0.1 Superposition Theorem

In any linear bilateral network containing more than one independent source, the response in any element is equal to the algebraic sum of all the responses due to each independent source acting independently, setting all other independent sources to zero.

All the voltage sources are replaced by its internal resistance or simply by a short circuit and remove the current source from the circuit.

Proof:

Proof) In the circuit shown in Figure 1, using superposition theorem find the current I_x.

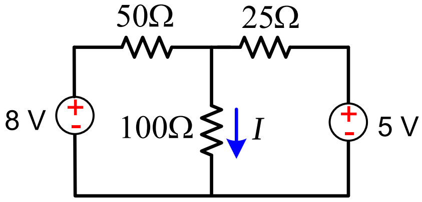

Figure 1

Solution:

By applying KVL for the circuit is as shown in Figure 2.

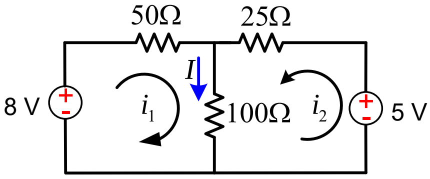

Figure 2

$$\begin{aligned} 50i_1 + 100(i_1 + i_2) - 8 &= 0 \\ 150i_1 + 100i_2 &= 8 \end{aligned}$$

$$\begin{aligned} 25i_2 + 100(i_2 + i_1) - 5 &= 0 \\ 100i_1 + 125i_2 &= 5 \end{aligned}$$

$$\begin{aligned} 150i_1 + 100i_2 &= 8 \\ 100i_1 + 125i_2 &= 5 \end{aligned}$$

Solving the above equations 57mA, -5.71mA The total Current in the 100 resistor is

$$I = i_1 + i_2 = 57mA - 5.71mA = 51.3mA$$

By considering a single voltage source, the circuit is redrawn and is as shown in Figure 2.

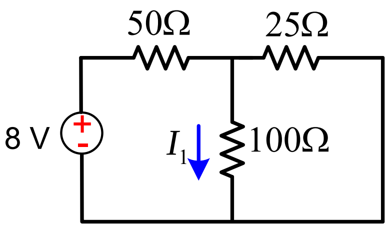

Figure 3

$$R_t = 50 + \frac{100 \times 25}{125} = 50 + 20 = 70$$

$$I_t = \frac{8}{70} = 0.114A$$

By Current division method

$$I_1 = 0.114\frac{25}{125} = 0.0228A$$

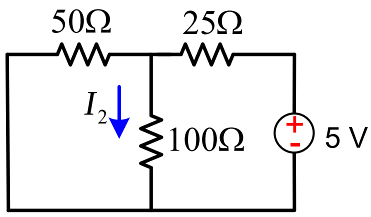

Figure 4

$$R_t = 25 + \frac{100 \times 50}{150} = 25 + 33.33 = 58.33$$

$$I_t = \frac{5}{58.33} = 0.0857A$$

By Current division method

$$I_2 = 0.0857\frac{50}{150} = 0.0285A$$

$$I_1 + I_2 = 0.0228A + 0.0285 = +0.0513A$$

Q 1) In the circuit shown in Figure 5, using superposition theorem find the voltage across V_2.

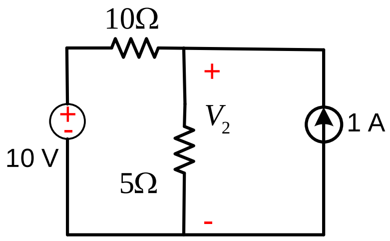

Figure 5

Solution:

By considering a single voltage source the circuit is redrawn and is as shown in Figure 6.

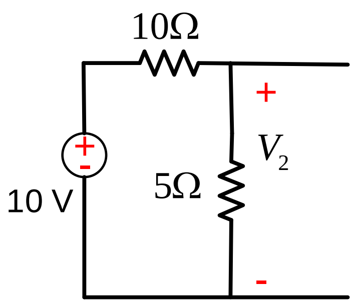

Figure 6

The voltage across 5 Ω resistor is

$$V_2' = 5 \times \frac{10}{15} = 3.333V$$

By considering a single current source the circuit is redrawn and is as shown in Figure 7

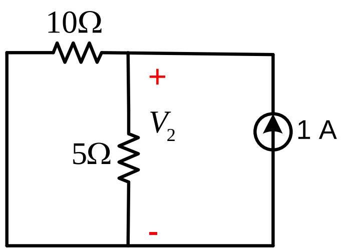

Figure 7

The current through 5 Ω resistor using current division method is

$$I = 1 \times \frac{10}{15} = 0.666A$$

The voltage across 5 Ω resistor is

$$V_2'' = 0.666A \times \frac{10}{5+10} = 3.333V$$

The total voltage across 5 Ω resistor is

$$V_2 = V_2' + V_2'' = 3.333 + 3.333 = 6.666V$$

Q 2) In the circuit shown in Figure 8, using superposition theorem find the voltage across v_0.

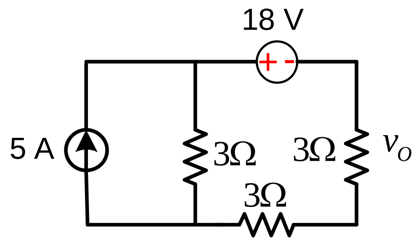

Figure 8

Solution:

By considering a single current source the circuit is redrawn and is as shown in Figure 9.

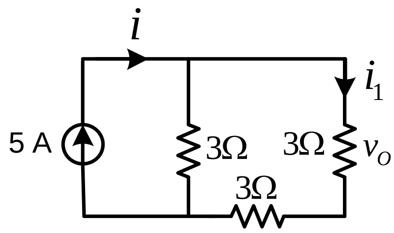

Figure 9

3 and 6 Ω resistors are in parallel the total resistance in the network is

$$R = \frac{3 \times 6}{3+6} = 2\Omega$$

The current i_1 is

$$i_1 = 5\frac{2}{6} = 1.667A$$

By considering a single voltage source the circuit is redrawn and is as shown in Figure 10.

