0.1 Thevenin/Norton Theorem

Q 2020-Aug) Find the Thevinin's and Norton's equivalent circuits at the terminals a-b for the circuit shown in Figure 1.

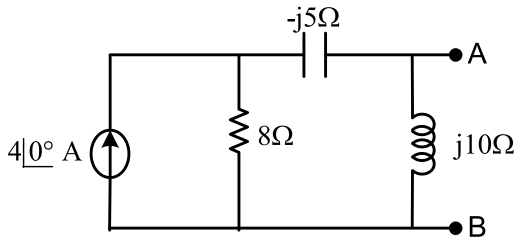

Figure 1

Solution:

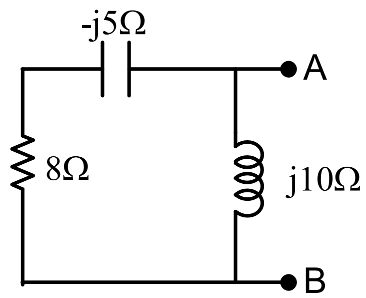

Figure 2

$$Z_{TH} = \frac{(8-j5)(j10)}{(8-j5)+(j10)} = 10\angle 26\Omega$$

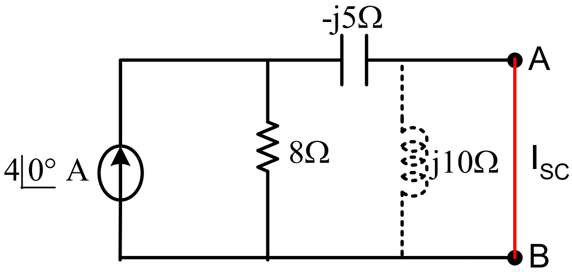

Figure 3

$$I_{SC} = I_N = 4\frac{8}{8-j5} = 3.39\angle 32\ A$$

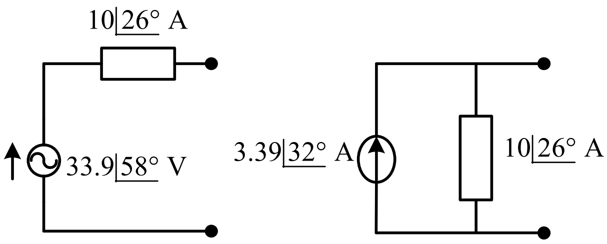

Figure 4

Q 2020-JUNE) Find the value of R_L for the network shown in Figure 5 that results in maximum power transfer. Also find the value of maximum power.

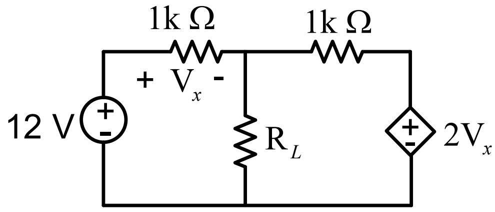

Figure 5

Solution:

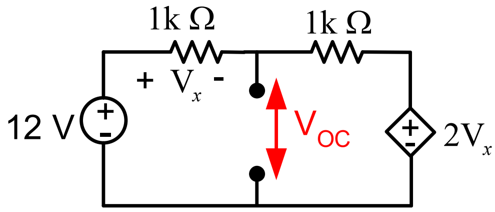

Figure 6

$$V_x = 1\times 10^3 i$$

Apply KVL around the loop

$$\begin{aligned} 2\times 10^3 i + 2V_x - 12 &= 0 \\ 2\times 10^3 i + 2(1\times 10^3 i) &= 12 \\ i &= \frac{12}{4\times 10^3} = 3mA \end{aligned}$$

The voltage V_{OC}

$$\begin{aligned} V_{OC} &= 12 - 1\times 10^3 i - \\ &= 12 - 1\times 10^3 3mA \\ &= 9V \end{aligned}$$

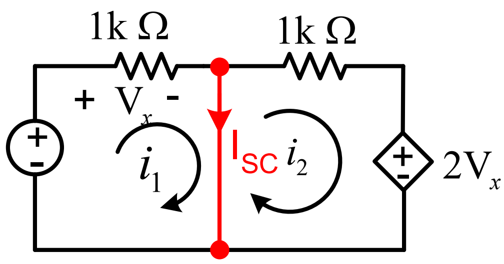

Figure 7

KVL for the mesh 1

$$\begin{aligned} 1\times 10^3 i_1 - 12 &= 0 \\ i_1 &= \frac{12}{1\times 10^3} = 12mA \end{aligned}$$

KVL for the mesh 2

$$\begin{aligned} 1\times 10^3 i_2 + 2V_x &= 0 \\ 1\times 10^3 i_2 + 2(1\times 10^3 i_1) &= 0 \\ 1\times 10^3 i_2 + 24 &= 0 \\ i_2 &= -\frac{24}{1\times 10^3} = 24mA \end{aligned}$$

The short circuit current is

$$\begin{aligned} I_{SC} &= i_1 - i_2 = 12mA - (-24mA) \\ &= 36mA \end{aligned}$$

The Thevenin's resistance is

$$\begin{aligned} R_{TH} &= \frac{V_{OC}}{I_{SC}} = \frac{9}{36mA} \\ &= 250\ \Omega \end{aligned}$$

Maximum power is transferred when $R_L = R_{TH}$. The current in the circuit is

$$i = \frac{9}{250 + 250} = 0.018A$$

Maximum power is

$$P = i^2 R_L = (0.018)^2 \times 250 = 81mW$$

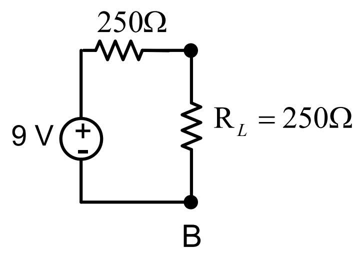

Figure 8

Q 2020-EE-JUNE) Determine the Thevenin's equivalent of the circuit shown in Figure ??

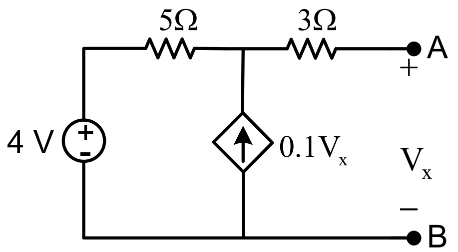

Figure 9

Solution:

$$\begin{aligned} \frac{V_x - 4}{5} - 0.1V_x &= 0 \\ 0.2V_x - 0.1V_x &= 0.8 \\ V_x &= \frac{0.8}{0.1} = 8V = V_{OC} \end{aligned}$$

By shorting the terminals

$$V_x = 0$$

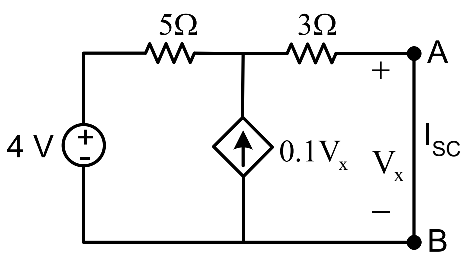

Figure 10

$$\begin{aligned} \frac{V_1 - 4}{5} - \frac{V_1}{3} &= 0 \\ 0.2V_1 - 0.8 - 0.33V_1 &= 0 \\ -0.1333V_1 &= 0.8 \\ V_1 &= -\frac{0.8}{0.1333} = 6V \end{aligned}$$

$$I_{SC} = \frac{V_1}{3} = \frac{6}{3} = 2A$$

$$Z_{SC} = \frac{V_{OC}}{I_{SC}} = \frac{8}{2} = 4\Omega$$

Q 2019-DEC) Find the Thevenin and Norton equivalent for the circuit shown in Figure ?? with respect terminals a-b.

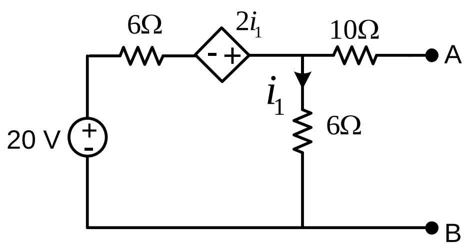

Figure 11

Solution:

Determine the Thevenin voltage V_{TH}. Apply KVL for the circuit shown in Figure 12.
By KVL around the loop

$$\begin{aligned} 6i - 2i + 6i - 20 &= 0 \\ 10i &= 20 \\ i &= 2A \end{aligned}$$

Voltage across AB $V_{OC} = V_{TH}$ is

$$V_{OC} = 6i = 6 \times 2 = 12V$$

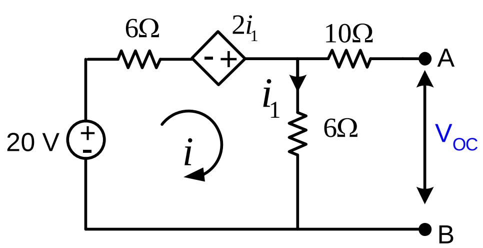

Figure 12

When dependant voltage sources are present then Thevenin Resistance R_{TH} is calculated

by determining the short circuit current at terminals AB:

