \\ -g(t - iT_c) & (a_i = 1) \end{cases}$$

$s(t) = \pm Re[g(t)e^{j2\pi f_c t}]$ $[e^{\pm jx} = cosx \pm jsinx]$

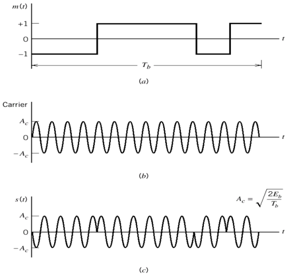


Figure: Illustrates the spreading process

The modulo-2 addition can also be implemented by multiplying two waveforms as follows. The coded sequence is multipied by

$$c_i(t) = (2c_i - 1)g(t - iT_c)$$

$$p_i(t) = (2b_i - 1)p(t - iT_c)$$

where $p(t)$ is the rectangular pulse of duration T_c. The equivalent low pass transmitted signal for the i^{th} code is

$$\begin{aligned} g_i(t) &= p_i(t)c_i(t) \\ &= (2b_i - 1)(2c_i - 1)g(t - iT_c) \end{aligned}$$

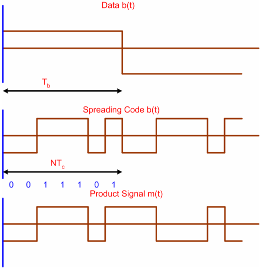


Figure: Illustrates the spreading process

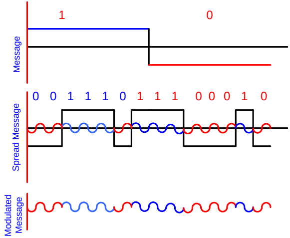


Figure: Illustrates the spreading process

 Direct sequence spread spectrum signals

The received equivalent low pass signal for the i^{th} code is

$$
\begin{aligned}
r_i(t) &= p_i(t)c_i(t) + z(t) \\
&= (2b_i - 1)(2c_i - 1)g(t - iT_c) + z(t)
\end{aligned}
$$

where $z(t)$ represents the interference or jamming signal.

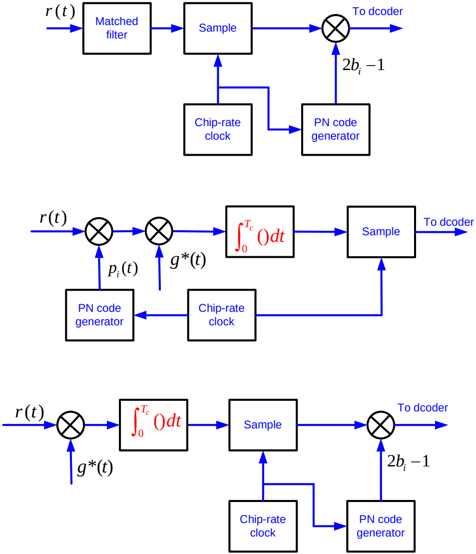


Figure: DSSS Demodulator

- Consider an information rate to the encoder is $R\ bits/s$ and the available channel bandwidth is W Hz.
- The reciprocal of the R denoted by T_b, i.e., $T_b = 1/R$ defines the duration of a pulse corresponding to the transmission time of an information bit.
- The PN code is at a rate of $W\ times/s$. The reciprocal of the W denoted by T_c, defines the duration of a pulse, which is called chip, T_c is called chip interval.
- The bandwidth expansion factor W/R may be expressed as

$$B_e = \frac{W}{R} = \frac{T_b}{T_c} \tag{1}$$

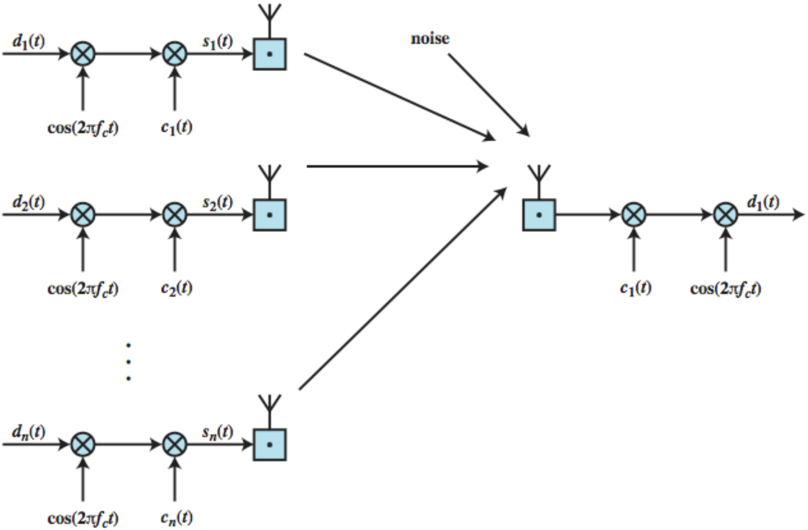
- In practical system the ratio T_b/T_c is an integer,

$$L_e = \frac{T_b}{T_c} \tag{2}$$

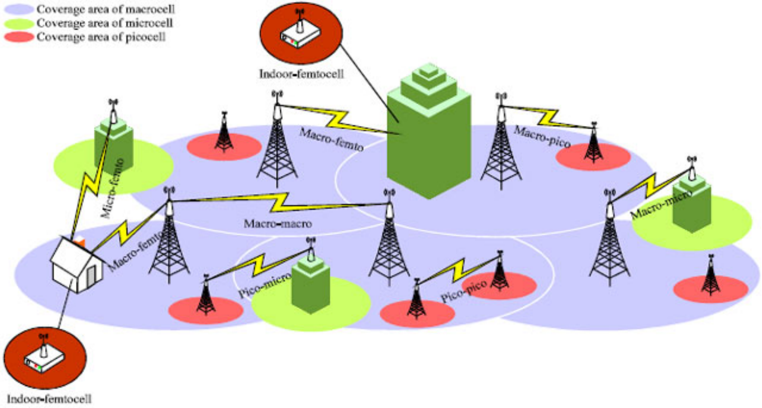
- which is the number of chips per information bit.