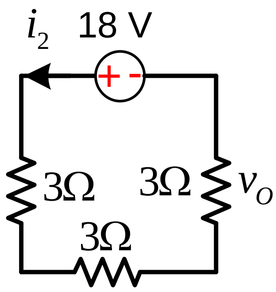

Figure 10

The current i_2 is

$$i_2 = \frac{18}{9} = 2A$$

2 A and 1.667A are in opposite directions therefore net current is $1.667A - 2A = -0.333A$. Then the voltage v_0 is

$$v_0 = -0.333A \times 3 = -1V$$

Q 3) In the circuit shown in Figure 11, using superposition theorem find the voltage across and current flowing through a $3.3k\Omega$ resistor.

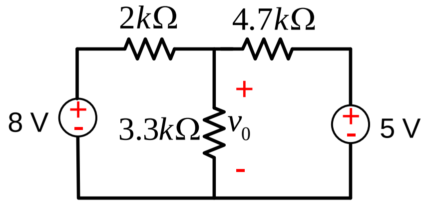

Figure 11

Solution:

By considering a single voltage source the circuit is redrawn and is as shown in Figure 12.

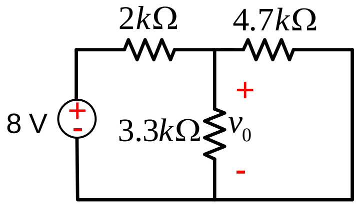

Figure 12

In the circuit $3.3k\Omega$ and $4.7k\Omega$ are in parallel, this combination is in series with $2k\Omega$ resistor.

$$R = \frac{4.7 \times 3.3}{4.7 + 3.3} = 1.938k\Omega$$

Total resistance is $2k\Omega + 1.938k\Omega = 3.938k\Omega$
current through using current division method is

$$I = \frac{8}{3.938k\Omega} = 2.03mA$$

$$I_1 = 2.03mA \times \frac{1.938}{3.3} = 1.192mA$$

$$v_1 = 1.192mA \times 3.3k\Omega = 3.934V$$

By considering a single voltage source the circuit is redrawn and is as shown in Figure 13.

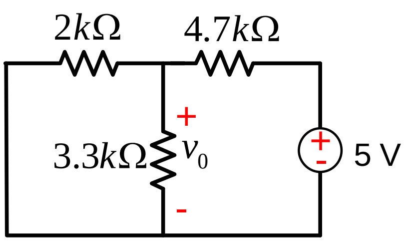

Figure 13

In the circuit $3.3k\Omega$ and $2k\Omega$ are in parallel, this combination is in series with $4.7k\Omega$ resistor.

$$R = \frac{2 \times 3.3}{2 + 3.3} = 1.245k\Omega$$

Total resistance is $4.7k\Omega + 1.245k\Omega = 5.945k\Omega$
current through using current division method is

$$I = \frac{5}{5.945k\Omega} = 0.841mA$$

$$I_2 = 0.841mA \times \frac{1.245}{3.3} = 0.317mA$$

$$v_2 = 0.317mA \times 3.3k\Omega = 1.0474V$$

The total voltage drop across $3.3k\Omega$ resistor is

$$V_0 = v_1 + v_2 = 3.934V + 1.0474V = 4.9774V$$

The total voltage current through $3.3k\Omega$ resistor is

$$I_0 = I_1 + I_2 = 1.192mA + 0.317mA = 1.509mA$$

Q 4) In the circuit shown in Figure 14, using superposition theorem find the voltage V_1.

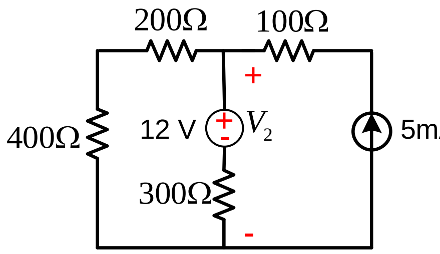

Figure 14

By considering a single voltage source 12V, the circuit is redrawn and is as shown in Figure 15.

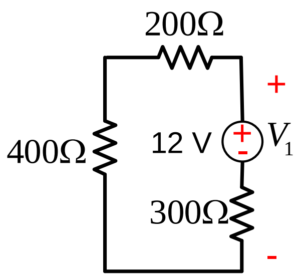

Figure 15

By KVL

$$12 - 900I_1 = 0$$

$$I_1 = \frac{12}{900} = 13.33mA$$

$$V_1 = 12 - 300 \times 13.33mA = 8V$$

By considering a single current source 5A, the circuit is redrawn and is as shown in Figure 16.

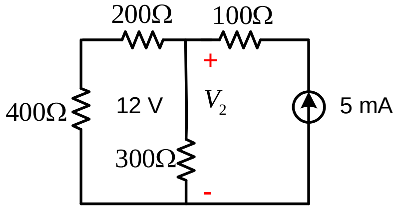

Figure 16

In the circuit 600Ω and 300Ω are in parallel, this combination is in series with 100Ω resistor.

$$R = \frac{600 \times 300}{600 + 300} = 200\Omega$$

current through using current division method is

$$I_2 = 5mA \times \frac{200}{300} = 3.333mA$$

$$v_2 = 3.333mA \times 300 = 1V$$

The total voltage V is

$$V_0 = v_1 + v_2 = 8V + 1V = 9V$$

Q 5) Using superposition theorem find the voltage drop across 2 Ω resistance of the circuit shown in Figure 17.

