2020-Aug ) Use superposition theorem to find $I_{O}$ in the circuit shown in Figure ?? .


Figure 1

## Solution:



Figure 2

$$
\begin{array}{r}
2 I_{2}+2\left(I_{1}+I_{2}\right)+6-12=0 \\
2 I_{1}+4 I_{2}+0 I_{3}=6 \\
2\left(I_{1}+I_{3}\right)+2 I_{3}-6=0 \\
2 I_{1}+0 I_{2}+4 I_{3}=6 \\
2 I_{1}+4 I_{2}+0 I_{3}=6 \\
2 I_{1}+0 I_{2}+4 I_{3}=6
\end{array}
$$

$$
I_{1}=2 m A
$$

$$
\begin{aligned}
& 4 I_{2}+0 I_{3}=2 \\
& 0 I_{2}+4 I_{3}=2
\end{aligned}
$$

On solving

$$
I_{1}=2 m A, I_{2}=0.5 I_{3}=0.5
$$

$$
I_{O}=I_{1}+I_{2}=2+0.5=2.5 m A
$$



Figure 3

$$
\begin{aligned}
2 I_{2}+2\left(I_{1}+I_{2}\right) & =0 \\
2 I_{1}+4 I_{2}+0 I_{3} & =0 \\
2\left(I_{1}+I_{3}\right)+2 I_{3} & =0 \\
2 I_{1}+0 I_{2}+4 I_{3} & =0 \\
& \\
2 I_{1}+4 I_{2}+0 I_{3} & =0 \\
2 I_{1}+0 I_{2}+4 I_{3} & =0
\end{aligned}
$$

$$
\begin{gathered}
I_{1}=2 m A \\
4 I_{2}+0 I_{3}=-4 \\
0 I_{2}+4 I_{3}=-4
\end{gathered}
$$

On solving

$$
I_{1}=2 m A, I_{2}=-1 I_{3}=-1
$$

$$
I_{O 1}=I_{1}+I_{2}=2-1=1 m A
$$



Figure 4

$$
I_{02}=\frac{12}{4}=3 m A
$$



Figure 5

$$
I_{03}=\frac{6}{4}=1.5 \mathrm{~mA}
$$

$$
I_{O}=I_{1}+I_{2}=1+3-1.5=2.5 \mathrm{~mA}
$$

2020-AugEE ) Use superposition theorem to find $I_{O}$ in the circuit shown in Figure 6.


Figure 6

## Solution:



Figure 7

$$
\begin{aligned}
\frac{V_{1}-3}{7}+\frac{V_{1}-V_{2}}{15}-2 & =0 \\
V_{1}[0.1428+0.067]-0.428-0.0 .067 V_{2}-2 & =0 \\
0.21 V_{1}-0.067 V_{2} & =2.428 \\
\frac{V_{2}}{5}+\frac{V_{2}-V_{1}}{15}-4 I_{X} & =0 \\
-0.067 V_{1}+V_{2}[0.5+0.067]-4 I_{X} & =0 \\
-0.067 V_{1}+0.567 V_{2}-4 I_{X} & =0
\end{aligned}
$$

$$
I_{X}=\frac{V_{2}}{5}=0.2 V_{2}
$$

$$
-0.067 V_{1}+0.567 V_{2}-4 I_{X}=0
$$

$$
-0.067 V_{1}+0.567 V_{2}-4\left(0.2 V_{2}\right)=0
$$

$$
-0.067 V_{1}+1.367 V_{2}=0
$$

$$
0.21 V_{1}-0.067 V_{2}=2.428
$$

$$
-0.067 V_{1}+1.367 V_{2}=0
$$

On Solving

$$
\begin{aligned}
V_{1} & =11.745 \quad V_{2}=0.575 \\
I_{X} & =\frac{V_{2}}{5}=\frac{0.575}{5}=0.115
\end{aligned}
$$



$$
\begin{aligned}
& \frac{V_{1}}{7}+\frac{V_{1}-V_{2}}{15}-2=0 \\
& V_{1}[0.1428+0.067]-0.067 V_{2}-2=0 \\
& 0.21 V_{1}-0.067 V_{2}= 2 \\
& \\
& 0.21 V_{1}-0.067 V_{2}=2 \\
&-0.067 V_{1}+1.367 V_{2}=0
\end{aligned}
$$