Code Division Multiple Access (CDMA)

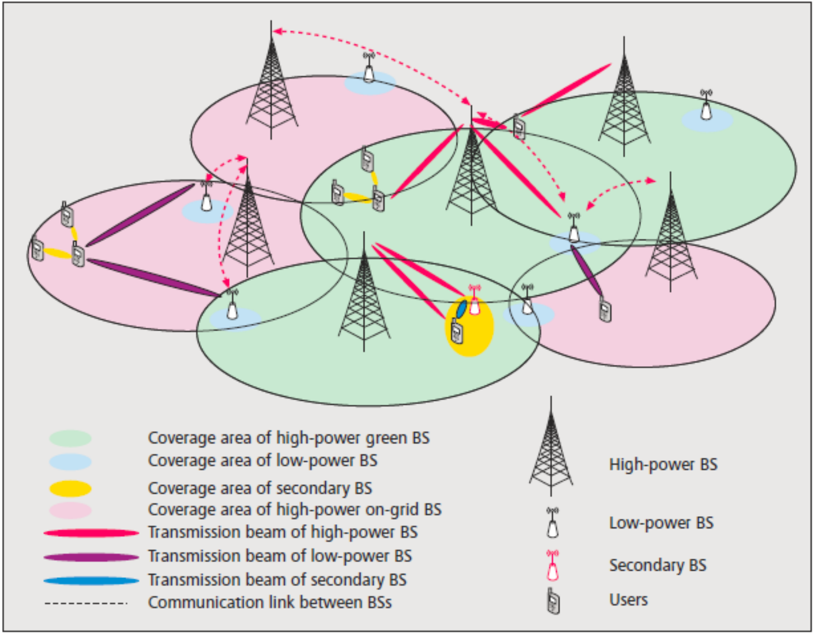
CDMA



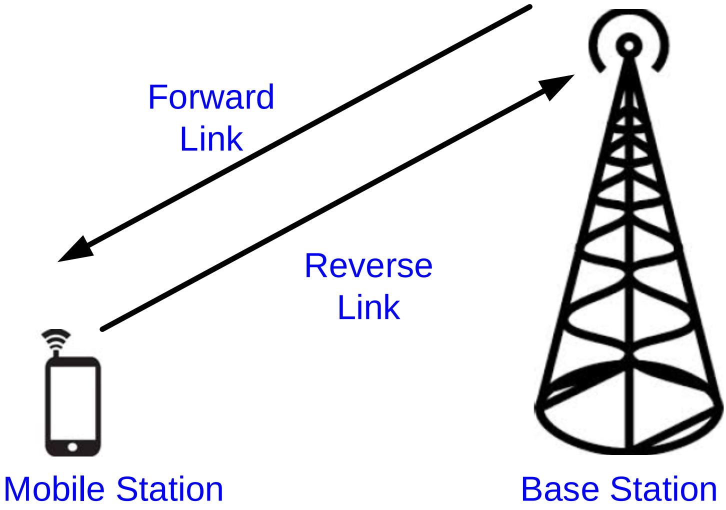
d1(t)
s1(t)
cos(2πfct)
c1(t)
noise
d2(t)
s2(t)
cos(2πfct)
c2(t)
d1(t)
c1(t)
cos(2πfct)
dn(t)
sn(t)
cos(2πfct)
cn(t)


Coverage area of macrocell
Coverage area of microcell
Coverage area of picocell
Indoor-femtocell
Macro-femto
Macro-pico
Micro-femto
Macro-macro
Macro-micro
Macro-femto
Pico-micro
Pico-pico
Indoor-femtocell

 Code Division Multiple Access (CDMA)



CDMA



- Initially adopted in North America, Proposed by Qualcomm, standardized as IS-95 by Telecommunication Industry Association (TIA) for use in 800 MHz and 1900 MHz freq bands
- Base station to Mobile (Forward (Down)link) and mobile to BS (Reverse (UP) link) channel bandwidth is of 1.25 M Hz.
- Both forward and reverse links are DS spread having a chip rate of $1.2288X10^6$ chips per second.

Forward Link

- CELP coder generates a variable data rate of 9600, 4800, 2400, and 1200 bits/s, (based speech activity) in the frame of 20 ms.
- Data is encoded by a rate of 1/2, constraint length K=9 convolutional encoder.
- For lower speech activity 4800, 2400, or 1200 bits/s the output symbols from the convolutional encoder are repeated either twice, four times, or eight times so as to maintain a constant bit rate of 9600 bits/s.
- Block interleaver is used to overcome the effects of burst errors that may occur during the transmission through the channel.
- Scrambler is used for Data Encryption to make call more secure.
- Scrambler will randomizes data and prevents the transition of a long series of 1's or 0's
- Block interleaver data rate of 19.2 kbps are scrambled by multiplication with long code with chip rate of 1.2288 M chips/s and is decimated by factor of 64 to 19.2 kchips/s.
- The long code is used to identify a call of a MS on the forward and reverse links.
- Hadmard or Walsh code sequence of length 64 is assigned to each channel.
- 64 orthogonal sequences are assigned to each BS (64 Channels), One channel is used to transmit pilot signal, which is used measure the channel characteristics (includes signal strength and the carrier phase offset).
- Another channel is used to provide time synchronization. one channel for paging activity.
- Remaining 61 channels for user.

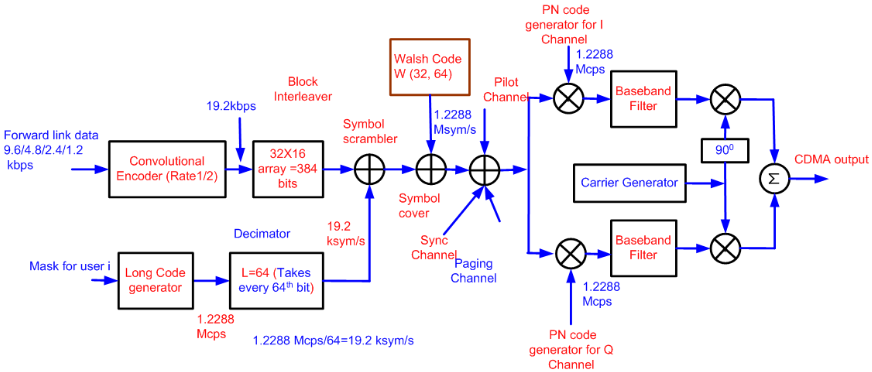


Figure: Block diagram of IS-95 forward link

- Each user data is multiplied by Hadmard sequence, the resulting sequence is spread by two PN sequences one in-phase and other in quadrature phase of length $N = 2^{15}$.
- Different BS are identified by different offsets of these PN sequences.

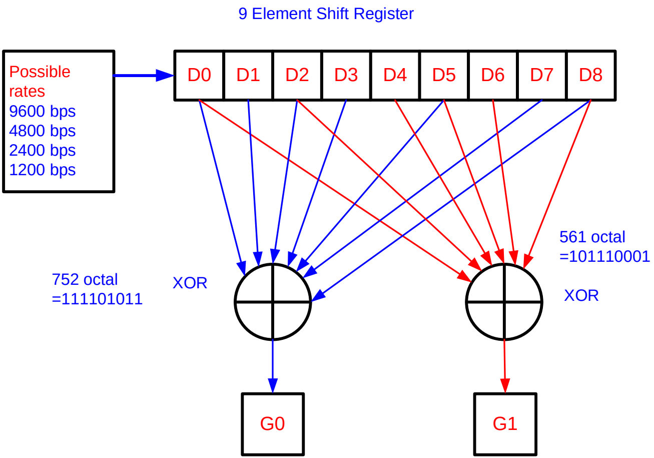


Figure: Convolution Encoder rate 1/2 (r=input bits/output bits)