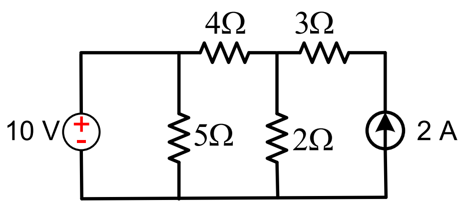

Figure 17

Solution:
By considering a single voltage source of 10 V the redrawn circuit is as shown in Figure 27. In the circuit 4 and 2 Ω resistors are in series which is in parallel with 5 Ω resistor.

$$R = \frac{5 \times 6}{5 + 6} = 2.727\ \Omega$$

$$I = \frac{10}{2.727} = 3.666\ A$$

Using Current division method, the current through 2 Ω resistor is

$$I_1 = 3.666\frac{5}{5 + 6} = 1.666\ A$$

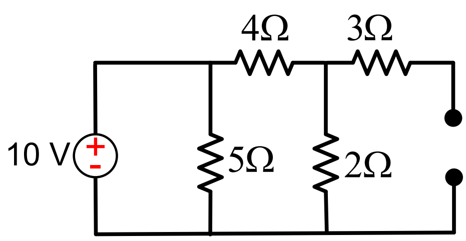

Figure 18

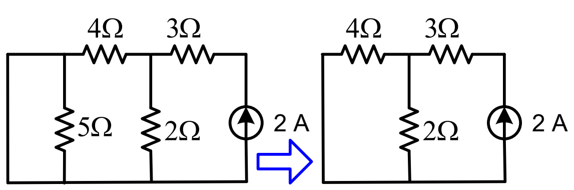

Figure 19

Network resistance is

$$R = \frac{4 \times 2}{6} = 1.333\ \Omega$$

Using Current division method, the current through 2 Ω resistor is

$$I_1 = 2\frac{1.333}{2} = 1.333\ A$$

Total Current in the 2 Ω resistor is

$$I = 1.666\ A + 1.333\ A \simeq 3A$$

Voltage across the 2 Ω resistor is

$$V = 2 \times 3 = 6V$$

Q 6) In the circuit shown in Figure 20, using superposition theorem find the current I_x.

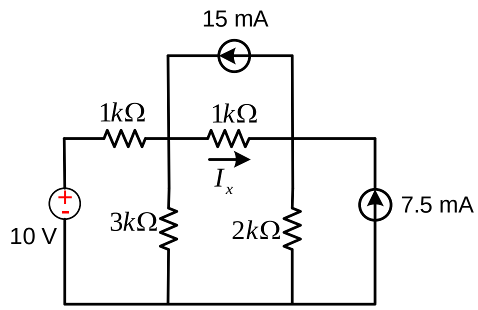

Figure 20

Solution:
By considering a single voltage source, the circuit is redrawn and is as shown in Figure 21.

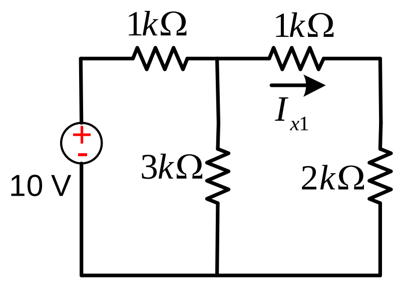

Figure 21

By Converting voltage source to current source the circuit is as shown in Figure 22

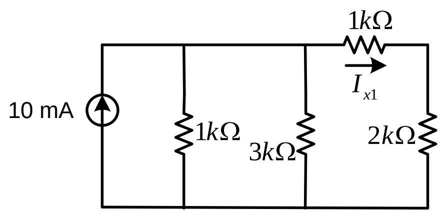

Figure 22

The total resistance of the network is is

$$\frac{1}{R_t} = \frac{1}{1} + \frac{1}{3} + \frac{1}{3} = 1.6667$$

$$R_t = 0.6k\Omega$$

The current Ix_1 is

$$Ix_1 = 10ma \times \frac{0.6k\Omega}{3k\Omega} = 2mA$$

By Considering 15mA current source then the redrawn circuit is as shown in Figure 23

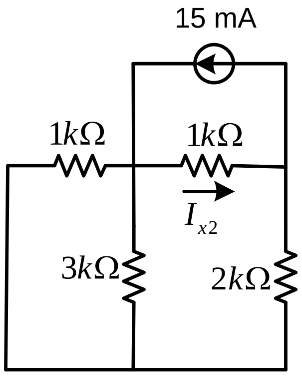

Figure 23

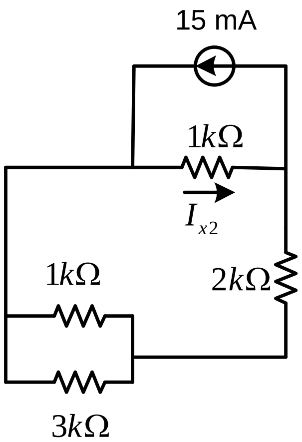

Figure 24

The 1kΩ and 3kΩ resistances are in parallel

$$\frac{1}{R_t} = \frac{1}{1} + \frac{1}{3} = 1.333$$

$$R_t = 0.750k\Omega$$

This resistance is in series with 2kΩ i.e 2.750kΩ

The current Ix_2 is

$$Ix_2 = 15ma \times \frac{2.750k\Omega}{3.750k\Omega} = 11mA$$

By Considering 7.5mA current source then the redrawn circuit is as shown in Figure ??

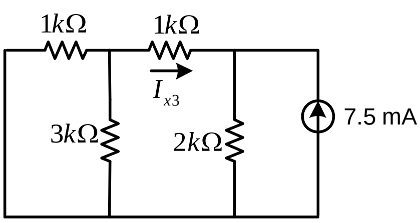

Figure 25

The 1kΩ and 3kΩ resistances are in parallel

$$\frac{1}{R_t} = \frac{1}{1} + \frac{1}{3} = 1.333$$

$$R_t = 0.750k\Omega$$

This resistance is in series with 1kΩ i.e 1.750kΩ

The current Ix_3 is

$$Ix_3 = -7.5ma \times \frac{2k\Omega}{3.750k\Omega} = -4mA$$

The total current Ix is

$$Ix_1 + Ix_1 + Ix_3 = 2ma + 11mA - 4mA = 9mA$$

Q 7) In the circuit shown in Figure 26, using superposition theorem find the current flowing through a load resistance $R_L = 10\Omega$.