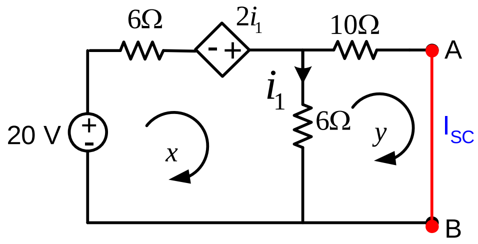

Figure 13

$$x - y = i_1$$

KVL for loop x

$$\begin{aligned} 12x - 2i_1 - 6y - 20 &= 0 \\ 12x - 2(x-y) - 6y &= 20 \\ 10x - 4y &= 20 \end{aligned}$$

KVL for loop y

$$\begin{aligned} -6x + 16y &= 0 \\ 6x - 16y &= 0 \end{aligned}$$

Solving the following simultaneous equations

$$\begin{aligned} 10x - 4y &= 20 \\ 6x - 16y &= 0 \end{aligned}$$

$$x = 2.353 \quad y = 0.882$$

$$I_{SC} = y = 0.882A$$

Thevenin's resistance is

$$R_{TH} = \frac{V_{TH}}{I_{SC}} = \frac{12}{0.882} = 13.6\Omega$$

Thevenin and Norton equivalent circuits as shown in Figure 14

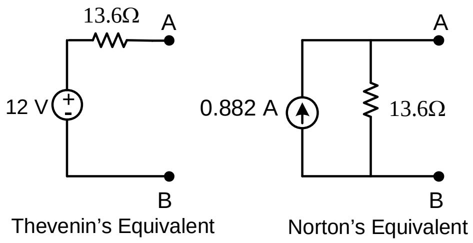

Figure 14

Q 2019-DEC) Determine the current through the load resistance using Norton's theorem for the circuit shown in Figure 15.

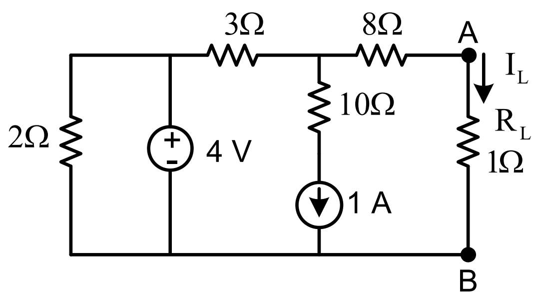

Figure 15

Solution:

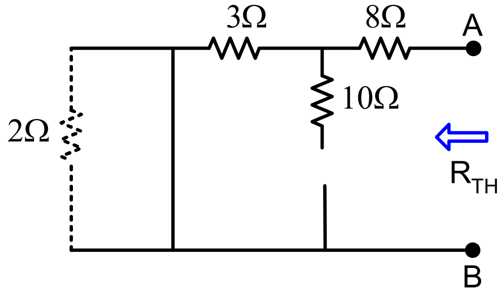

Figure 16

$$R_{TH} = 11\ \Omega$$

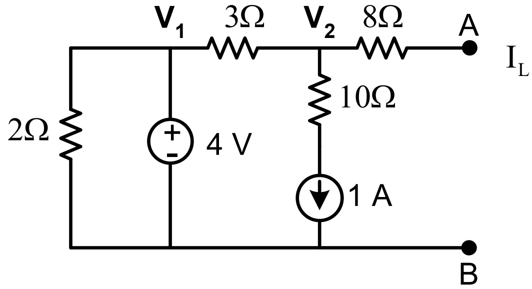

Figure 17

$$\begin{aligned} V_1 &= 4 \\ \frac{V_2 - V_1}{3} + 1 &= 0 \\ \frac{V_2 - 4}{3} + 1 &= 0 \\ V_2 - 4 + 3 &= 0 \\ V_2 &= 1 \end{aligned}$$

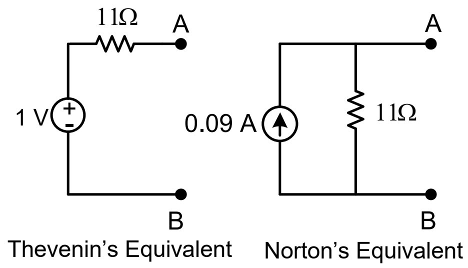

Figure 18

Q 2019-Dec) Obtain the Thevenin's equivalent network for the circuit shown in Figure 19.

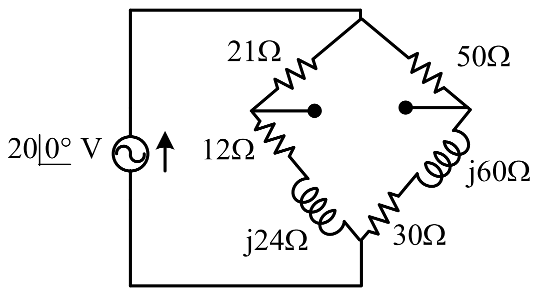

Figure 19

Solution:

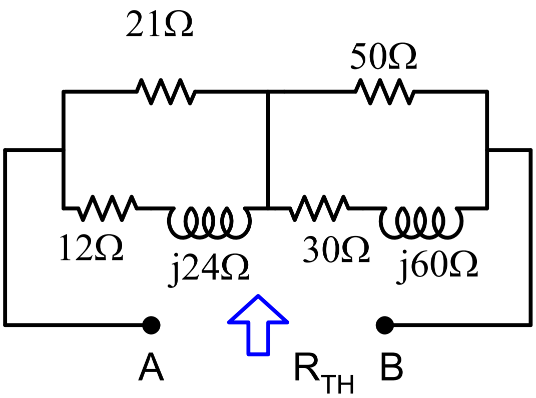

Figure 20

$$
\begin{aligned}
R_{TH} &= \frac{21(12+j24)}{21+12+j24} + \frac{50(30+j60)}{50+30+j60} \\
&= (12.26+j6.356) + (30+j15)) \\
&= 42.26 + j21.356\Omega
\end{aligned}
$$

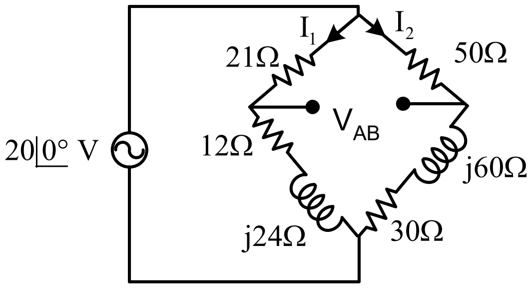

Figure 21

$$
\begin{aligned}
I_1 &= \frac{20}{33+j24} = 0.49\angle-36 \\
I_2 &= \frac{20}{80+j60} = 0.2\angle-36.87
\end{aligned}
$$

$$
\begin{aligned}
V_{AB} &= I_1 \times 21 - I_2 \times 50 \\
&= 0.49\angle-36 \times 21 - 0.2\angle-36.87 \times 50 \\
&= 0.49\angle-36 \times 21 - 0.2\angle-36.87 \times 50 \\
&= 0.328\angle-8.457
\end{aligned}
$$

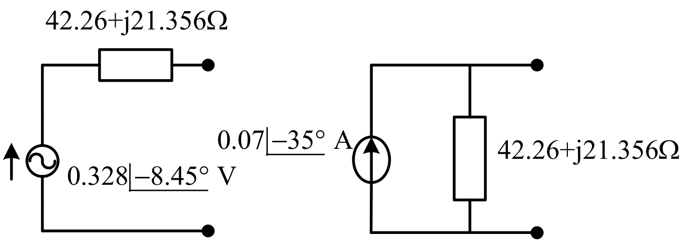

Figure 22

Q 2019-JUNE) Obtain the Thevenin's equivalent across A and B for the circuit shown in Figure 23.

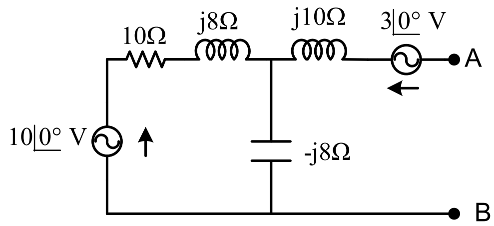

Figure 23

Solution:

$$
\begin{aligned}
R_{TH} &= j10 + \frac{(10+j8)(-j8)}{(10+j8-j8)} \\
&= j10 + (6.4 - j8) \\
&= 6 + j2\Omega
\end{aligned}
$$

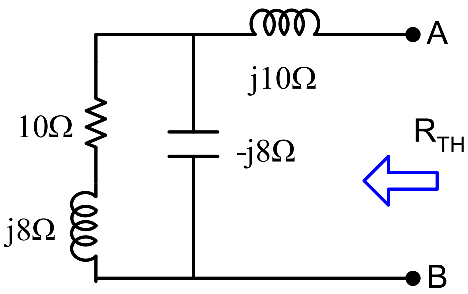

Figure 24

$$
\frac{V_1 - 10}{(10+j8)} + \frac{V_1}{(-j8)} + \frac{V_1 - 3}{(j10)} = 0
$$

$$
\begin{aligned}
V_1[0.078\angle-38.66 + 0.125\angle90 + 0.1\angle-90] \\
+0.78\angle141 + 0.3\angle90 &= 0
\end{aligned}
$$

$$
\begin{aligned}
0.0653\angle-21.28V_1 &= -0.9964\angle127.46 \\
V_1 &= \frac{-0.9964\angle127.46}{0.0653\angle-21.28} \\
V_1 &= 15.25\angle-31.26
\end{aligned}
$$