Reverse Link

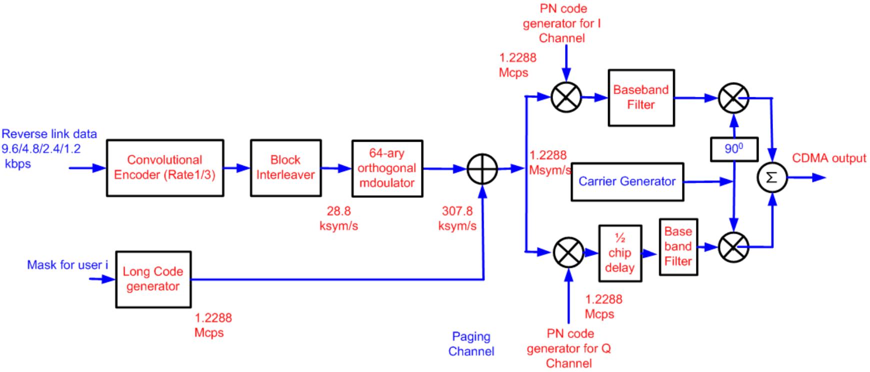


Figure: Block diagram of IS-95 reverse link

Orthogonal Modulation

- 6 bit block of data is mapped into 64 Hadamard seq $=> (64/6) * 28.8 = 307.8$
- 64-ary orthogonal modulation using the same Walsh function in the forward link
- Contrary to the forward link, used for orthogonal data modulation
- One Walsh function is transmitted for six coded bits
- Modulated symbol rate: $28.8kbps * 64chips/6codedbits = 307.2kcps$
- Increase interference tolerance

Processing Gain and Jamming Margin: Some Basic Relations:

- Information rate (Data rate) is R bits/s $\therefore$ Bit duration $T_b = 1/R$
- PN code generator data rate is W times/s $\therefore$ Chip duration $T_c = 1/W$
- Signal energy per bit $E_b = P_{av} T_b = P_{av}/R$
- The average jamming power (J_{av})is $J_{av} = J_0 W$ where J_0 PSD of jamming signal $\therefore$ $J_0 = J_{av}/W$

$$\frac{E_b}{J_0} = \frac{P_{av}/R}{J_{av}/W} = \frac{W/R}{J_{av}/P_{av}} = \left(\frac{P_S/R}{P_I/W} = \frac{W/R}{P_I/P_S} \right)$$

- $P_{av}/J_{av}$is the jamming to signal power ratio which is greater than unity, J_{av}/P_{av} is jamming margin.
- The ratio $W/R = T_b/T_c = B_e = Lc$ is the bandwidth expansion factor or number of chips per info bit is called processing gain.
- $R_c d_{min}$ is the coding gain and all these are related by:

$$(SNR)_{dB} = (W/R)_{dB} + (R_c d_{min})_{dB} - (J_{av}/P_{av})_{dB}$$

- Probability of error for binary signaling system is

$$P2 = Q\left(\sqrt{\frac{2E_b}{J_0}}\right) = Q\left(\sqrt{\frac{2W/R}{J_{av}/P_{av}}}\right)$$

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \quad and \quad erfc(u) = 1 - erf(u)$$

- If there are N_u simultaneous users then the desired signal-to-nose interference ratio at a given receiver is $(P_{av} = P_S \ J_{av} = P_N)$