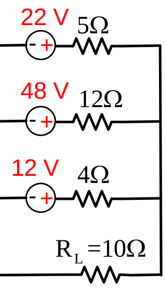

Figure 26

Solution:

By considering a single voltage source 22 V the redrawn circuit is as shown in Figure 27. In the circuit 12, 4 and 10 Ω resistors are in parallel.

$$\frac{1}{R} = \frac{1}{12} + \frac{1}{4} + \frac{1}{10} = 0.433$$
$$R = 2.3\Omega$$

2.3 Ω resistor is in series with 5 Ω. The net resistance in the circuit is 7.3 Ω

The current I in the circuit is

$$I = \frac{22}{7.3} = 3.01A$$

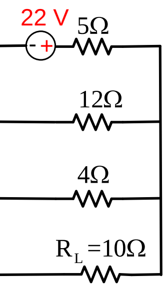

Figure 27

The current through $R_L = 10\Omega$ using current division method is

$$I = 3.01 \times \frac{2.3}{10} = 0.692A$$

By considering a single voltage source 48 V the redrawn circuit is as shown in Figure 28. In the circuit 5, 4 and 10 Ω resistors are in parallel.

$$\frac{1}{R} = \frac{1}{5} + \frac{1}{4} + \frac{1}{10} = 0.55$$
$$R = 1.818\Omega$$

1.818 Ω resistor is in series with 12 Ω. The net resistance in the circuit is 13.818 Ω

The current I in the circuit is

$$I = \frac{48}{13.818} = 3.473A$$

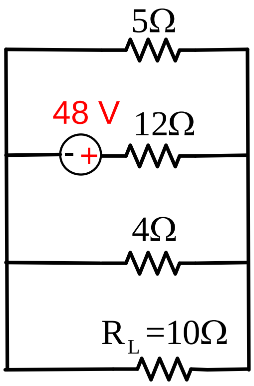

Figure 28

The current through $R_L = 10\Omega$ using current division method is

$$I = 3.473 \times \frac{1.818}{10} = 0.631A$$

By considering a single voltage source 12 V the redrawn circuit is as shown in Figure 29. In the circuit 5, 12 and 10 Ω resistors are in parallel.

$$\frac{1}{R} = \frac{1}{5} + \frac{1}{12} + \frac{1}{10} = 0.383$$
$$R = 2.611\Omega$$

2.611 Ω resistor is in series with 4 Ω. The net resistance in the circuit is 6.611 Ω

The current I in the circuit is

$$I = \frac{12}{6.611} = 1.815A$$

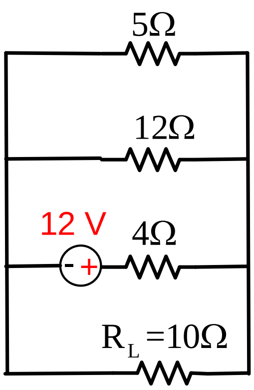

Figure 29

The current through $R_L = 10\Omega$ using current division method is

$$I = 1.815 \times \frac{2.611}{10} = 0.474A$$

The net current through $R_L = 10\Omega$ is

$$I = 0.692 + 0.631 + 0.474 = 1.797A$$

Q 8) In the circuit shown in Figure 30, using superposition theorem find the current i.

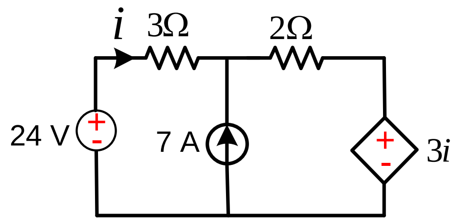

Figure 30

Solution:

By considering a single voltage source the circuit is redrawn and is as shown in Figure 31.

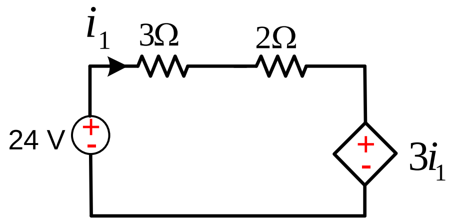

Figure 31

By applying KVL, around the loop

$$24 - (3+2)i_1 - 3i_1 = 0$$
$$i_1 = 3A$$

By considering a single current source 7A, the circuit is redrawn and is as shown in Figure 32.

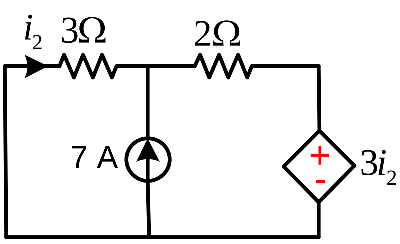

Figure 32

By applying node voltage analysis at node a

$$-i_2 - 7 + \frac{(V_A - 3i_2)}{2} = 0$$

Also we have

$$-i_2 = \frac{V_A}{3}$$

$$V_A = -3i_2$$

$$-i_2 7 + \frac{(-3i_2 - 3i_2)}{2} = 0$$

$$-i_2 = \frac{-7}{4}$$

$$i = i_1 + i_2 = 1.25A$$

0.2 Question Papers

2020-Aug) Use superposition theorem to find I_O in the circuit shown in Figure 26 .