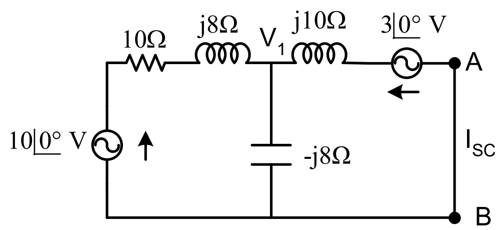

Figure 25

$$
\begin{aligned}
I_{SC} &= \frac{V_1 - 3}{(j10)} \\
&= \frac{(15.25\angle-31.26) - 3}{j10} \\
&= 1.278\angle-128.25
\end{aligned}
$$

$$
\begin{aligned}
V_{OC} &= I_{SC} Z_N \\
&= 1.278\angle -128.25(6+j2) \\
&= 8.08\angle -109.815
\end{aligned}
$$

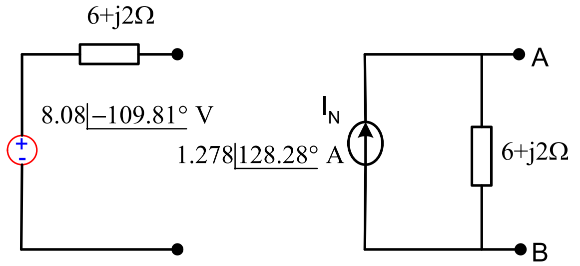

Figure 26

Q 2019-JUNE) Find the value of Z_L in the circuit shown in Figure 27 using maximum power transfer theorem and hence the maximum power.

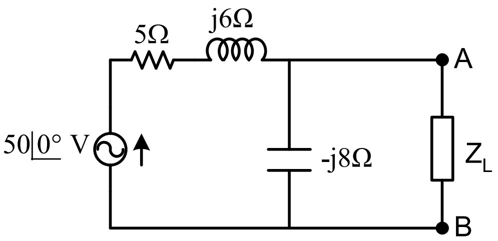

Figure 27

Solution:

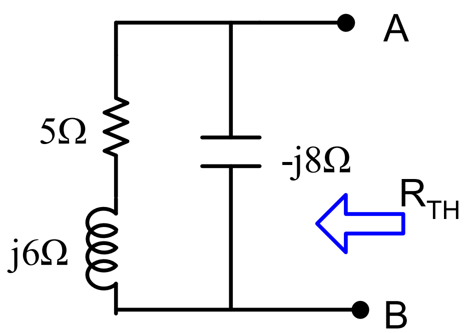

Figure 28

$$
\begin{aligned}
R_{TH} &= \frac{(5+j6)(-j8)}{(5+j6-j8)} \\
&= 11 - j3.586\Omega
\end{aligned}
$$

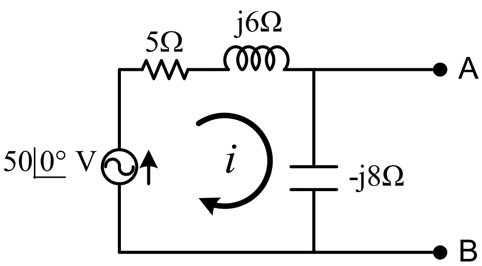

Figure 29

$$
\begin{aligned}
i &= \frac{50}{(5+j6-j8)} \\
&= 9.28\angle 21.8
\end{aligned}
$$

$$
\begin{aligned}
V_{OC} &= i(-j8) \\
&= 9.28\angle 21.8(-j8) \\
&= 74.24\angle -68.2
\end{aligned}
$$

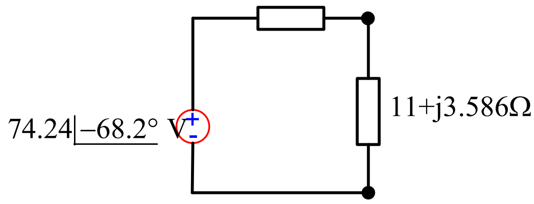

Figure 30

Maximum Power is transferred when

$$
\begin{aligned}
R_{TH} &= R_L \\
11 - j3.586\Omega &= 11 + j3.586\Omega
\end{aligned}
$$

Current through the load is

$$
\begin{aligned}
i_L &= \frac{74.24\angle -68.2}{(11-j3.586)+(11+j3.586)} \\
&= 3.374A
\end{aligned}
$$

Maximum Power transferred through the load is

$$
\begin{aligned}
P_L &= i_L^2 R_L = (3.374)^2(11+j3.586) \\
&= 131.7\angle 18
\end{aligned}
$$

Q 2019-JAN) Find the value of R for which the power transferred across AB of the circuit shown in Figure 31 is maximum and the maximum power power transferred.

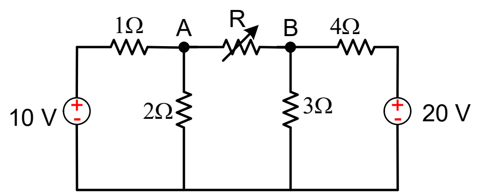

Figure 31

Solution:

First remove the R from the network and determine the V_{TH} and R_{TH} the details are as shown in Figure 32. The voltage across AB is the potential difference between AB.

$$
i_1 = \frac{10}{3}
$$

The potential at A is

$$
V_A = \frac{10}{3} \times 2\Omega = 6.667V
$$

$$i_2 = \frac{20}{7}$$

The potential at B is

$$V_B = \frac{20}{7} \times 3 = 8.571V$$

The potential at B is

$$V_{AB} = V_A - V_B = 6.667V - 8.571V = -1.9V$$

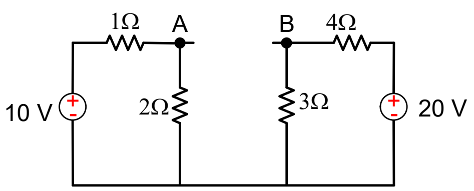

Figure 32

To determine R_{TH} the details are as shown in Figure 75. The 10Ω and 5Ω are in parallel which is in series with 2Ω.

$$R_{TH} = (1||2) + (3||4) = 0.667 + 1.714 = 2.381\Omega$$

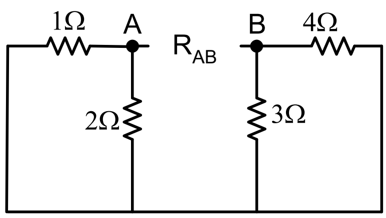

Figure 33

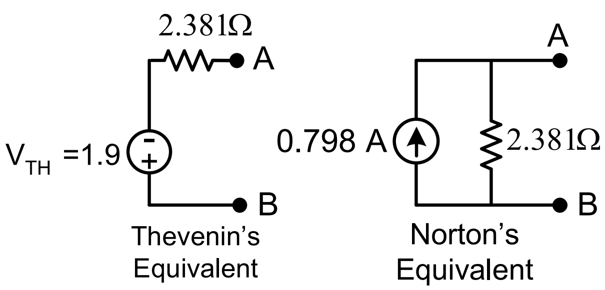

Figure 34

$$I_L = \frac{1.9V}{2.381 + 2.381} = 0.4A$$

$$P_L = (0.4)^2 \times 2.381 = 0.381W$$

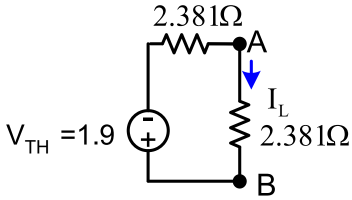

Figure 35

2018 Dec JUNE 2013-JUNE MARCH-2000) Find the current through 6 Ω resistor using Norton's theorem for the circuit shown in Figure 36.

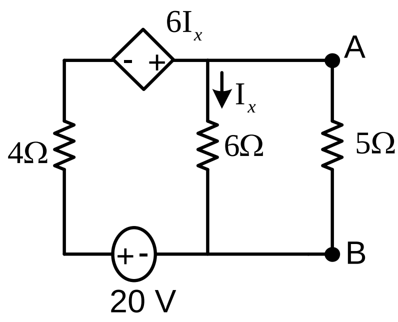

Figure 36

Solution:

Determine the V_{OC} at the terminal AB. When the resistor is removed from the terminals AB then the circuit is as shown in Figure 37. Apply KVL around the loop

$$\begin{aligned} 4I_x - 6I_x + 6I_x - 20 &= 0 \\ I_x &= \frac{20}{4} = 5A \\ V_{OC} &= 5A \times 6 = 30V \end{aligned}$$