$$\frac{P_S}{P_N} = \frac{P_S}{(N_u - 1)P_S} = \frac{1}{N_u - 1}$$

Complementary Error Function Table

x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)	x	erfc(x)
0	1.000000	0.5	0.479500	1	0.157299	1.5	0.033895	2	0.004678	2.5	0.000407	3	0.00002209
0.01	0.988717	0.51	0.470756	1.01	0.153190	1.51	0.032723	2.01	0.004475	2.51	0.000386	3.01	0.00002074
0.02	0.977435	0.52	0.462101	1.02	0.149162	1.52	0.031587	2.02	0.004281	2.52	0.000365	3.02	0.00001947
0.03	0.966159	0.53	0.453536	1.03	0.145216	1.53	0.030484	2.03	0.004094	2.53	0.000346	3.03	0.00001827
0.04	0.954889	0.54	0.445061	1.04	0.141350	1.54	0.029414	2.04	0.003914	2.54	0.000328	3.04	0.00001714
0.05	0.943628	0.55	0.436677	1.05	0.137564	1.55	0.028377	2.05	0.003742	2.55	0.000311	3.05	0.00001608
0.06	0.932378	0.56	0.428384	1.06	0.133856	1.56	0.027372	2.06	0.003577	2.56	0.000294	3.06	0.00001508
0.07	0.921142	0.57	0.420184	1.07	0.130227	1.57	0.026397	2.07	0.003418	2.57	0.000278	3.07	0.00001414
0.08	0.909922	0.58	0.412077	1.08	0.126674	1.58	0.025453	2.08	0.003266	2.58	0.000264	3.08	0.00001326
0.09	0.898719	0.59	0.404064	1.09	0.123197	1.59	0.024538	2.09	0.003120	2.59	0.000249	3.09	0.00001243
0.1	0.887537	0.6	0.396144	1.1	0.119795	1.6	0.023652	2.1	0.002979	2.6	0.000236	3.1	0.00001165
0.11	0.876377	0.61	0.388319	1.11	0.116467	1.61	0.022793	2.11	0.002845	2.61	0.000223	3.11	0.00001092
0.12	0.865242	0.62	0.380589	1.12	0.113212	1.62	0.021962	2.12	0.002716	2.62	0.000211	3.12	0.00001023
0.13	0.854133	0.63	0.372954	1.13	0.110029	1.63	0.021157	2.13	0.002593	2.63	0.000200	3.13	0.00000958
0.14	0.843053	0.64	0.365414	1.14	0.106918	1.64	0.020378	2.14	0.002475	2.64	0.000189	3.14	0.00000897
0.15	0.832004	0.65	0.357971	1.15	0.103876	1.65	0.019624	2.15	0.002361	2.65	0.000178	3.15	0.00000840
0.16	0.820988	0.66	0.350623	1.16	0.100904	1.66	0.018895	2.16	0.002253	2.66	0.000169	3.16	0.00000786
0.17	0.810008	0.67	0.343372	1.17	0.098000	1.67	0.018190	2.17	0.002149	2.67	0.000159	3.17	0.00000736
0.18	0.799064	0.68	0.336218	1.18	0.095163	1.68	0.017507	2.18	0.002049	2.68	0.000151	3.18	0.00000689
0.19	0.788160	0.69	0.329160	1.19	0.092392	1.69	0.016847	2.19	0.001954	2.69	0.000142	3.19	0.00000644
0.2	0.777297	0.7	0.322199	1.2	0.089686	1.7	0.016210	2.2	0.001863	2.7	0.000134	3.2	0.00000603
0.21	0.766478	0.71	0.315335	1.21	0.087045	1.71	0.015593	2.21	0.001776	2.71	0.000127	3.21	0.00000564
0.22	0.755704	0.72	0.308567	1.22	0.084466	1.72	0.014997	2.22	0.001692	2.72	0.000120	3.22	0.00000527
0.23	0.744977	0.73	0.301896	1.23	0.081950	1.73	0.014422	2.23	0.001612	2.73	0.000113	3.23	0.00000493
0.24	0.734300	0.74	0.295322	1.24	0.079495	1.74	0.013865	2.24	0.001536	2.74	0.000107	3.24	0.00000460
0.25	0.723674	0.75	0.288845	1.25	0.077100	1.75	0.013328	2.25	0.001463	2.75	0.000101	3.25	0.00000430
0.26	0.713100	0.76	0.282463	1.26	0.074764	1.76	0.012810	2.26	0.001393	2.76	0.000095	3.26	0.00000402
0.27	0.702582	0.77	0.276179	1.27	0.072486	1.77	0.012309	2.27	0.001326	2.77	0.000090	3.27	0.00000376
0.28	0.692120	0.78	0.269990	1.28	0.070266	1.78	0.011826	2.28	0.001262	2.78	0.000084	3.28	0.00000351
0.29	0.681717	0.79	0.263897	1.29	0.068101	1.79	0.011359	2.29	0.001201	2.79	0.000080	3.29	0.00000328
0.3	0.671373	0.8	0.257899	1.3	0.065992	1.8	0.010909	2.3	0.001143	2.8	0.000075	3.3	0.00000306
0.31	0.661092	0.81	0.251997	1.31	0.063937	1.81	0.010475	2.31	0.001088	2.81	0.000071	3.31	0.00000285
0.32	0.650874	0.82	0.246189	1.32	0.061935	1.82	0.010057	2.32	0.001034	2.82	0.000067	3.32	0.00000266
0.33	0.640721	0.83	0.240476	1.33	0.059985	1.83	0.009653	2.33	0.000984	2.83	0.000063	3.33	0.00000249
0.34	0.630635	0.84	0.234857	1.34	0.058086	1.84	0.009264	2.34	0.000935	2.84	0.000059	3.34	0.00000232
0.35	0.620618	0.85	0.229332	1.35	0.056238	1.85	0.008889	2.35	0.000889	2.85	0.000056	3.35	0.00000216
0.36	0.610670	0.86	0.223900	1.36	0.054439	1.86	0.008528	2.36	0.000845	2.86	0.000052	3.36	0.00000202
0.37	0.600794	0.87	0.218560	1.37	0.052688	1.87	0.008179	2.37	0.000803	2.87	0.000049	3.37	0.00000188
0.38	0.590991	0.88	0.213313	1.38	0.050984	1.88	0.007844	2.38	0.000763	2.88	0.000046	3.38	0.00000175
0.39	0.581261	0.89	0.208157	1.39	0.049327	1.89	0.007521	2.39	0.000725	2.89	0.000044	3.39	0.00000163
0.4	0.571608	0.9	0.203092	1.4	0.047715	1.9	0.007210	2.4	0.000689	2.9	0.000041	3.4	0.00000152
0.41	0.562031	0.91	0.198117	1.41	0.046148	1.91	0.006910	2.41	0.000654	2.91	0.000039	3.41	0.00000142
0.42	0.552532	0.92	0.193232	1.42	0.044624	1.92	0.006622	2.42	0.000621	2.92	0.000036	3.42	0.00000132
0.43	0.543113	0.93	0.188437	1.43	0.043143	1.93	0.006344	2.43	0.000589	2.93	0.000034	3.43	0.00000123
0.44	0.533775	0.94	0.183729	1.44	0.041703	1.94	0.006077	2.44	0.000559	2.94	0.000032	3.44	0.00000115
0.45	0.524518	0.95	0.179109	1.45	0.040305	1.95	0.005821	2.45	0.000531	2.95	0.000030	3.45	0.00000107
0.46	0.515345	0.96	0.174576	1.46	0.038946	1.96	0.005574	2.46	0.000503	2.96	0.000028	3.46	0.00000099
0.47	0.506255	0.97	0.170130	1.47	0.037627	1.97	0.005336	2.47	0.000477	2.97	0.000027	3.47	0.00000092
0.48	0.497250	0.98	0.165769	1.48	0.036346	1.98	0.005108	2.48	0.000453	2.98	0.000025	3.48	0.00000086
0.49	0.488332	0.99	0.161492	1.49	0.035102	1.99	0.004889	2.49	0.000429	2.99	0.000024	3.49	0.00000080

Figure: Block diagram of IS-95 reverse link

Table I. Values of Erf(x) and Erfc(x)
(for positive values of x)

Value x	Erf (x)	Erfc(x)
0	0	1
0.05	0.0563720	0.9436280
0.1	0.1124629	0.8875371
0.15	0.1679960	0.8320040
0.2	0.2227026	0.7772974
0.25	0.2763264	0.7236736
0.3	0.3286268	0.6713732
0.35	0.3793821	0.6206179
0.4	0.4283924	0.5716076
0.45	0.4754817	0.5245183
0.5	0.5204999	0.4795001
0.55	0.5633234	0.4366766
0.6	0.6038561	0.3961439
0.65	0.6420293	0.3579707
0.7	0.6778012	0.3221988
0.75	0.7111556	0.2888444
0.8	0.7421010	0.2578990
0.85	0.7706681	0.2293319
0.9	0.7969082	0.2030918
0.95	0.8208908	0.1791092
1	0.8427008	0.1572992
1.1	0.8802051	0.1197949
1.2	0.9103140	0.0896860
1.3	0.9340079	0.0659921
1.4	0.9522851	0.0477149
1.5	0.9661051	0.0338949
1.6	0.9763484	0.0236516
1.7	0.9837905	0.0162095
1.8	0.9890905	0.0109095
1.9	0.9927904	0.0072096
2	0.9953223	0.0046777
2.1	0.9970205	0.0029795
2.2	0.9981372	0.0018628
2.3	0.9988568	0.0011432
2.4	0.9993115	0.0006885
2.5	0.9995930	0.0004070
2.6	0.9997640	0.0002360
2.7	0.9998657	0.0001343
2.8	0.9999250	0.0000750
2.9	0.9999589	0.0000411
3	0.9999779	0.0000221
3.1	0.9999884	0.0000116
3.2	0.9999940	0.0000060
3.3	0.9999969	0.0000031
3.4	0.9999985	0.0000015
3.5	0.9999993	0.0000007

- 13.5 A rate of 1/2 convolutional code with $d_{min} = 10$ is used to encode a data sequence occuring at a rate of 1000 bits/s. The modulation is binary PSK. The DS spread spectrum sequence has a chip rate of 10 MHz

  a) Determine the coding gain
  b) Determine the processing gain
  c) Determine the Jamming margin assuming an $E_b/J_0 = 10$

Solution:

a) The Coding gain is

$$R_c d_{min} = 1/2 * 10 = 5 = 10log(5) = 6.989\ db$$

b) The processing gain is W/R

$$\frac{W}{R} = \frac{10^7}{2 * 10^3} = 5 * 10^3 = 10log(5 * 10^3) = 37\ db$$

c) The Jamming margin is

$$\frac{P_{av}}{J_{av}} = \frac{(W/R)(R_c d_{min})}{(E_b/J_0)} db$$

$$\begin{aligned} \frac{P_{av}}{J_{av}} &= (W/R)_{dB} + (CG_{dB}) - (E_b/J_0)_{dB} \\ &= 37 + 7 - 10 = 34dB \end{aligned}$$

- 13.6 A total of 30 equal-power users are to share a common communication channel by CDMA. Each user transmits information at a rate of 10 kbps via DS spread spectrum and binary PSK. Determine the minimum chip rate in order to obtain a bit-error probability of 10^{-5}. Additive noise at the receiver may be ignored in this computation.