On Solving

$$
\begin{aligned}
V_{1} & =9.675 \quad V_{2}=0.474 \\
I_{X 1} & =\frac{V_{2}}{5}=\frac{0.474}{5}=0.095
\end{aligned}
$$



Figure 9

$$
\begin{aligned}
\frac{V_{1}-3}{22}+\frac{V_{1}}{5}-4 I_{x} & =0 \\
V_{1}[0.045+0.2]-0.136-4 \frac{V_{1}}{5} & =0 \\
0.245 V_{1}-0.8 V_{1} & =0.136 \\
-0.555 V_{1} & =0.136 \\
V_{1} & =-\frac{0.136}{0.555}=-0.245
\end{aligned}
$$

$$
I_{X 2}=\frac{V_{1}}{5}=\frac{-0.245}{5}=-0.049
$$

2018-DEC-17Scheme ) For the circuit shown in Figure find the current $I_{X} 10$ using superposition theorem.


Figure 10
Solution:


Figure 11

$$
\begin{aligned}
20-6 I_{X}-2 I_{X} & =0 \\
I_{X} & =\frac{20}{8}=2.5 \mathrm{~A}
\end{aligned}
$$



Figure 12

$$
\begin{array}{r}
\frac{V_{1}}{4}+\frac{V_{1}-2 I_{x}}{2}-5=0 \\
(0.25+0.5) V_{1}-I_{x}=5
\end{array}
$$

$$
I_{x}=\frac{V_{1}}{4}=0.25 V_{1}
$$

$$
\begin{aligned}
{[0.75] V_{1}-0.25 V_{1} } & =5 \\
V_{1} & =\frac{5}{0.5}=10
\end{aligned}
$$

$$
I_{x}=\frac{V_{1}}{4}=\frac{10}{4}=2.5
$$

## Verification



Figure 13

$$
\begin{aligned}
\frac{V_{1}-20}{4}+\frac{V_{1}-2 I_{x}}{2}-5 & =0 \\
(0.25+0.5) V_{1}-I_{x} & =10
\end{aligned}
$$

$$
\begin{aligned}
I_{x}= & \frac{V_{1}}{4} \\
{[0.75] V_{1}-0.25 V_{1} } & =10 \\
V_{1} & =\frac{10}{0.5}=20
\end{aligned}
$$

$$
I_{x}=\frac{V_{1}}{4}=\frac{20}{4}=5
$$

2017-June ) Using superposition theorem find the current in $6 \Omega$ resistor in circuit shown in Figure 14.


Figure 14

## Solution:



Figure 15

$$
18-1 I+2 V_{X}-6 I=0
$$

$$
V_{X}=I
$$

$$
\begin{aligned}
& 18-1 I+2 V_{X}-6 I=0 \\
& 18-1 I+2(I)-6 I=0
\end{aligned}
$$

$$
I=\frac{18}{5}=3.6
$$



Figure 16

$$
\begin{array}{r}
\frac{V_{1}}{1}+\frac{V_{1}+2 V_{x}}{6}-3=0 \\
(1+0.166) V_{1}+0.333 V_{x}=3
\end{array}
$$

$$
\begin{gathered}
V_{x}=V_{1} \\
1.166 V_{1}+0.33 V_{1}=3 \\
V_{1}=\frac{3}{1.5}=2 \\
I_{x}=\frac{V_{1}}{4}=\frac{10}{4}=2.5
\end{gathered}
$$

Figure 17
2016-June ) Using superposition theorem find the current $I$ in circuit shown in Figure 18.


Figure 18
Solution:


Figure 19

$$
\begin{aligned}
24-5 I-3 I & =0 \\
I & =\frac{24}{8}=3
\end{aligned}
$$



Figure 20

$$
\begin{aligned}
\frac{V_{1}}{3}+\frac{V_{1}-3 I}{2}-7 & =0 \\
(0.333+0.5) V_{1}-1.5 I & =7
\end{aligned}
$$

$$
\begin{gathered}
I=\frac{V_{1}}{3} \\
{[0.8333] V_{1}-0.5 V_{1}=7} \\
V_{1}=\frac{7}{0.333}=21 \\
I=\frac{V_{1}}{3}=\frac{21}{3}=8
\end{gathered}
$$