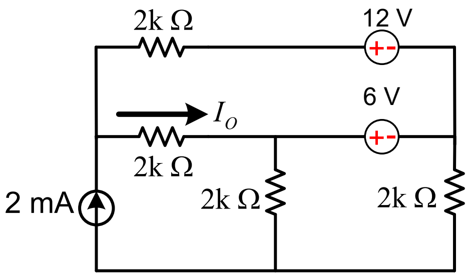

Figure 33

Solution:

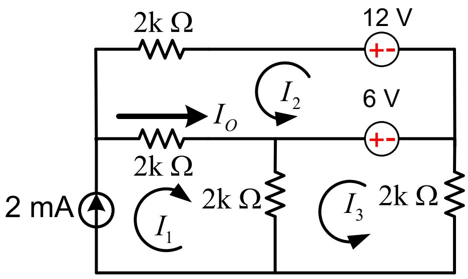

Figure 34

$$\begin{aligned} 2I_2 + 2(I_1 + I_2) + 6 - 12 &= 0 \\ 2I_1 + 4I_2 + 0I_3 &= 6 \end{aligned}$$

$$\begin{aligned} 2(I_1 + I_3) + 2I_3 - 6 &= 0 \\ 2I_1 + 0I_2 + 4I_3 &= 6 \end{aligned}$$

$$\begin{aligned} 2I_1 + 4I_2 + 0I_3 &= 6 \\ 2I_1 + 0I_2 + 4I_3 &= 6 \end{aligned}$$

$$I_1 = 2mA$$

$$\begin{aligned} 4I_2 + 0I_3 &= 2 \\ 0I_2 + 4I_3 &= 2 \end{aligned}$$

On solving

$$I_1 = 2mA, \ I_2 = 0.5 \ I_3 = 0.5$$

$$I_O = I_1 + I_2 = 2 + 0.5 = 2.5mA$$

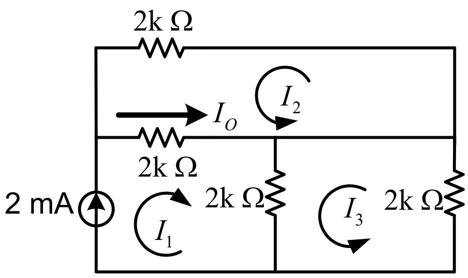

Figure 35

$$\begin{aligned} 2I_2 + 2(I_1 + I_2) &= 0 \\ 2I_1 + 4I_2 + 0I_3 &= 0 \end{aligned}$$

$$\begin{aligned} 2(I_1 + I_3) + 2I_3 &= 0 \\ 2I_1 + 0I_2 + 4I_3 &= 0 \end{aligned}$$

$$\begin{aligned} 2I_1 + 4I_2 + 0I_3 &= 0 \\ 2I_1 + 0I_2 + 4I_3 &= 0 \end{aligned}$$

$$I_1 = 2mA$$

$$\begin{aligned} 4I_2 + 0I_3 &= -4 \\ 0I_2 + 4I_3 &= -4 \end{aligned}$$

On solving

$$I_1 = 2mA, \ I_2 = -1 I_3 = -1$$

$$I_{O1} = I_1 + I_2 = 2 - 1 = 1mA$$

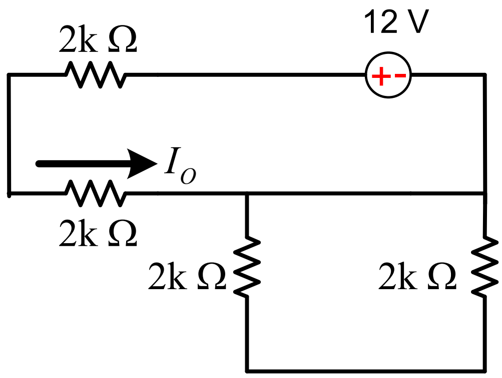

Figure 36

$$I_{02} = \frac{12}{4} = 3mA$$

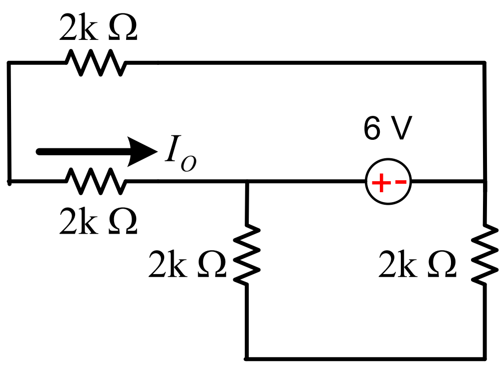

Figure 37

$$I_{03} = \frac{6}{4} = 1.5mA$$

$$I_O = I_1 + I_2 = 1 + 3 - 1.5 = 2.5mA$$

2020-AugEE) Use superposition theorem to find I_O in the circuit shown in Figure 38 .

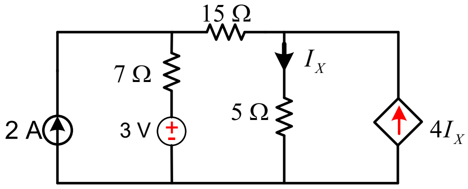

Figure 38

Solution:

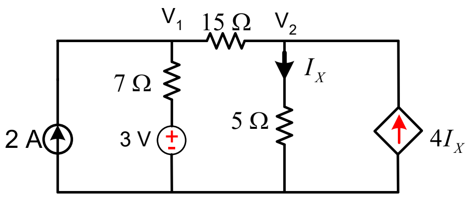

Figure 39

$$\begin{aligned}
\frac{V_1 - 3}{7} + \frac{V_1 - V_2}{15} - 2 &= 0 \\
V_1[0.1428 + 0.067] - 0.428 - 0.0.067V_2 - 2 &= 0 \\
0.21V_1 - 0.067V_2 &= 2.428
\end{aligned}$$

$$\begin{aligned}
\frac{V_2}{5} + \frac{V_2 - V_1}{15} - 4I_X &= 0 \\
-0.067V_1 + V_2[0.5 + 0.067] - 4I_X &= 0 \\
-0.067V_1 + 0.567V_2 - 4I_X &= 0
\end{aligned}$$

$$I_X = \frac{V_2}{5} = 0.2V_2$$

$$\begin{aligned}
-0.067V_1 + 0.567V_2 - 4I_X &= 0 \\
-0.067V_1 + 0.567V_2 - 4(0.2V_2) &= 0 \\
-0.067V_1 + 1.367V_2 &= 0
\end{aligned}$$

$$\begin{aligned}
0.21V_1 - 0.067V_2 &= 2.428 \\
-0.067V_1 + 1.367V_2 &= 0
\end{aligned}$$