Solution:

$$P_e = 10^{-5} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \quad \therefore\ erfc\left(\sqrt{\frac{E_b}{N_0}}\right) = 2 * 10^{-5}$$

$$erf(u) = 1 - erfc(u) = 1 - 2 * 10^{-5} = 0.99998$$

From the error function table $u = \sqrt{E_b/N_0} = 3.0$ for a value of 0.99998

$$u = \sqrt{\frac{E_b}{N_0}} \simeq 3.0 \qquad \therefore \quad E_b/N_0 = 9$$

To achieve an error probability of 10^{-5}, the required $E_b/J_0 = 10$ Then, by using the relation in and we have

$$\frac{W/R}{P_N/P_S} = \frac{W/R}{N_u - 1} = \frac{E_b}{J_0} \qquad \therefore \quad W/R = \frac{E_b}{J_0}(N_u - 1)$$

$$W = R\frac{E_b}{J_0}(N_u - 1)$$

$R = 10^4$ bps, $N_u = 30$ and $E_b/J_0 = 10$

$$W = 10^4 * 10(30 - 1) = 2.9 * 10^6 Hz$$

The minimum chip rate is $1/Tc = W = 2.9 * 10^6$ chips/sec

13.7 A CDMA system is designed based on DS spread spectrum with a processing gain of 1000 and binary PSK modulation. Determine the number of users, if each user has equal power and the desired level of performance is an error probability of 10^{-6}. Repeat the computation if the processing gain is changed to 500.

Solution:

$$P_e = 10^{-6} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad i.e., 2*10^{-6} = erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$erfc(u) = 2*10^{-6}$$

From the complementary error function table $u = \sqrt{E_b/N_0} \simeq 3.36$ for a value of $2*10^{-6}$ $\therefore E_b/N_0 = 11.3$
Then, the number of users of the CDMA system is

$$N_u = \frac{W/R_b}{E_b/J_0} + 1$$

$$N_u = \frac{W/R_b}{E_b/J_0} + 1 = N_u = \frac{1000}{11.3} + 1 = 89 \ users$$

If the processing gain is reduced to $W/R_b = 500$,then

$$N_u = \frac{500}{11.3} + 1 = 45 \ users$$

13.8 A DS spread-spectrum system transmits at a rate of 1000 bps in the presence of tone interference. The interference power is 20 dB greater than the desired signal and the required to achieve satisfactory performance is 10 dB.

a Determine the spreading bandwidth required to meet the specifications.

b In the case of pulse interference, determine the pulse duty cycle that results in worst-case performance and the corresponding probability of error.

Solution:

a $(P_J/P_S)_{dB} = 20dB \ R = 1000$ bps and $(E_b/J_0)_{dB} = 10dB$

$$\left(\frac{W}{R}\right)_{db} = \left(\frac{P_J}{P_S}\right)_{db} + \left(\frac{E_b}{J_0}\right)_{db} = 30dB$$

$$\frac{W}{R} = 1000$$

$$W = 1000R = 1000 * 1000 = 10^6 Hz$$

b The duty cycle of pulse jammer for worst - case jamming is

$$\alpha = \frac{0.7}{E_b/J_0} = \frac{0.7}{10} = 0.07$$

The corresponding probability of error for this works case jamming is

$$P_2 = \frac{0.082}{E_b/J_0} = \frac{0.082}{10} = 8.2 * 10^{-3}$$

13.9 CDMA system consists of 15 equal power users that transmit information at a rate of 10,000 bits/s, each using a DS spread spectrum signal operating at a chip rate of 1 MHz. The modulation is binary PSK.

- a) Determine the E_b/J_o where J_o is the spectral density of the combined interference.
- b) What is processing gain?
- c) How much should the processing gain be increased to allow for doubling the number of users without affecting the output SNR?

Solution:

- a) We have $N_u = 15$ users transmitting at a rate of 10, 000 bps each, in a bandwidth of W = 1 MHz The E_b/J_0 is

$$\frac{E_b}{J_0} = \frac{W/R}{N_u - 1} = \frac{10^6/10^4}{14} = \frac{100}{14} = 7.14(8.54dB)$$

- b) Processing gain PG $= \frac{W_c}{R_b} \frac{1Mbps}{10Kbps} = 100$
- c) With $N_u = 30$ and $E_b/J_0 = 7.14$, the processing gain should be increased to

$$W/R = (7.14)(29) = 207$$

Hence, the bandwidth must be increased to W = 2.07MHz

13.14 An m=10 ML shift register is used to generate the pseudorandom sequence in a DS SS. The chip duration is $T_c = 1 microsec$ and the bit duration is $T_b = NT_c$ where N is the length of the m sequences

1. Determine the processing gain of the system in dB
2. Determine the jamming margin if the required $E - b/J_0 = 10$ and the jammer is tone jammer with an average power J_{av}

Solution:

1. The period of the maximum length shift register sequence is

$$N = 2^{10} - 1 = 1023$$

Since $T_b = NT_c$, then the processing gain is

$$N\frac{Tb}{Tc} = 1023(30dB)$$

2. According to jamming margin is

$$\frac{J_{av}}{P_{av}}_{dB} = \frac{W}{R}_{dB} - \frac{E_b}{J_0}_{dB}$$

= 30-10 = 20dB
where $Jav = J_0 W \approx J_0/T_c = J_0 * 10^6$

13.15 An FH binary orthogonal FSK system employs a m = 15 stage linear feedback shift register that generates a maximal length sequence. Each state of the shift register selects one of N nonoverlapping frequency bands in the hopping pattern. The bit rate is 100 bits/sec and the hop rate is once/bit. The demodulator employs noncoherent detection.

1. Determine the hopping bandwidth for this channel.
2. What is the processing gain?
3. What is the probability of error in the presence of AWGN?

Solution:

1. The length of the shift-register sequence is

$$L = 2^m - 1 = 2^{15} - 1 = 32767 \quad bits$$

   For binary FSK modulation, the minimum frequency separation is 2/T, where 1/T is the symbol (bit) rate. The hop rate is 100 hops/ sec. Since the shift register has N = 32767 states and each state utilizes a bandwidth of 2/T = 200 Hz, then the total bandwidth for the FH signal is 6.5534 MHz.

2. The processing gain is W/R. We have,

$$\frac{W}{R} = \frac{6.5534 * 10^6}{100} = 6.5534 \times 10^4 bps$$

3. If the noise is AWG with power spectral density N0, the probability of error expression

$$P_2 = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q\left(\sqrt{\frac{W/R}{P_N/P_S}}\right)$$

13.16 Consider the FH binary orthogonal FSK system described in Problem 13.15. Suppose that the hop rate is increased to 2 hops/bit. The receiver uses squarelaw combining to combine the signal over the two hops.