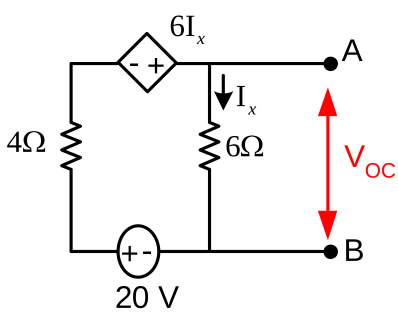

Figure 37

When the terminals AB short circuited then 6 Ω resistor is also shorted and no current flows through resistor hence $I_x = 0$ hence $6I_x = 0$. The circuit is as shown in Figure 38. The Norton current is

$$I_{SC} = I_N = \frac{20}{4} = 5\ A$$

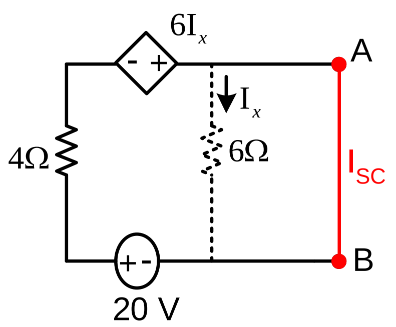

Figure 38

$$Z_N = \frac{V_{OC}}{I_{SC}} = \frac{30}{5} = 6\Omega$$

The Thevenin and Norton circuits are as shown in Figure 39

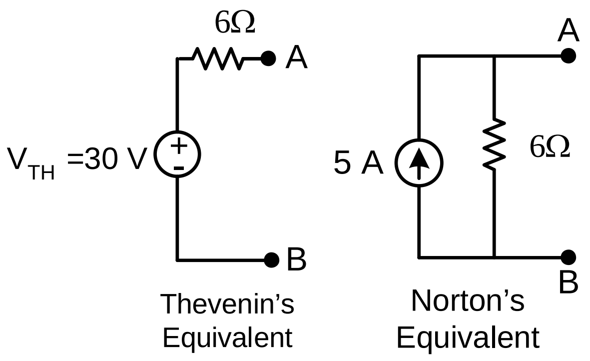

Figure 39

Current through 5 Ω resistor is

$$I_5 = 5\ A \frac{6}{6+5} \simeq 2.72\ A$$

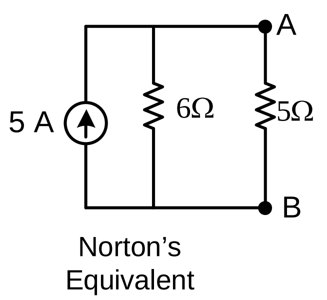

Figure 40

2018 Dec 2011-JULY) Find the value of Z_L for which maximum power is transfer occurs in the circuit shown in Figure 41.

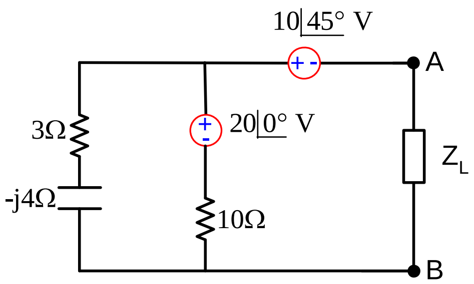

Figure 41

Solution:

Determine the V_{OC} at the terminal AB. When the resistor is removed from the terminals AB then the circuit is as shown in Figure 42. Apply KVL around the loop

$$\begin{aligned} I &= \frac{20\angle 0^0}{10+3-j4} = \frac{20\angle 0^0}{13.6\angle -17.1^0} \\ &= 1.47\angle 17.1^0 A \\ V_{OC} &= [1.47\angle 17.1^0 \times (3-j4)] - 10\angle 45^0 \\ &= [1.47\angle 17.1^0 \times 5\angle -53.13^0] - 10\angle 45^0 \\ &= [7.35\angle -36.3^0] - 10\angle 45^0 \\ &= [5.923 - j4.5] - 7.07 - j7.07 \\ &= -1.147 - j11.42 \\ &= 11.47V\angle 95.73^0 \end{aligned}$$

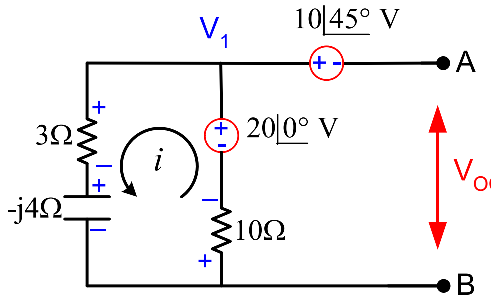

Figure 42

$$Z_{TH} = \frac{(3-j4)\times 10}{3-j4+10} = 2.973 - j2.162\Omega$$

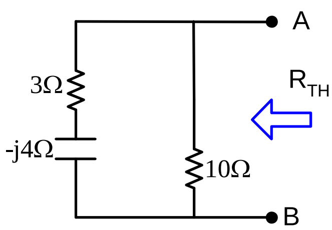

Figure 43

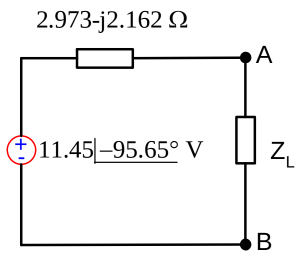

Figure 44

$$Z_{TH} = \frac{(3-j4)\times 10}{3-j4+10} = 2.973 - j2.162\Omega$$

The maximum power is delivered when the load impedance is complex conjugatae of the network impedance. Thus

$$Z_L = Z_{TH}^* = 2.973 + j2.162\Omega$$

The current flowing in the load impedance is

$$\begin{aligned} I_L &= \frac{11.47V\angle 95.73^0}{Z_{TH}+Z_L} \\ &= \frac{11.47V\angle 95.73^0}{2.973-j2.162+2.973+j2.162} \\ &= \frac{11.47V\angle 95.73^0}{2.973+2.973} \\ &= \frac{11.47V\angle 95.73^0}{5.946} = 1.929\angle 95.73^0 \end{aligned}$$

The power delivered in the load impedance is

$$P_L = I_L^2 \times R_L = 1.922^2 \times 2.973 = 11.46W$$

2018 Jan) Find the Thevenin equivalent for the circuit shown in Figure 45 with respect terminals a-b

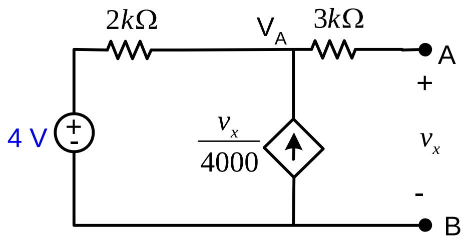

Figure 45

Solution:
Determine the Thevenin voltage V_{TH} for circuit shown in Figure 46. Apply KCL for the node V_1

$$V_x = V_A$$

$$\begin{aligned} \frac{V_A-4}{2k\Omega} - \frac{V_x}{4k\Omega} &= 0 \\ 0.5\times 10^{-3}V_A - 0.25\times 10^{-3}V_A &= 2mA \\ V_A &= 8V \\ V_{OC} &= 8V \end{aligned}$$

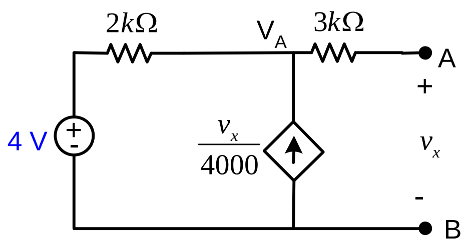

Figure 46

Determine the short circuit by shorting the output terminals AB for circuit shown in Figure 47. Apply KCL for the node V_1. It is observed that $V_x = 0\ V$, hence dependent current source becomes zero.

$$\begin{aligned} \frac{V_A-4}{2k\Omega} - \frac{V_x}{4k\Omega} + \frac{V_A}{3k\Omega} &= 0 \\ 0.5\times 10^{-3}V_A + 0.333\times 10^{-3}V_A &= 2mA \\ 0.833V_A &= 2 \\ V_A &= 2.4V \end{aligned}$$

$$I_{SC} = \frac{2.4V}{3k\Omega} = 0.8mA$$

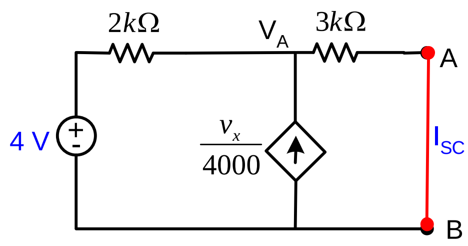

Figure 47

$$Z_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{8}{0.8mA} = 10k\Omega$$

Thevenin and Norton circuits are as shown in Figure 48

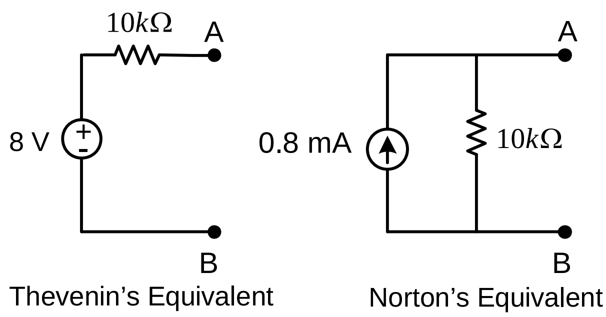

Figure 48

Q 2017-Jan) What value of impedance Z_L results in maximum power transfer condition for the network shown in Figure 49. Also determine the corresponding power.