On Solving

$$V_1 = 11.745 \quad V_2 = 0.575$$

$$I_X = \frac{V_2}{5} = \frac{0.575}{5} = 0.115$$

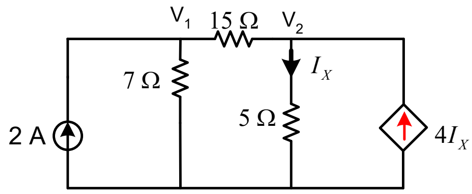

Figure 40

$$\begin{aligned}
\frac{V_1}{7} + \frac{V_1 - V_2}{15} - 2 &= 0 \\
V_1[0.1428 + 0.067] - 0.067V_2 - 2 &= 0 \\
0.21V_1 - 0.067V_2 &= 2
\end{aligned}$$

$$\begin{aligned}
0.21V_1 - 0.067V_2 &= 2 \\
-0.067V_1 + 1.367V_2 &= 0
\end{aligned}$$

On Solving

$$V_1 = 9.675 \quad V_2 = 0.474$$

$$I_{X1} = \frac{V_2}{5} = \frac{0.474}{5} = 0.095$$

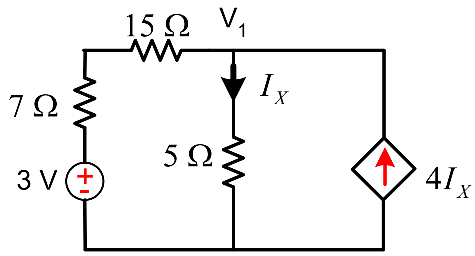

Figure 41

$$\begin{aligned}
\frac{V_1 - 3}{22} + \frac{V_1}{5} - 4I_x &= 0 \\
V_1[0.045 + 0.2] - 0.136 - 4\frac{V_1}{5} &= 0 \\
0.245V_1 - 0.8V_1 &= 0.136 \\
-0.555V_1 &= 0.136 \\
V_1 &= -\frac{0.136}{0.555} = -0.245
\end{aligned}$$

$$I_{X2} = \frac{V_1}{5} = \frac{-0.245}{5} = -0.049$$

2018-DEC-17Scheme) For the circuit shown in Figure find the current I_X 42 using superposition theorem.

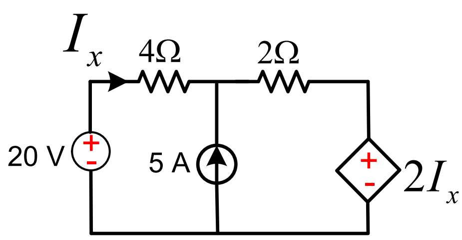

Figure 42

Solution:

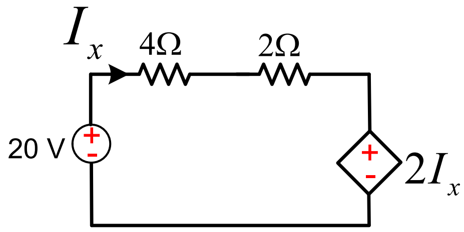

Figure 43

$$
\begin{aligned}
20 - 6I_X - 2I_X &= 0 \\
I_X &= \frac{20}{8} = 2.5A
\end{aligned}
$$

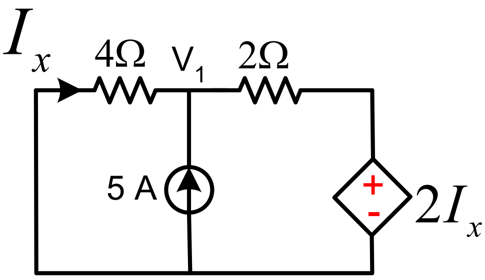

Figure 44

$$
\begin{aligned}
\frac{V_1}{4} + \frac{V_1 - 2I_x}{2} - 5 &= 0 \\
(0.25 + 0.5)V_1 - I_x &= 5
\end{aligned}
$$

$$
I_x = \frac{V_1}{4} = 0.25V_1
$$

$$
\begin{aligned}
[0.75]V_1 - 0.25V_1 &= 5 \\
V_1 &= \frac{5}{0.5} = 10
\end{aligned}
$$

$$
I_x = \frac{V_1}{4} = \frac{10}{4} = 2.5
$$

Verification

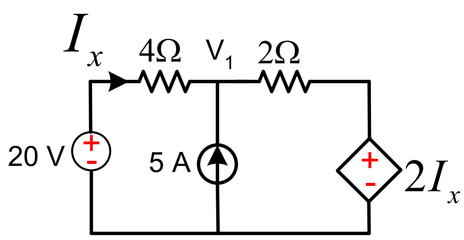

Figure 45

$$
\begin{aligned}
\frac{V_1 - 20}{4} + \frac{V_1 - 2I_x}{2} - 5 &= 0 \\
(0.25 + 0.5)V_1 - I_x &= 10
\end{aligned}
$$

$$
I_x = \frac{V_1}{4}
$$

$$
\begin{aligned}
[0.75]V_1 - 0.25V_1 &= 10 \\
V_1 &= \frac{10}{0.5} = 20
\end{aligned}
$$

$$
I_x = \frac{V_1}{4} = \frac{20}{4} = 5
$$

2017-June) Using superposition theorem find the current in 6 Ω resistor in circuit shown in Figure 46.