1. Determine the hopping bandwidth for the channel.
2. What is the processing gain?
3. What is the error probability in the presence of AWGN?

Frequency Hopping Spread Spectrum Modulation

- The type of spread spectrum in which carrier hops randomly from one frequency to another is called Frequency Hop (FH) spread spectrum.
- Frequencies are shifted for every Tc seconds.
- Duration of signal element is Tb seconds.
- FH/SS are of two types:
  1. Slow Frequency Hopping (SFH):
  2. Fast Frequency Hopping (FFH):

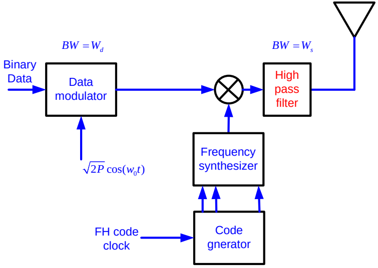


Figure: Transmitter

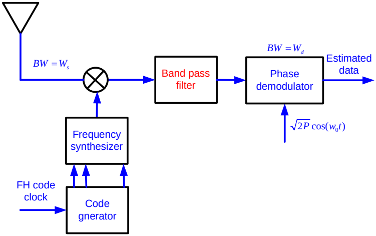
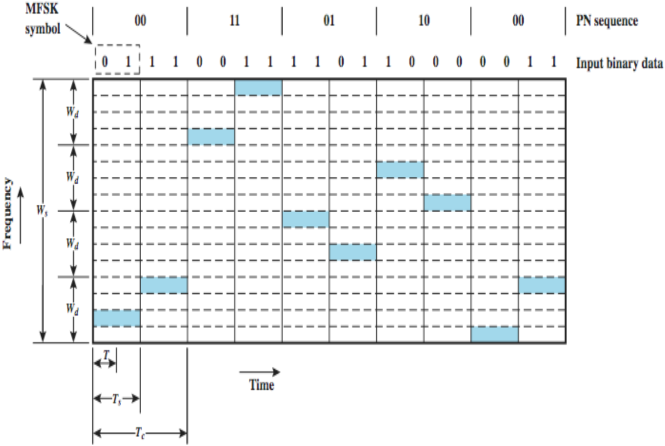


Figure: Receiver

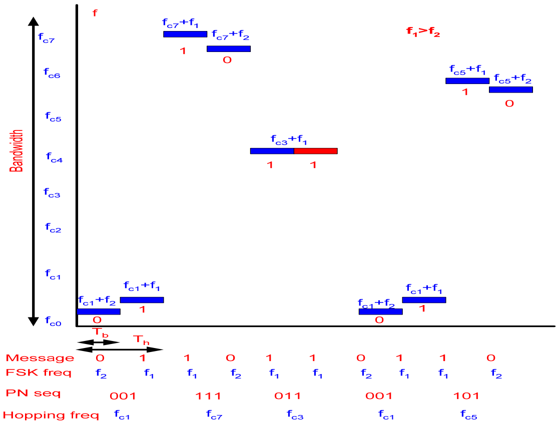
Slow Frequency Hopping (SFH):

- In slow frequency hopping the symbol rate of the input signal is an integer multiple of the frequency hopping rate.
- That is several symbols are transmitted on each frequency hop.
- Slow FHSS has $Tc \geq Tb$



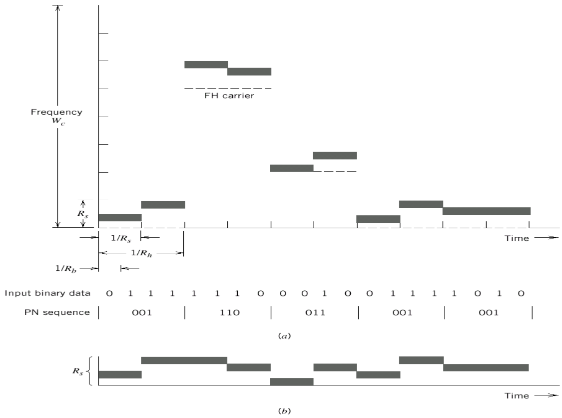
Slow Frequency Hopping (SFH):

- Number of bits per MFSK symbol = 2 ⇒ M = 4
- Rs = Rb/2 Rc = max(Rh, Rs) = Rs
- Length of PN segment per hop = 3 Total number of frequency hops = 2^3 = 8



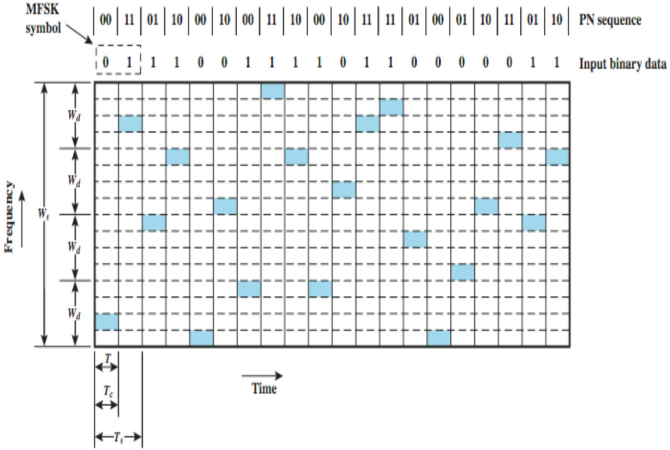
Slow Frequency Hopping (SFH):

- Number of bits per MFSK symbol = 2 ⇒ M = 4
- Rs = Rb/2 Rc = max(Rh, Rs) = Rs
- Length of PN segment per hop = 3 Total number of frequency hops = $2^3 = 8$



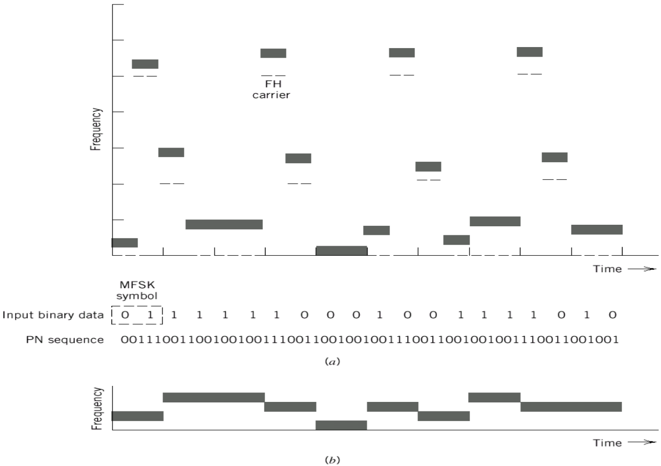
Fast Frequency Hopping (FFH):

- In fast frequency hopping the frequency hopping rate is an integer multiple of the input symbol rate.
- That is the carrier frequency will change or hop several times during the transmission of the one symbol.
- Fast FHSS has $Tc < Tb$.