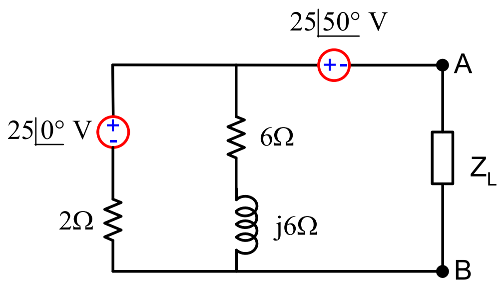

Figure 49

Solution:

$$
\begin{aligned}
Z_{TH} &= \frac{V_{OC}}{I_{SC}} = 2||(6+j6)\Omega \\
&= \frac{2(6+j6)}{(2+6+j6)} = 1.68 + j0.24
\end{aligned}
$$

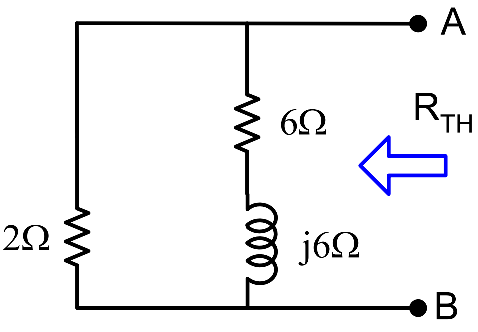

Figure 50

$$
\begin{aligned}
i &= \frac{25}{8+j6} = 2.5\angle -36.87 \\
V_1 &= i \times (6+j6) = 2.5\angle -36.87 \times (6+j6) \\
&= 21.21\angle 8.31V
\end{aligned}
$$

$$
\begin{aligned}
V_{OC} = V_{AB} &= V_1 - 25\angle 50 \\
&= 21.21\angle 8.31 - 25\angle 50 \\
&= 16.82\angle -73
\end{aligned}
$$

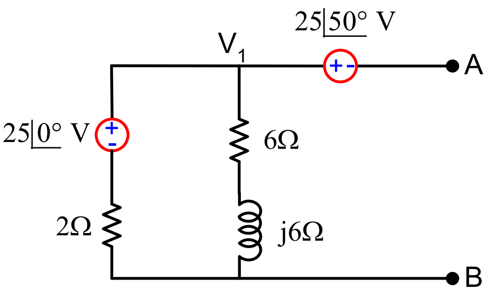

Figure 51

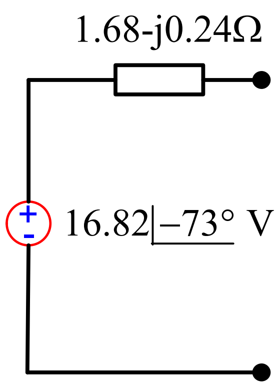

Figure 52

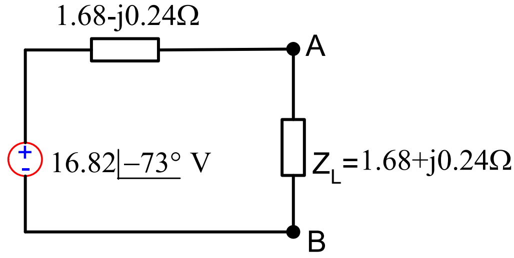

Figure 53

2017 Jan, 2014-JAN) Find the Thevenin's equivalent of the network as shown in Figure 54

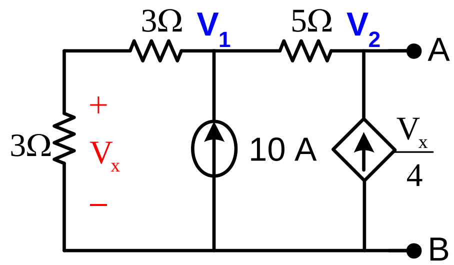

Figure 54

Solution:

Using node analysis the following equations are written

$$
\begin{aligned}
\frac{V_1}{6} + \frac{V_1 - V_2}{5} - 10 &= 0 \\
V_1[0.166 + 0.2] - 0.2V_2 &= 10 \\
0.366V_1 - 0.2V_2 &= 10
\end{aligned}
$$

$$
\begin{aligned}
\frac{V_2 - V_1}{5} - \frac{V_x}{4} &= 0 \\
-0.2V_1 + 0.2V_2 - 0.25V_x &= = 0
\end{aligned}
$$

$$
V_x = \frac{V_1}{6} \times 3 = 0.5V_1
$$

$$
\begin{aligned}
-0.2V_1 + 0.2V_2 - 0.25V_x &= 0 \\
-0.2V_1 + 0.2V_2 - 0.25 \times 0.5V_1 &= \\
-0.325V_1 + 0.2V_2 &= 0
\end{aligned}
$$

$$
\begin{aligned}
0.366V_1 - 0.2V_2 &= 10 \\
-0.325V_1 + 0.2V_2 &= 0
\end{aligned}
$$

$$
V_1 = 243.93\ V \quad V_2 = 396.3\ V
$$

$$
V_{TH} = V_2 = 396.3\ V
$$

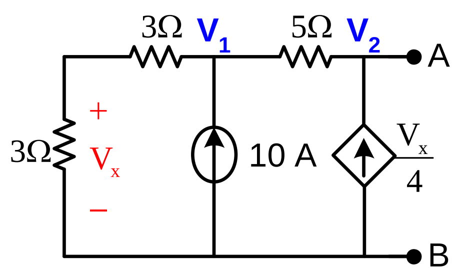

Figure 55

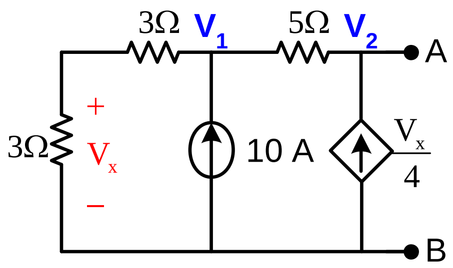

Figure 56

$$V_x = \frac{10 \times 5}{11} \times 3 = 13.636$$

$$\begin{aligned} I_{SC} &= \frac{V_x}{4} + 10 \times \frac{6}{11} \\ &= \frac{13.636}{4} + 5.45 = 3.41 + 5.45 \\ &= 8.86A \end{aligned}$$

$$R_{TH} = \frac{V_{TH}}{I_{SC}} = \frac{396.3}{13.636} = 44.01\Omega$$

Q 2016-JUNE) Obtain the Thevenin's equivalent of the circuit shown in Figure 57 and thereby find current through 5Ω resistor connected between terminals A and B.

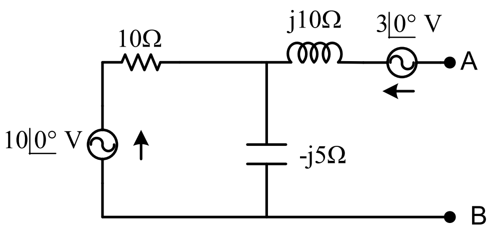

Figure 57

Solution:

$$\begin{aligned} R_{TH} &= j10 + \frac{(10)(-j5)}{(10-j5)} \\ &= j10 + (2-j4) \\ &= 2 + j6\Omega \end{aligned}$$

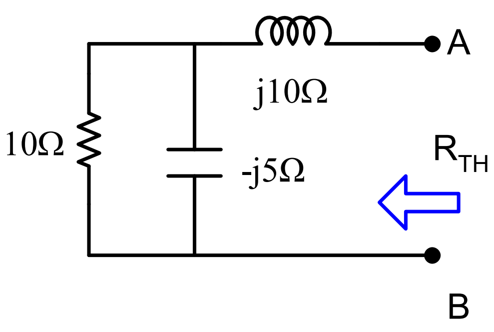

Figure 58

$$\frac{V_1 - 10}{(10)} + \frac{V_1}{(-j5)} + \frac{V_1 - 3}{(j10)} = 0$$

$$\begin{aligned} V_1[0.1 + 0.2\angle 90 + 0.1\angle -90] & \\ -1 + 0.3\angle 90 &= 0 \end{aligned}$$

$$\begin{aligned} 0.141\angle -45 V_1 &= 1.04\angle 163.3 \\ V_1 &= \frac{1.04\angle 163.3}{0.141\angle -45} \\ V_1 &= 7.37\angle -151.7 \end{aligned}$$

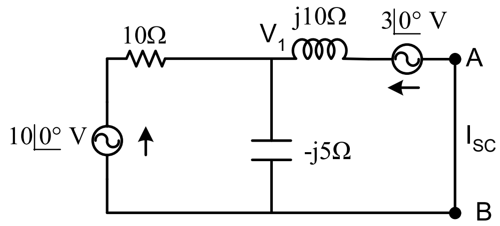

Figure 59

$$\begin{aligned} I_{SC} &= \frac{V_1 - 3}{(j10)} \\ &= \frac{(7.37\angle -151.7) - 3}{j10} \\ &= 1.01\angle 110 \end{aligned}$$

$$\begin{aligned} V_{OC} &= I_{SC} Z_N \\ &= 1.01\angle 110(2 + j6) \\ &= 6.38\angle -17 \end{aligned}$$

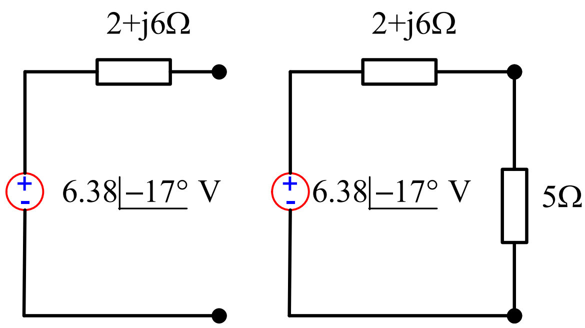

Figure 60

$$\begin{aligned} I &= j\frac{6.38\angle -17}{(7+j6)} \\ &= 0.69\angle -57.6 \end{aligned}$$

Q 2015-Jan) For the network shown in Figure 61 draw the Thevenin's equivalent circuit.