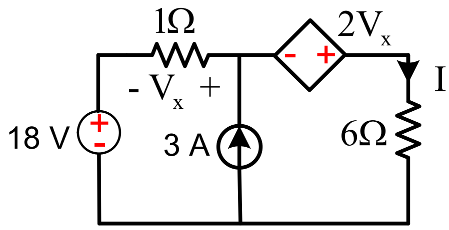

Figure 46

Solution:

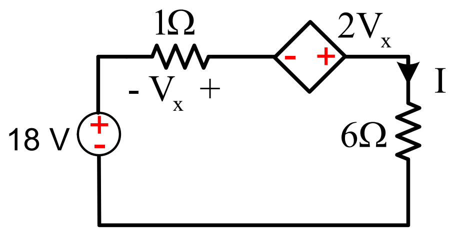

Figure 47

$$
18 - 1I + 2V_X - 6I = 0
$$

$$
V_X = I
$$

$$
\begin{aligned}
18 - 1I + 2V_X - 6I &= 0 \\
18 - 1I + 2(I) - 6I &= 0 \\
I &= \frac{18}{5} = 3.6
\end{aligned}
$$

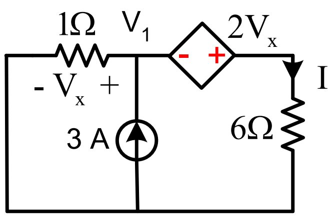

Figure 48

$$
\begin{aligned}
\frac{V_1}{1} + \frac{V_1 + 2V_x}{6} - 3 &= 0 \\
(1 + 0.166)V_1 + 0.333V_x &= 3
\end{aligned}
$$

$$
\begin{aligned}
V_x &= V_1 \\
1.166V_1 + 0.33V_1 &= 3 \\
V_1 &= \frac{3}{1.5} = 2
\end{aligned}
$$

$$
I_x = \frac{V_1}{4} = \frac{10}{4} = 2.5
$$

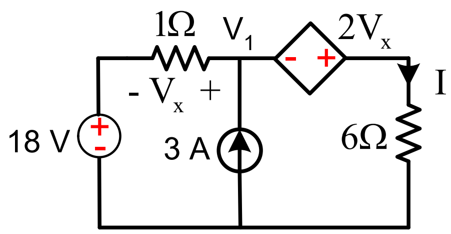

Figure 49

2016-June) Using superposition theorem find the current I in circuit shown in Figure 50.

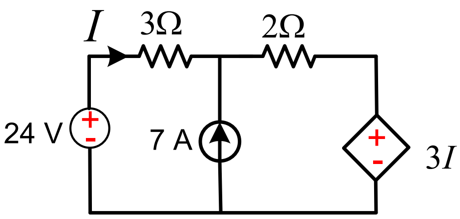

Figure 50

Solution:

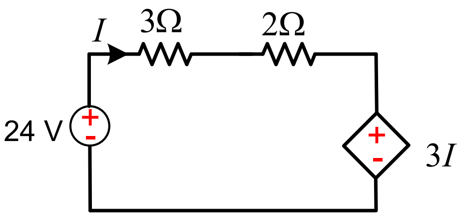

Figure 51

$$
\begin{aligned}
24 - 5I - 3I &= 0 \\
I &= \frac{24}{8} = 3
\end{aligned}
$$

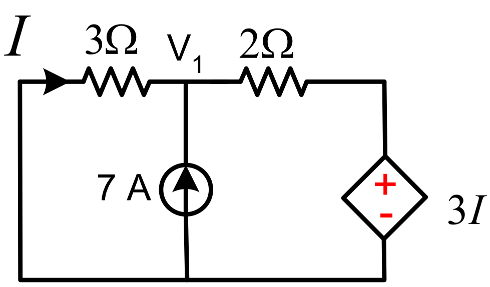

Figure 52

$$
\begin{aligned}
\frac{V_1}{3} + \frac{V_1 - 3I}{2} - 7 &= 0 \\
(0.333 + 0.5)V_1 - 1.5I &= 7
\end{aligned}
$$

$$
I = \frac{V_1}{3}
$$

$$
\begin{aligned}
[0.8333]V_1 - 0.5V_1 &= 7 \\
V_1 &= \frac{7}{0.333} = 21
\end{aligned}
$$

$$
I = \frac{V_1}{3} = \frac{21}{3} = 8
$$

Total current I in the circuit is

$$
I = 3 - 8 = -5A
$$

Which is flowing opposite to the direction.

Verification

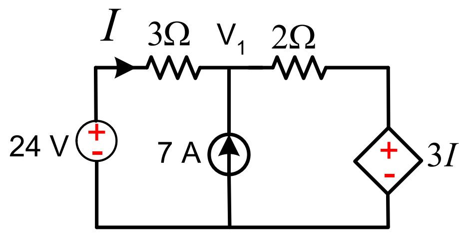

Figure 53

$$
\begin{aligned}
\frac{V_1 - 24}{3} + \frac{V_1 - 3I}{2} - 7 &= 0 \\
(0.333 + 0.5)V_1 - 1.5I &= 15
\end{aligned}
$$

$$
I = \frac{V_1 - 24}{3} = 0.333V_1 - 8
$$

$$
\begin{aligned}
(0.833)V_1 - 1.5(0.333V_1 - 8) &= 15 \\
(0.833)V_1 - 0.5V_1 + 12 &= 15 \\
0.333V_1 &= 3 \\
V_1 &= \frac{3}{0.333} = 9
\end{aligned}
$$

$$
I = \frac{V_1 - 24}{3} = \frac{9 - 24}{3} = -5
$$

2015-Dec) Find I_X for the circuit shown in Figure 54 using superposition theorem.