Fast Frequency Hopping (FFH):

- Number of bits per MFSK symbol = 2 ⇒ M = 4
- Rs = Rb/2 Rc = max(Rh, Rs) = Rh
- Length of PN segment per hop = 3 Total number of frequency hops = $2^3 = 8$



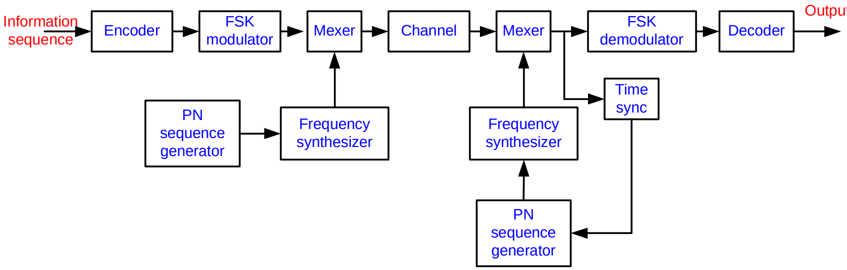


Figure: Block diagram of FH spread spectrum system

- The initial modulation is usually binary or M-ary FSK.
- FSK signal is translated by random frequency which is synthesized by frequency synthesizer from PN codes.
- Synthesized frequency is mixed with the output of the modulator.
- The m bits from the PN generater will generate $2^m - 1$ frequencies.
- At the receiver an identical PN generator will generate m bits and these m bits will be used synthesize corresponding $2^m - 1$ frequencies.

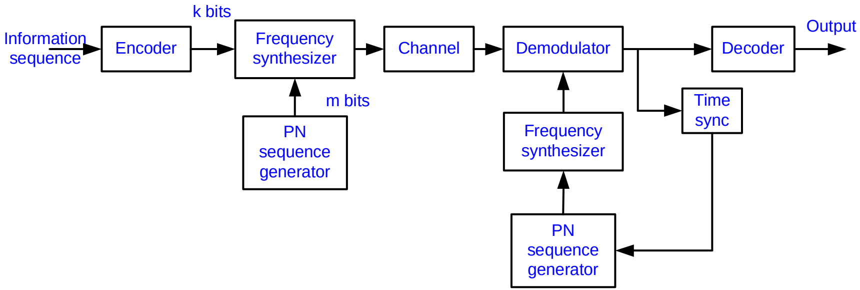


Figure: Block diagram of an independent tone FH spread spectrum system

- Independent tone hopping is less vulnerable to some jamming strategies.
- In this m bits from the PN generator and the k bits are used to specify the frequency slot.
- Synthesized frequency is mixed with the output of the modulator.
- The FH rate is usually equal to or faster than symbol rate.
- If there multiple hops it is called as fast hopped FHSS and if the hopping is performed at symbol rate is called slow FHSS.

Time Hopping Spread Spectrum Modulation

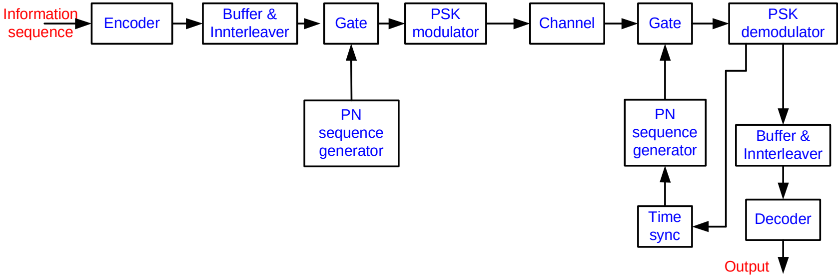


Figure: Block diagram of an independent tone FH spread spectrum system

- In time hop (TH) a time interval which is selected to be much larger than the reciprocal of the information rate.
- The coded information are transmitted in pseudorandomly selected time slot as a block of one or more code words.
- The PSK modulation is used to transmit the coded bits.
- Because of the burst characteristics buffer storage is used at the transmitter.
- Buffer storage is used at the receiver to provide a uniform data stream to the user.

Fast Frequency Hopping (FFH):

- In an FH system an FH tone of short duration is referred as a chip.
- The chip rate Rc for an FH system is defined by:

$$R_c = \max(R_h, R_s)$$

- where R_h is the hop rate and R_s is symbol rate
- Let J be the average Jammers signal over the entire frequency hopped spectrum of bandwidth Wc Hz, then the average power spectral density N0/2 of the equivalent AWGN is:

$$\frac{N_0}{2} = \frac{1}{2}\frac{J}{W_c}$$

$$N_0 = \frac{J}{W_c}$$

- Thus, the average energy E to noise density ratio is

$$\frac{E}{N_0} = \frac{P/J}{W_c/R_S}$$

- Where P/J is reciprocal of the jamming margin

Synchronization

Synchronization

- In spread spectrum system, de-spreading is done in order to retrieve the message signal. [3, 4, 5, 6].
- In order to de-spread the received signal, the locally generated receiver spreading code must be in synchronous to the transmitter spreading code.
- Time synchronization of the receiver to the received spread-spectrum signal may be achieved in two distinct phases:

1. Initial Acquisition (Coarse synchronization).
2. Tracking (Fine synchronization).

Phase for Direct sequence spread spectrum:

- The initial synchronization is the one in which synchronize the receiver clock to the transmitter clock.
- There is always an initial timing uncertainty that is due to propagation delay in the transmission of the signal through the channel.
- The initial synchronization is achieved by transmitting a known pseudorandom sequence to the receiver.
- The receiver is continuously in a search mode looking for this sequence in order to establish initial synchronization.
- Suppose that the initial timing uncertainty is T_u seconds and the chip duration is T_c.
- Dwell time $T_d = NT_c$ is required to test synchronism at each time instant.
- If the search over the time uncertainty interval in (coarse) time steps of $T_c/2$, then the time required to establish initial synchronization is

$$T_{init\_sync} = \frac{T_u}{T_c/2} NT_c = 2NT_u$$

- The transmitted sequence must be at least as long as $2NT_u$ in order to perform necessary search .

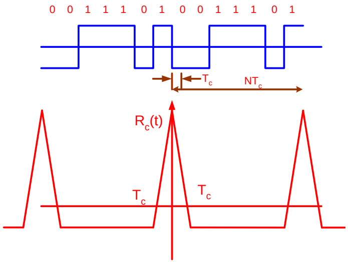


Figure: Plot of correlation.

Serial Search sliding correlator:

The correlator cycles in $T_c/2$ discrete-time intervals. The crosscorrelation is performed over NT_c interval where N is the number of chips, and the correlator output is compared with a threshold to determine if the known signal sequence is present.
If the threshold is not exceeded, the known reference sequence is advanced by $T_c/2$ sec and the correlation process is repeated.
These operations are performed until a signal is detected.
Consider a BPSK modulated DS-SS signal

$$v(t) = g(t)s(t) = \sqrt{2P_s}g(t)d(t)\cos w_0 t$$

where g(t) is spreading code, d(t) binray baseband data and w_0 is IF carrier frequency During initial synchronization, the baseband data signal is set to constant value d(t)=1.

$$v(t) = \sqrt{2P_s}g(t)\cos(w_0 t + \theta)$$

Prior to acquisition, the transmit and receive PN codes are not in synchronism, that is, the time position of the chip patterns is not aligned. The output of the multiplier is given by

$$\begin{aligned} v_r(t) &= v_i(t).g(t - iT_c) \\ &= \sqrt{2P_s}g(t)\cos(w_0 t + \theta).g(t - iT_c) \end{aligned}$$

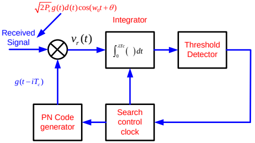


Figure: A sliding correlator.