Total current I in the circuit is

$$
I=3-8=-5 A
$$

Which is flowing opposite to the direction.
Verification


Figure 21

$$
\begin{aligned}
& \frac{V_{1}-24}{3}+\frac{V_{1}-3 I}{2}-7=0 \\
& (0.333+0.5) V_{1}-1.5 I=15 \\
& I=\frac{V_{1}-24}{3}=0.333 V_{1}-8
\end{aligned}
$$

$$
\begin{aligned}
(0.833) V_{1}-1.5\left(0.333 V_{1}-8\right) & =15 \\
(0.833) V_{1}-0.5 V_{1}+12 & =15 \\
0.333 V_{1} & =3 \\
V_{1} & =\frac{3}{0.333}=9
\end{aligned}
$$

$$
I=\frac{V_{1}-24}{3}=\frac{9-24}{3}=-5
$$

2015-Dec ) Find $I_{X}$ for the circuit shown in Figure 22 using superposition theorem.


Figure 22
Solution:


Figure 23

$$
\begin{aligned}
\frac{V-12}{2}+\frac{V}{4}+\frac{V}{4} & =0 \\
V & =6
\end{aligned}
$$

$$
I=\frac{V}{4}=\frac{6}{4}=1.5
$$



Figure 24

$$
\begin{aligned}
\frac{V}{2}+\frac{V-12}{4}+\frac{V}{4} & =0 \\
V & =3
\end{aligned}
$$

$$
I=\frac{V-12}{4}=\frac{3-12}{4}=-2.25
$$

$2 \Omega$


Figure 25

$$
\begin{aligned}
\frac{V}{2}+\frac{V}{4}+\frac{V-8}{4} & =0 \\
V & =2
\end{aligned}
$$

$$
I=\frac{V}{4}=\frac{2}{4}=0.5
$$

Total current $I_{X}$ is

$$
I=1.5-2.25+0.5=-0.25
$$

## Verification



Figure 26

$$
\begin{aligned}
& \frac{V-12}{2}+\frac{V-12}{4}+\frac{V-8}{4}=0 \\
& V=11 \\
& I=\frac{V-12}{4}=\frac{11-12}{4}=-0.25
\end{aligned}
$$

2011-June ) Find $V$ using the principle of superposition in network shown in Figure 27.


Figure 27
Solution:


Figure 28

$$
\begin{aligned}
& \frac{V_{1}-4}{5}+\frac{V_{1}}{5}+\frac{V_{1}-2 V_{A}}{1}=0 \\
& V_{1}(0.2+0.2+1)-2 V_{A}=0.8 \\
& V_{A}=3\left(\frac{V_{1}-4}{5}\right)=0.6 V_{1}-2.4
\end{aligned}
$$

$$
\begin{aligned}
1.4 V_{1}-2\left(0.6 V_{1}-2.4\right) & =0.8 \\
1.4 V_{1}-1.2 V_{1}+4.8 & =0.8 \\
0.2 V_{1} & =-4 \\
V_{1} & =-\frac{4}{0.2}=-20
\end{aligned}
$$

$$
V_{A}=3\left(\frac{V_{1}-4}{5}\right)=0.6(-20)-2.4=-14.4
$$



Figure 29

$$
\begin{gathered}
\frac{V_{1}}{3}+\frac{V_{1}-V_{2}}{2}-2=0 \\
(0.33+0.5) V_{1}-0.5 V_{2}=2 \\
(0.833) V_{1}-0.5 V_{2}=2 \\
\frac{V_{2}-V_{1}}{2}+\frac{V_{2}}{5}+\frac{V_{2}-2 V_{A}}{1}=0 \\
-0.5 V_{1}+(0.2+0.5+1) V_{1}-2 V_{A}= \\
-0.5 V_{1}+1.7 V_{1}-2 V_{A}= \\
V_{A}=V_{1} \\
-2.5 V_{1}+1.7 V_{1}=0 \\
0.833 V_{1}-0.5 V_{2}=2 \\
-2.5 V_{1}+1.7 V_{1}=0 \\
V_{1}=20.46 V_{2}=30.1
\end{gathered}
$$

Total Voltage $V_{A}$ is

$$
V_{A}=20.46 \quad V_{2}=-14.4+20.46=6.06 \mathrm{~V}
$$

2014-July Find the voltage across $3 \Omega$ resistor using superposition theorem for the circuit shown in Figure 30