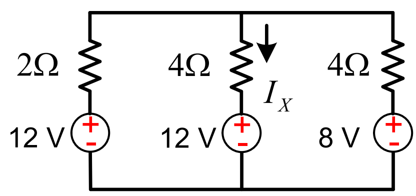

Figure 54

Solution:

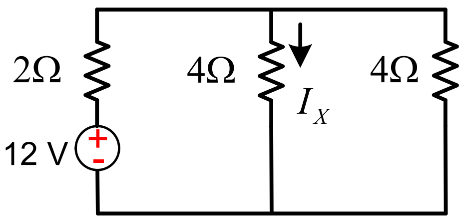

Figure 55

$$
\begin{aligned}
\frac{V-12}{2}+\frac{V}{4}+\frac{V}{4} &= 0 \\
V &= 6
\end{aligned}
$$

$$
I = \frac{V}{4} = \frac{6}{4} = 1.5
$$

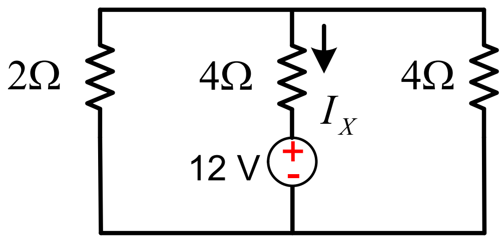

Figure 56

$$
\begin{aligned}
\frac{V}{2}+\frac{V-12}{4}+\frac{V}{4} &= 0 \\
V &= 3
\end{aligned}
$$

$$
I = \frac{V-12}{4} = \frac{3-12}{4} = -2.25
$$

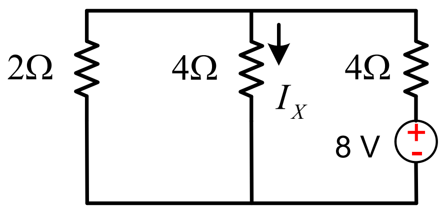

Figure 57

$$
\begin{aligned}
\frac{V}{2}+\frac{V}{4}+\frac{V-8}{4} &= 0 \\
V &= 2
\end{aligned}
$$

$$
I = \frac{V}{4} = \frac{2}{4} = 0.5
$$

Total current I_X is

$$
I = 1.5-2.25+0.5 = -0.25
$$

Verification

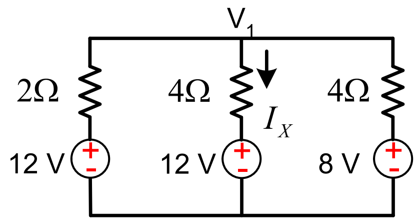

Figure 58

$$
\begin{aligned}
\frac{V-12}{2}+\frac{V-12}{4}+\frac{V-8}{4} &= 0 \\
V &= 11
\end{aligned}
$$

$$
I = \frac{V-12}{4} = \frac{11-12}{4} = -0.25
$$

2011-June) Find V using the principle of superposition in network shown in Figure 59 .

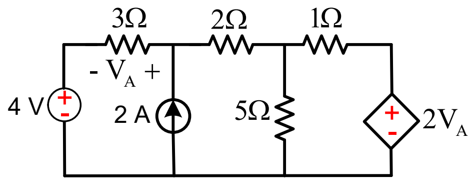

Figure 59

Solution:

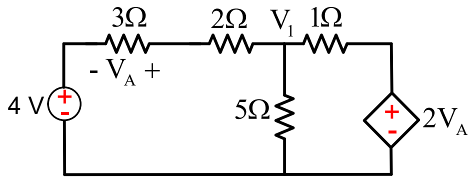

Figure 60

$$
\begin{aligned}
\frac{V_1-4}{5}+\frac{V_1}{5}+\frac{V_1-2V_A}{1} &= 0 \\
V_1(0.2+0.2+1)-2V_A &= 0.8
\end{aligned}
$$

$$
V_A = 3\left(\frac{V_1-4}{5}\right) = 0.6V_1-2.4
$$

$$
\begin{aligned}
1.4V_1-2(0.6V_1-2.4) &= 0.8 \\
1.4V_1-1.2V_1+4.8 &= 0.8 \\
0.2V_1 &= -4 \\
V_1 &= -\frac{4}{0.2} = -20
\end{aligned}
$$

$$
V_A = 3\left(\frac{V_1-4}{5}\right) = 0.6(-20)-2.4 = -14.4
$$

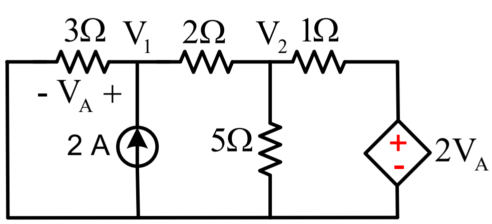

Figure 61

$$\begin{aligned}
\frac{V_1}{3}+\frac{V_1-V_2}{2}-2 &= 0\\
(0.33+0.5)V_1-0.5V_2 &= 2\\
(0.833)V_1-0.5V_2 &= 2
\end{aligned}$$

$$\begin{aligned}
\frac{V_2-V_1}{2}+\frac{V_2}{5}+\frac{V_2-2V_A}{1} &= 0\\
-0.5V_1+(0.2+0.5+1)V_1-2V_A &= \\
-0.5V_1+1.7V_1-2V_A &=
\end{aligned}$$

$$V_A = V_1$$

$$-2.5V_1+1.7V_1 = 0$$

$$\begin{aligned}
0.833V_1-0.5V_2 &= 2\\
-2.5V_1+1.7V_1 &= 0
\end{aligned}$$

$$V_1 = 20.46 \quad V_2 = 30.1$$

Total Voltage V_A is

$$V_A = 20.46 \quad V_2 = -14.4+20.46 = 6.06V$$