The repetitive trial-and-error process eventually leads to a state in which the received chipped signal g(t) is aligned with $g(t - iT_c)$ that is, the relative shift.

$$g(t).g(t - iT_c) = g(t).g(t - 0.T_c) = 1$$

This is the received "pilot" signal is despread and becomes

$$v_r(t) = \sqrt{2P_s}\cos(w_0 t + \theta)$$

The integrator output transfers the complete received signal power to the envelope detector. This leads to a high comparator output state, and acquisition or crude synchronization is now established.

Tracking:

- The tracking maintains the PN code generator at the receiver in synchronism with the received signal i.e., fine-chip synchronization.
- In this tracking loop, the received signal is applied to two multipliers, where it is multiplied by two outputs from the local PN code generator which are delayed relative to each other by an amount of $2\delta \leq T_c$
- The product signals are the crosscorrelations between the received signal and the PN sequence at the two values of delay.

$v_D(t) = \sqrt{2P_s} d(t) g(t) g(t+\tau-T_c/2)\cos(w_0 t+\theta)$ $\quad$ $v_A(t) = \sqrt{2P_s} d(t) g(t) g(t+\tau+T_c/2)\cos(w_0 t+\theta)$

$v_{DF}(t) = \sqrt{2P_s} d(t) \cos(w_0 t+\theta)\overline{[g(t)g(t+\tau-T_c/2)]}$ $\quad$ $v_{AF}(t) = \sqrt{2P_s} d(t) \cos(w_0 t+\theta)\overline{[g(t)g(t+\tau+T_c/2}$

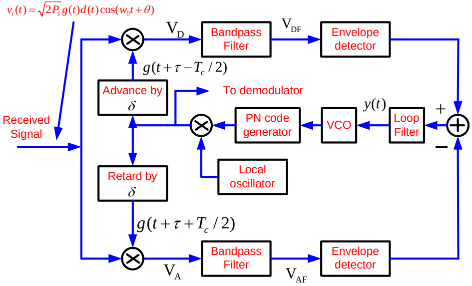


Figure: Simple PLL.

- The average value of the product of the received PN sequence and a shifted version of the same sequence is the autocorrelation function defined by

$$R_g(\tau \pm T_c/2) = \overline{g(t).g(t+\tau \pm T_c/2)}$$

- The envelope detectors extract the envelopes of and and removes the data . Then

$$V_D(t) = |R_g(\tau - T_c/2)| \qquad V_A(t) = |R_g(\tau + T_c/2)|$$

- The input to VCO is y(t) given by

$$y(t) = |R_g(\tau - T_c/2) - R_g(\tau + T_c/2)|$$

- If τ is positive, a positive voltage appears at the VCO input and the VCO frequency is increased.
- This increased VCO frequency reduces τ.
- For negative values of τ a negative y(t) voltage is generated at the VCO input decreasing the VCO rate and thereby increasing τ .

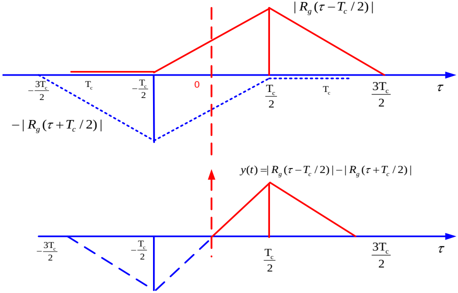


Figure: Autocorrelation function and tracking error signal for DLL.

Tau-dither loop (TDL):

- Tau-dither loop (TDL), employs only a single "arm" instead of the two "arms".
- In this case, the crosscorrelator output is regularly sampled at two values of delay, by stepping the code clock forward and backward in time by an amount δ.
- The envelope of the crosscorrelation that is sampled at $\pm\delta$ has an amplitude modulation whose phase relative to the tau-dither modulator determines the sign of the tracking error.
- The error signal is low pass filtered and then applied to a voltage controlled oscillator that controls (symbol waveform generator) the charging and discharging instants of the correlators.
- The instantaneous frequency of the local clock is advanced or retarded in an iterative manner until the equilibrium point is reached, and symbol synchronization is thereby established.

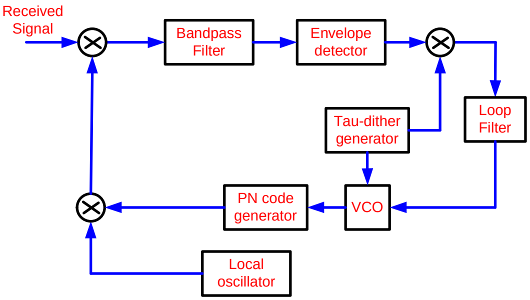


Figure: Tau-dither loop (TDL).

Acquisition Phase for FH spread-spectrum system:

- Synchronize the PN code sequence generated at the receiver that controls the hopped frequency pattern.
- Bank of matched filters tuned to the transmitted frequencies in the known pattern may be employed.
- Their outputs properly delayed, envelope or square-law detected, and added to produce the signal output which is compared with a threshold.
- A signal present (signal acquisition) is declared when the threshold is exceeded.
- The search process is usually performed continuously in time until a threshold is exceeded.

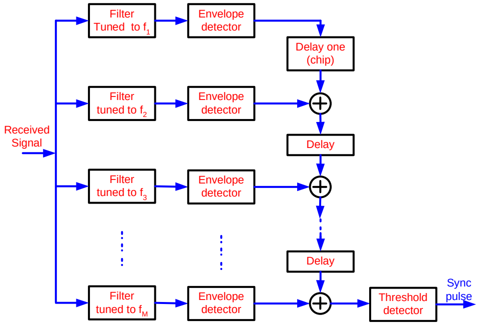


Figure: Tau-dither loop (TDL).

Acquisition Phase for FH spread-spectrum system:

- It is based on a serial search and is similar to the sliding correlator for DS spread-spectrum signals.
- It is of single matched-filter and an envelope detector preceded by a frequency hopping pattern generator and followed by a threshold detector.

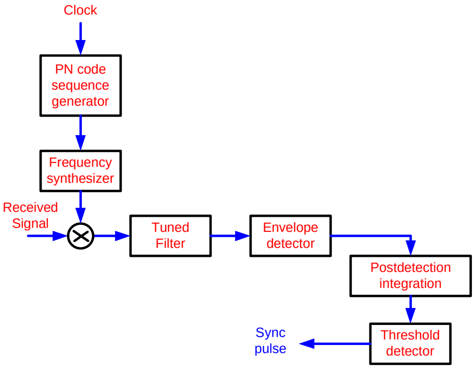


Figure: Alternative for acquisition for FH signal.

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