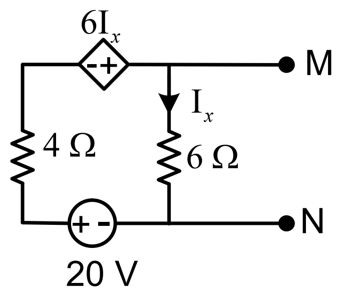

Figure 61

Solution:

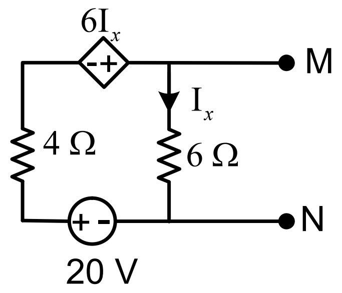

Figure 62

$$
\begin{aligned}
-6I_x + 6I_x - 20 + 4I_x &= 0 \\
I_x &= \frac{20}{4} \\
&= 5A
\end{aligned}
$$

$$
\begin{aligned}
V_{OC} &= I_x \times 6 = 5 \times 6 \\
&= 30V
\end{aligned}
$$

$$I_{SC} = I_N = \frac{20}{4} = 5\ A$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{30}{5} = 6\ \Omega$$

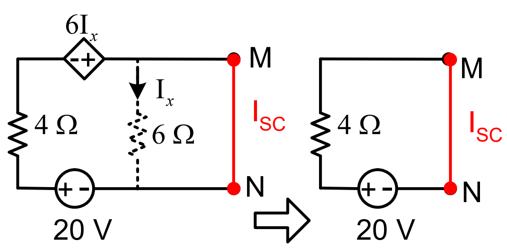

Figure 63

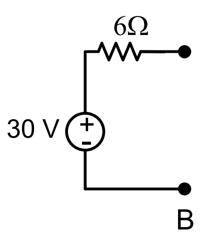

Figure 64: Thevinin Circuit

Q 2014-JUNE) Find the value of load resistance when maximum power is transferred across it and also find the value of maximum power transferred for the network of the circuit shown in Figure 65.

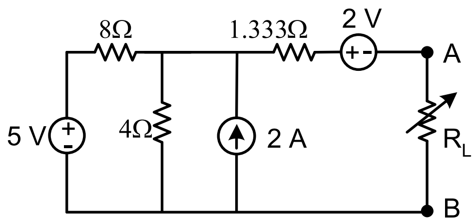

Figure 65

Solution:

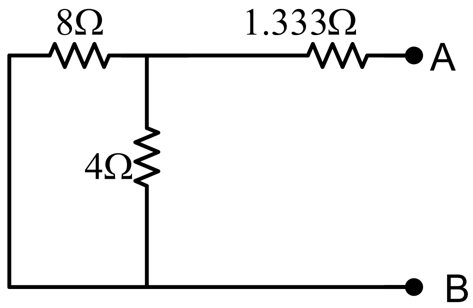

Figure 66

$$Z_{TH} = 1.333 + \frac{8 \times 4}{8+4} = 1.333 + 2.6667 = 4\Omega$$

$$
\begin{aligned}
\frac{V_1 - 5}{8} + \frac{V_1}{4} + \frac{V_1 - 2}{1.333} - 2 &= 0 \\
V_1[0.125 + 0.25 + 0.75] - 0.625 - 1.5 - 2 &= 0 \\
V_1[0.125 + 0.25 + 0.75] - 0.625 - 1.5 - 2 &= \frac{4.125}{1.125} \\
V_1 &= 3.666
\end{aligned}
$$

$$I_{SC} = \frac{V_1 - 2}{1.333} = \frac{3.6366 - 2}{1.333} = 1.25$$

$$V_{OC} = I_{SC} Z_{TH} = 1.25 \times 4 = 5$$

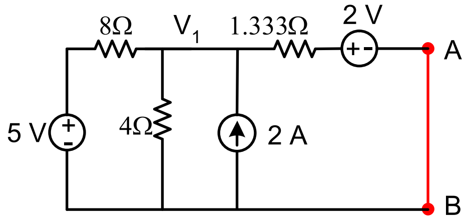

Figure 67

$$P_{max} = \frac{V_{OC}^2}{R_L} = \frac{5^2}{4} = 6.25W$$

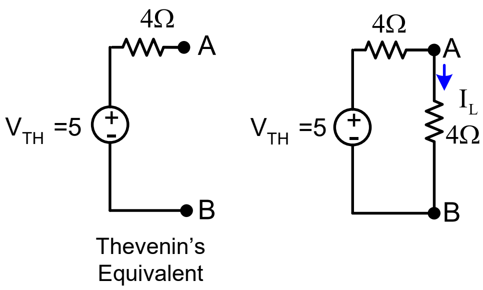

Figure 68

Q 2014-JUNE) Find the current through 16 Ω resistor using Nortons theorem for the circuit shown in Figure 69.

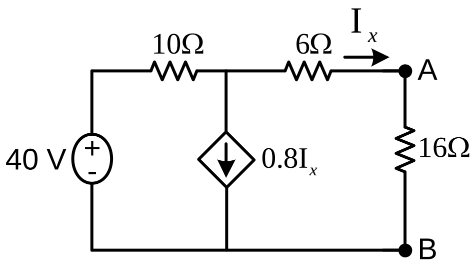

Figure 69

Solution:

Determine the V_{OC} at the terminal AB. When the resistor is removed from the terminals AB then $I_x = 0$

$$V_{OC} = 40\ V$$

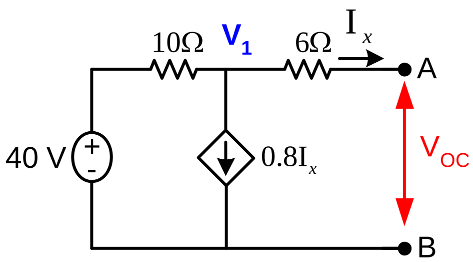

Figure 70

Determine the V_{OC} at the terminal AB by shorting output terminals AB. Apply node analysis for the circuit shown in Figure 71.

$$I_x = \frac{V_1}{6}$$

$$\begin{aligned} \frac{V_1 - 40}{10} + \frac{V_1}{6} + 0.8 I_x &= 0 \\ \frac{V_1}{10} + \frac{V_1}{6} + 0.8\frac{V_1}{6} &= 0 \\ 0.4 V_1 &= 4 \\ V_1 &= 10 \end{aligned}$$

$$I_{SC} = I_N = \frac{10}{6} = 1.666\ A$$

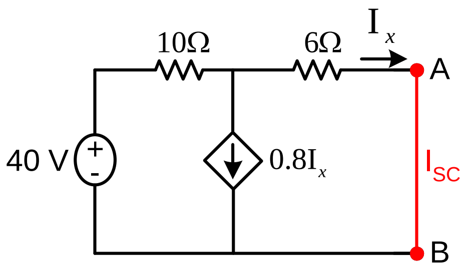

Figure 71

$$Z_N = \frac{V_{OC}}{I_{SC}} = \frac{40}{1.666} = 24\Omega$$

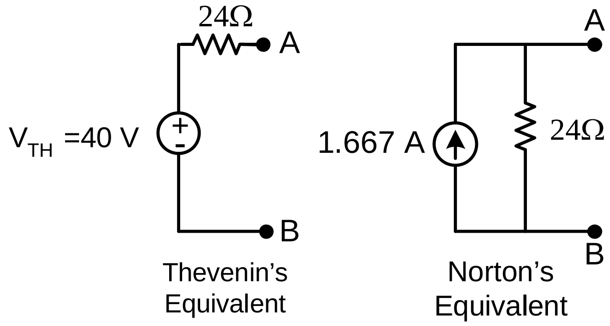

Figure 72

Current through 16 Ω resistor is

$$I_{16} = 1.666\ A \frac{24}{24 + 16} \simeq 1\ A$$

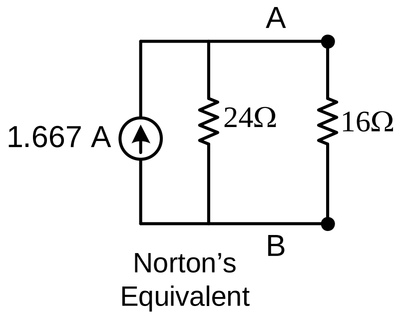

Figure 73

Q 2014-JAN) State maximum power transfer theorem. For the circuit shown in Figure 74 what should be the value of R such that maximum power transfer can take place from the rest of the network. Obtain the amount of this power

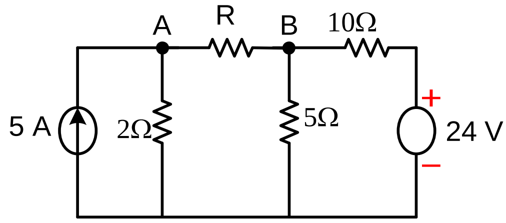

Figure 74

Solution:

First remove the R from the network and determine the V_{TH} and R_{TH} the details are as shown in Figure 75. The voltage across AB is the potential difference between AB.