Figure 30: 2014-July-Question Paper
Solution:
By considering 6 Volt supply the circuit is redrawn which is as shown in Figure 31


Figure 31
Voltage across $3 \Omega$ resistor is

$$
V_{1}=\frac{6}{3+1.5} 3=4 V
$$

By considering 18 Volt supply the circuit is redrawn which is as shown in Figure 32


Figure 32
The network resistance is

$$
R_{t}=6+\frac{2 \times 3}{2+3}=7.2
$$

The total current flowing in the network is

$$
I=\frac{18}{R_{t}}=2.5 A
$$

Current through $3 \Omega$ resistor is

$$
I=2.5 \frac{1.2}{3}=1 \mathrm{~A}
$$

Voltage across $3 \Omega$ resistor is

$$
V_{2}=1 \times 3=3 \mathrm{~V}
$$

By considering 2A current source the circuit is redrawn which is as shown in Figure 33


Figure 33


Figure 34
The network resistance is

$$
R_{t}=\frac{1}{6}+\frac{1}{2}+\frac{1}{3} \approx 1 \Omega
$$

Current through $3 \Omega$ resistor is

$$
I=1 \frac{2}{3}=0.6666 \mathrm{~A}
$$

Voltage across $3 \Omega$ resistor is

$$
V_{3}=-0.666 \times 3=-2 V
$$

The overall Voltage across $3 \Omega$ resistor by considering all the voltage sources is

$$
V=V_{1}+V_{2}+V_{3}=4+3-2=5 V
$$

2012-DEC Using superposition theorem obtain the response I for the circuit shown in Figure 35


Figure 35: 2012-DEC-Question Paper
Solution:
By considering Voltage source the circuit is redrawn which is as shown in Figure 36


Figure 36
There is no current flows in Voltage across $2 \Omega$ resistor. Current through inductor is

By KVL

$$
\begin{gathered}
I_{1}(j 2-j 1)+8 \angle 135=0 \\
I_{1}=-\frac{8 \angle 135}{j 2-j 1} \times j 2=-\frac{8 \angle 135}{j 1} \times j 2 \\
I_{1}=-8 \angle 135
\end{gathered}
$$

By considering Current source of 2 A the circuit is redrawn which is as shown in Figure 37


Figure 37

By considering 20 V Voltage source the circuit is redrawn which is as shown in Figure 36


Figure 40

By using current division method the current through $10 \Omega$ resistance is

$$
\begin{aligned}
& \frac{V_{1}}{10}+\frac{V_{1}}{-j 5}+\frac{V_{1}}{j 15}=\frac{20}{j 15} \\
& V_{1}[0.1+j 0.2-j 0.0666]=-j 1.333 \\
& V_{1}[0.1+j 0.1334]=-j 1.333 \\
& 0.1666 \angle 53.14 V_{1}=-1.333 \angle 90 \\
& V_{1}=\frac{-1.333 \angle 90}{0.1666 \angle 53.14} \\
&=-8 \angle 36.86 \\
& I=\frac{-8 \angle 36.86}{10}=-0.8 \angle 36.86=-0.64-j 0.48
\end{aligned}
$$



Figure 41
By using current division method the current through $10 \Omega$ resistance is

$$
\begin{aligned}
\frac{V_{2}}{10}+\frac{V_{2}}{-j 5}+\frac{V_{2}}{j 15} & =\frac{10 \angle 90}{-j 5} \\
V_{2}[0.1+j 0.2-j 0.0666] & =2 \angle 180 \\
V_{2}[0.1+j 0.1334] & =-2 \\
0.1666 \angle 53.14 V_{2} & =-2 \\
V_{2} & =\frac{-2}{0.1666 \angle 53.14} \\
& =-12 \angle-53.14
\end{aligned}
$$

The total current by considering both the sources is $I_{2}=\frac{-12 \angle-53.14}{10}=-1.2 \angle-53.14=-0.72+j 0.96$

$$
\begin{aligned}
I & =I_{1}+I_{2}=-0.64-j 0.48-0.72+j 0.96 \\
& =-1.36+j 0.48=1.422 \angle 160.56
\end{aligned}
$$

2011-December Determine the current through $Z_{3}$ using superposition theorem for the circuit shown in Figure 42


Figure 42: 2011-December-Question Paper
Solution:
By considering single voltage $10 \angle 0$, the circuit is redrawn which is as shown in Figure 36


Figure 43
By applying node voltage method

$$
\begin{aligned}
& V_{1}\left[\frac{1}{1+1 j}+\frac{1}{1+j 2}+\frac{1}{1-j 1}\right]=\frac{10}{1+j 1} \\
& V_{1}\left[\frac{1-j 1}{2}+\frac{1+j 1}{2}+\frac{1-j 2}{3}\right]=\frac{10}{1+j 1}
\end{aligned}
$$

$V_{1}[0.5-j 0.5+0.5+j 0.5+0.33-j 0.666]=\frac{10}{1+j 1}$

$$
\begin{gathered}
V_{1}[1.333-j 0.666]=7.07 \angle-45 \\
V_{1} 1.5 \angle-26.54=7.07 \angle-45 \\
V_{1}=\frac{7.07 \angle-45}{1.5 \angle-26.54}=4.713 \angle-18.46
\end{gathered}
$$

Current through $Z_{3}$ is

$$
\begin{gathered}
I_{1}=\frac{V_{1}}{1+j 2}=\frac{4.713 \angle-18.46}{2.23 \angle 63.43} \\
I_{1}=2.1 \angle-81.89
\end{gathered}
$$

By considering single voltage $10 \angle 0$, the circuit is redrawn which is as shown in Figure ??


Figure 44
By applying node voltage method

$$
\begin{gathered}
V_{2}\left[\frac{1}{1+1 j}+\frac{1}{1+j 2}+\frac{1}{1-j 1}\right]=\frac{10 \angle-60}{1+j 1} \\
V_{2}\left[\frac{1-j 1}{2}+\frac{1+j 1}{2}+\frac{1-j 2}{3}\right]=\frac{10 \angle-60}{1+j 1} \\
V_{2}[0.5-j 0.5+0.5+j 0.5+0.33-j 0.666]=\frac{10 \angle-60}{1+j 1} \\
V_{2}[1.333-j 0.666]=7.07 \angle-15
\end{gathered}
$$

$$
\begin{gathered}
V_{2} 1.5 \angle-26.54=7.07 \angle-15 \\
V_{2}=\frac{7.07 \angle-15}{1.5 \angle-26.54}=4.713 \angle 11.53
\end{gathered}
$$

Current through $Z_{3}$ is

$$
\begin{gathered}
I_{2}=\frac{V_{2}}{1+j 2}=\frac{4.713 \angle 11.53}{2.23 \angle 63.43} \\
I_{2}=2.1 \angle-51.9
\end{gathered}
$$

The total current by considering both the sources is

$$
\begin{gathered}
I_{2}=\frac{-12 \angle-53.14}{10}=-1.2 \angle-53.14=-0.72+j 0.96 \\
I=I_{1}+I_{2}=2.1 \angle-81.89+2.1 \angle-51.9 \\
\quad=0.29-j 2+1.29-j 1.65 \\
\quad=1.58-j 3.65=3.977 \angle-66.6
\end{gathered}
$$

2011-June Determine $V_{A}$ using superposition theorem for the circuit shown in Figure 45


Figure 45: 2011-December-Question Paper
Solution:
By considering single voltage 4 volts, the circuit is redrawn which is as shown in Figure 36


Figure 46
By applying KVL for the loops

$$
\begin{gathered}
5 i_{1}+5\left(i_{1}-i_{2}\right)-4=0 \\
10 i_{1}-5 i_{2}=4 \\
\\
V_{A}=-3 i_{1}
\end{gathered}
$$

$$
\begin{aligned}
1 i_{2}+5\left(i_{2}-i_{1}\right)+2 V_{A} & =0 \\
-5 i_{1}+6 i_{2}+2\left(-3 i_{1}\right) & =0 \\
-11 i_{1}+6 i_{2} & =0 \\
11 i_{1}-6 i_{2} & =0
\end{aligned}
$$

$$
\Delta=\left|\begin{array}{ll}
10 & -5 \\
11 & -6
\end{array}\right|=-60+55=-5
$$

$$
i_{1}=\frac{\left|\begin{array}{rr}
4 & -5 \\
0 & -6
\end{array}\right|}{\Delta}=\frac{-24}{-5}=4.8 A
$$

$$
V_{A}=-3 i_{1}=-3 \times 4.8=14.4 V
$$

By considering single current source 2A, the circuit is redrawn which is as shown in Figure 37


Figure 47
By applying KCL (Node analysis)
For Node $V_{1}$

$$
\begin{aligned}
V_{1}\left[\frac{1}{3}+\frac{1}{2}\right]-\frac{V_{2}}{2}-2 & =0 \\
0.833 V_{1}-0.5 V_{2} & =2
\end{aligned}
$$

For Node $V_{2}$
For Node $V_{A}=V_{1}$

$$
\begin{aligned}
V_{2}\left[\frac{1}{1}+\frac{1}{5}+\frac{1}{2}\right]-\frac{V_{1}}{2}-\frac{2 V_{A}}{1} & =0 \\
V_{2}[1+0.2+0.5]-0.5 V_{1}-\frac{2 V_{1}}{1} & =0 \\
-2.5 V_{1}+1.7 V_{2} & =0 \\
2.5 V_{1}-1.7 V_{2} & =0
\end{aligned}
$$

Simultaneous equations are

$$
\begin{gathered}
0.833 V_{1}-0.5 V_{2}=2 \\
2.5 V_{1}-1.7 V_{2}=0 \\
\Delta=\left|\begin{array}{cc}
0.833 & -0.5 \\
2.5 & -1.7
\end{array}\right|=-1.416+1.25=-0.166 \\
V_{1}=\frac{\left|\begin{array}{cc}
2 & -0.5 \\
0 & -1.7
\end{array}\right|}{\Delta}=\frac{-3.4}{-0.166}=20.4 \\
\text { By Superposition theorem }
\end{gathered}
$$

$$
V_{1}=-14.4+20.4=6 \mathrm{~V}
$$

2000-August Find the current through $R_{L}=7.5 \Omega$, using superposition theorem as shown in Figure 48


Figure 48: 2000-August-Question Paper
Solution:
Replace the voltage source by short circuit. $2 \Omega$ and $2 \Omega$ are in parallel which is in series with $4 \Omega$. The details are as shown in Figure 49


Figure 49

$$
\begin{gathered}
2 \Omega \| 2 \Omega=1 \Omega \\
4 \Omega+1 \Omega=5 \Omega
\end{gathered}
$$

$5 \Omega$ and $5 \Omega$ are in parallel


Figure 50
The current through $5 R_{L}$ using current division method is

$$
I_{L 1}=10 A \frac{2.5}{2.5+7.5}=2.5 A
$$

By removing the current source the circuit is as shown in Figure 51


Figure 51
Apply KVL and solve for loop currents

$$
I_{L}=10 A \frac{2.5}{2.5+7.5}=2.5 A
$$

$$
\begin{aligned}
12.5 I_{1}-7.5 I_{2}+0 I_{3} & =0 \\
-7.5 I_{1}+13.5 I_{2}-2 I_{3} & =0 \\
0 I_{1}-2 I_{2}+4 I_{3} & =20
\end{aligned}
$$

$$
\Delta=\left|\begin{array}{ccc}
12.5 & -7.5 & 0 \\
-7.5 & 13.5 & -2 \\
0 & -2 & 4
\end{array}\right|
$$

$$
12.5(54-4)+7.5(-30)=625-225=400
$$

$$
I_{1}=\frac{\left|\begin{array}{ccc}
0 & -7.5 & 0 \\
0 & 13.5 & -2 \\
20 & -2 & 4
\end{array}\right|}{\Delta}
$$

$20(15)=300$

$$
I_{1}=\frac{300}{400}=0.75
$$

$$
I_{1}=\frac{\left|\begin{array}{ccc}
12.5 & 0 & 0 \\
-7.5 & 0 & -2 \\
0 & 20 & 4
\end{array}\right|}{\Delta}
$$

$12.5(40)=500$

$$
I_{2}=\frac{500}{400}=1.25
$$

Current through $R_{L}$ is

$$
I_{2}-I_{1}=1.25-0.75=0.5
$$

Current through $R_{L}$ by considering both the sources is

$$
I_{L}=2.5+0.5=3 A
$$