The potential at A is

$$V_A = 5A \times 2\Omega = 10V$$

The potential at B is

$$V_B = \frac{24}{15} \times 5 = 8V$$

The potential at B is

$$V_{AB} = V_A - V_B = 10V - 8V = 2V$$

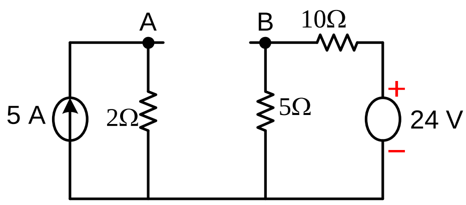

Figure 75

To determine R_{TH} the details are as shown in Figure 75. The 10Ω and 5Ω are in parallel which is in series with 2Ω.

$$R_{TH} = 2 + (10||5) = 2 + 3.333 = 3.333\Omega$$

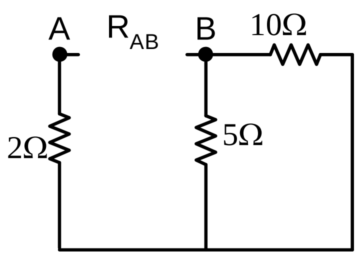

Figure 76

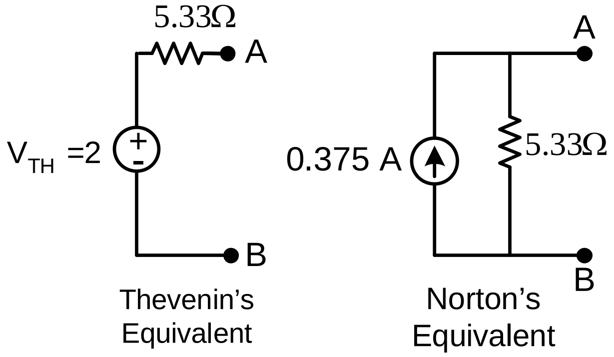

Figure 77

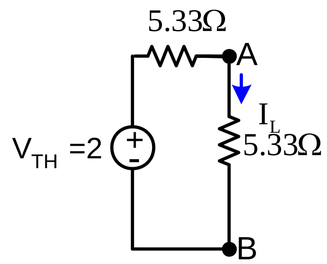

Figure 78

Q 2012-JUNE) State Thevenin's theorem. For the circuit shown in Figure 79 find the current through R_L using Thevenin's theorem.

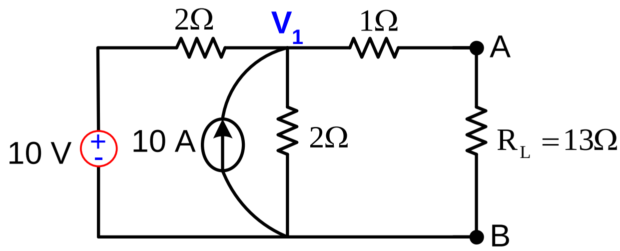

Figure 79

Solution:

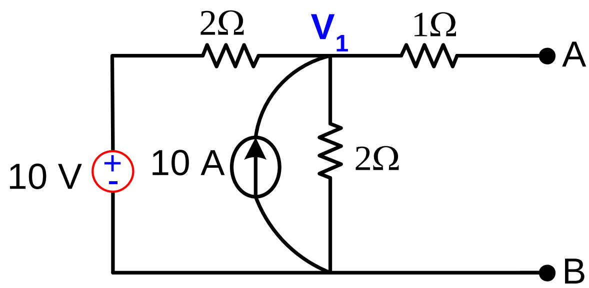

Figure 80

By node analysis V_{TH} is

$$\begin{aligned} \frac{V_1 - 10}{2} + \frac{V_1}{2} - 10 &= 0 \\ V_1 &= 10 + 5 = 15V = E_{TH} \end{aligned}$$

R_{TH} is

$$R_{TH} = 1 + \frac{2 \times 2}{2 + 2} = 2\Omega$$

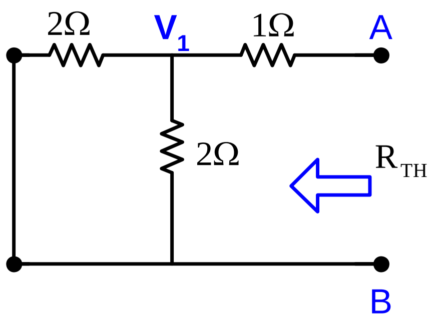

Figure 81

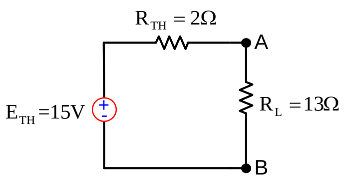

Figure 82

The current through I_L is

$$I_L = \frac{E_{TH}}{R_{TH} + R_L} = \frac{15}{2+13} \quad = \quad 1A$$

Q 2001-Aug, 2011-JAN) Obtain Thevenin and Norton equivalent circuit at terminals AB for the network shown in Figure 83 Find the current through 10 Ω resistor across AB.

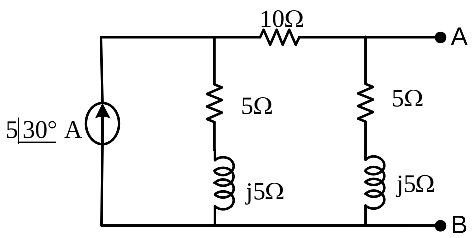

Figure 83

Solution:

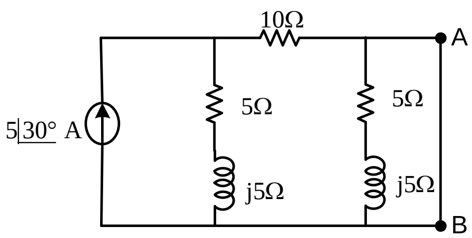

Figure 84

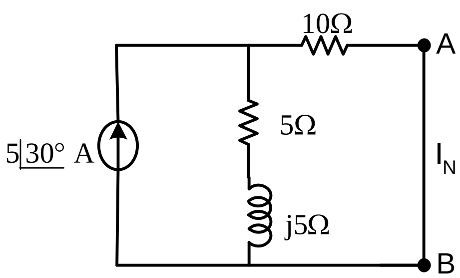

Figure 85

The Norton's current I_N is

$$I_N = 5\angle 30 \times \frac{5+j5}{10+5+j5} = 5\angle 30 \times \frac{7.07\angle 45}{15.81\angle 18.43}$$
$$= 2.236\angle 56.57^\circ A$$

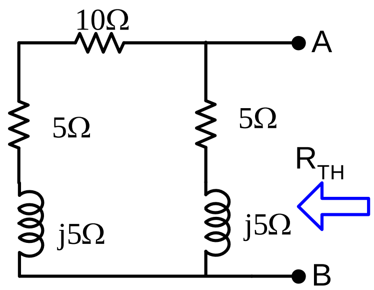

Figure 86

$$Z_N = \frac{(5+j5)\times(15+j5)}{5+j5+15+j5}$$
$$= \frac{7.07\angle 45 \times 15.81\angle 18.43}{22.36\angle 26.56}$$
$$= 5\angle 36.87^\circ \Omega$$

The Norton Equivalent circuit is as shown in Figure 87

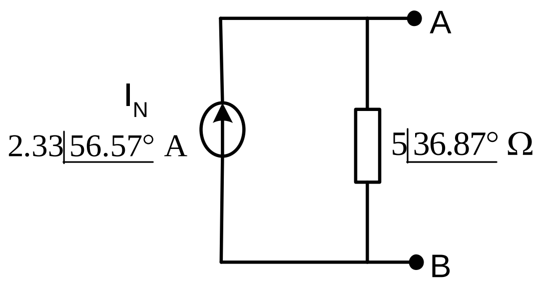

Figure 87

The Thevenin's Equivalent

$$V_{TH} = I_N \times Z_N = 2.236\angle 56.57^\circ A \times 5\angle 36.87^\circ \Omega$$
$$= 11.18\angle 93.44^\circ \Omega$$

$$V_{TH} = I_N \times Z_N = 2.236\angle 56.57^\circ Atimes 5\angle 36.87^\circ \Omega$$

The Thevenin's Equivalent circuit is as shown in Figure 88

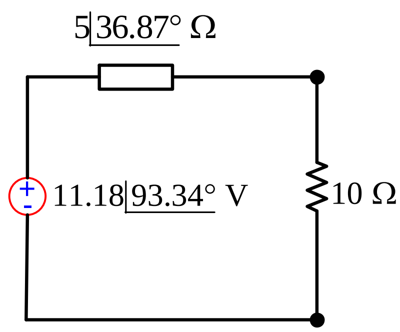

Figure 88

Current through load 10 is Ω

$$I_L = \frac{11.18\angle 93.43}{5\angle 36.87+10} = \frac{11.18\angle 93.43}{4+j3+10}$$
$$= \frac{11.18\angle 93.43^\circ}{14.31\angle 12.1}$$
$$= 0.781\angle 81.34^\circ$$

Q 2000-July) Obtain Thevenin and Norton equivalent circuit at terminals AB for the network shown in Figure 89. Find the current through 10 Ω resistor across AB

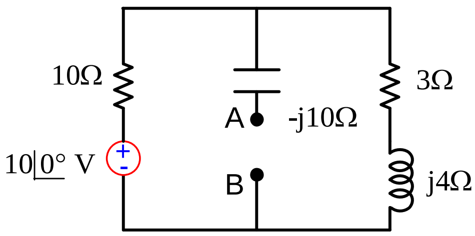

Figure 89

Solution:

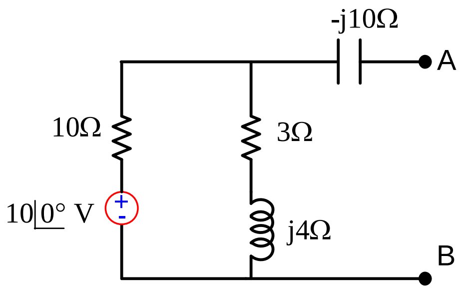

Figure 90

$$I(13+j4) - 10 = 0$$
$$I(13.6\angle 17.1) = 10$$
$$I = \frac{10}{13.6\angle 17.1}$$
$$I = 0.7352\angle -17.1$$

$$V_{TH} = I\times(3+j4) = (0.7352\angle -17.1)(5\angle 53.13)$$
$$= 3.676\angle 36.03$$

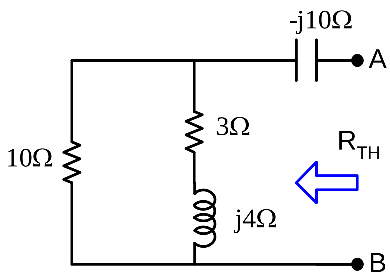

Figure 91

$$Z_{TH} = -j10 + \frac{10\times(3+j4)}{10+3+j4}$$
$$= -j10 + \frac{30+j40}{13+j4} = 10 + \frac{50\angle 53.13}{13.6\angle 17.027}$$
$$= -j10 + 3.6762\angle 36.027^\circ\Omega$$
$$= -j10 + 2.9731 + j2.1622\Omega$$
$$= 2.9731 - j7.8378\Omega$$
$$= 8.3828\angle -69.227\Omega$$

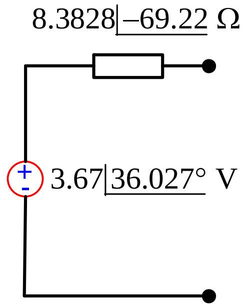

Figure 92

Nortons equivalent circuit is I_{SC} and Z_{TH} which are as shown in Figure 93

$$I_{SC} = \frac{V_{TH}}{Z_{TH}} = \frac{3.676\angle 36.03}{8.3828\angle -69.227}$$
$$= 0.4385\angle 105.257$$

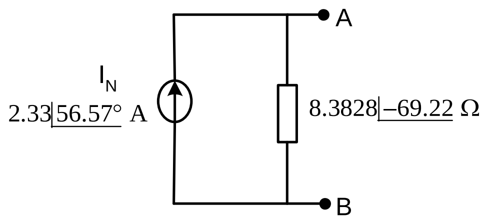

Figure 93

Q 2001-March) For the circuit shown in Figure 94 determine the load current I_L using Norton's theorem .

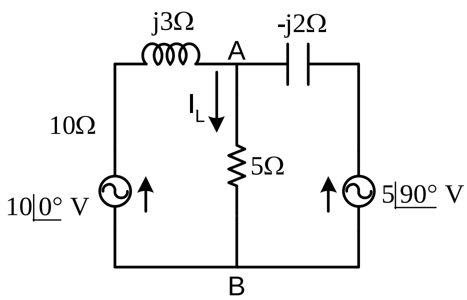

Figure 94

Solution:

Determine the open circuit voltage V_{TH} for the circuit is as shown in Figure 101. Apply KVL around the loop

$$\begin{aligned}
x(j3-j2)+5\angle 90^o-10\angle 0^o &= \\
jx+j5-10 &= \\
jx &= 10-j5 \\
x &= -j(10-j5) \\
x &= -5-j10 \\
x &= 11.18\angle-116.56^o
\end{aligned}$$

The voltage V_{TH} is the voltage between AB

$$\begin{aligned}
V_{TH} &= 10\angle 0^o - j3x \\
&= 10-j3(-5-j10) \\
&= -20+j15 \\
&= 25\angle 143.13^o
\end{aligned}$$

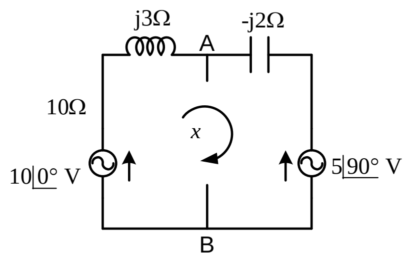

Figure 95

The Thevenin impedance Z_{TH} for the circuit is as shown in Figure **??** is

$$\begin{aligned}
Z_{TH} &= \frac{j3\times(-j2)}{j1} \\
&= \frac{6}{j1} \\
&= 6\angle-90^o
\end{aligned}$$

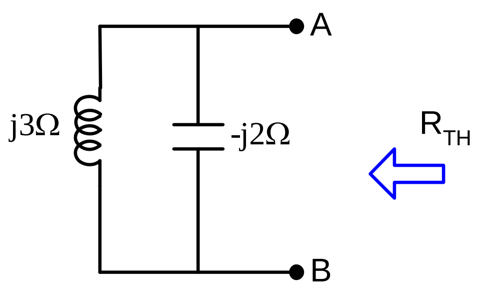

Figure 96

The Thevenin circuit is as shown in Figure 97. The current through the load is

$$\begin{aligned}
I_L &= \frac{25\angle 143.13^o}{5-j6} = \frac{25\angle 143.13^o}{7.8\angle-50.2^o} \\
&= 3.2\angle 193.3^o \\
P_L &= I_L^2 R_L \\
P_L &= (3.2)^2\times 5 \\
&= 51.2W
\end{aligned}$$

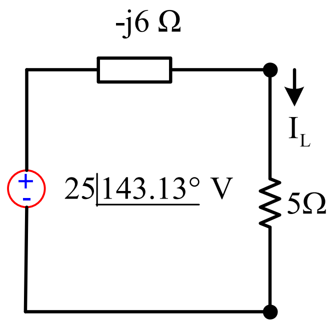

Figure 97

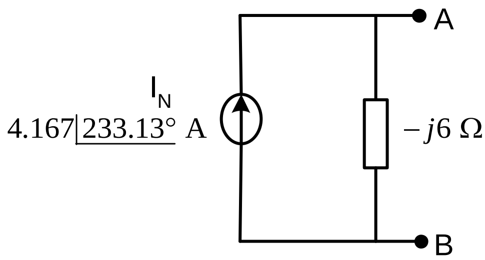

Figure 98

Q 2000-March) What should be the value of pure resistance to be connected across the terminals A and B in the network shown in Figure 99 so that maximum power is transferred to the load? What is the maximum power?

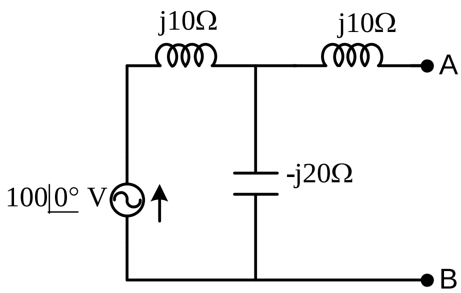

Figure 99

Solution:

The open circuit voltage V_{TH} for the circuit is as shown in Figure 101 is

$$\begin{aligned}
x(j10-j20)-100\angle 0^o &= \\
x &= \frac{100\angle 0^o}{-j10} \\
x &= j10 \\
V_{TH} &= j10\times -j20 \\
&= 200V
\end{aligned}$$

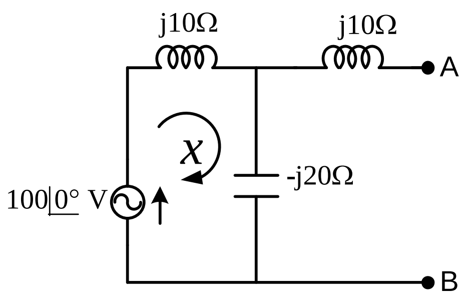

Figure 100

The Thevenin impedance Z_{TH} for the circuit is as shown in Figure ?? is

$$\begin{aligned} Z_{TH} &= j10 + \frac{j10 \times (-j20)}{j10 - j10} \\ &= j10 + \frac{200}{-j10} \\ &= j10 + \frac{100\angle 0^o}{-j10} \\ &= j10 + j20 \\ &= j30 \end{aligned}$$

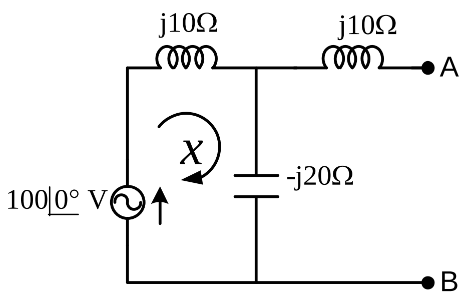

Figure 101

The Thevenin circuit is as shown in Figure 102. The current through the load is

$$\begin{aligned} I_L &= \frac{200}{30 + j30} = \frac{200}{42.43\angle 45^o} \\ &= 4.714\angle - 45^o \\ P_L &= I_L^2 R_L \\ &= (4.714)^2 \times 30 \\ &= 666.6W \end{aligned}$$

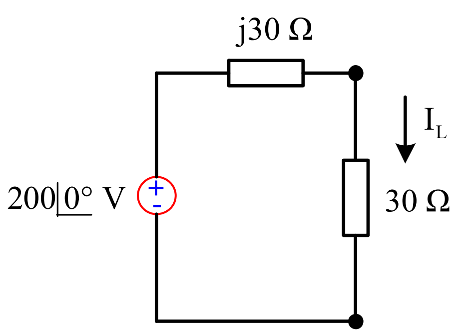

Figure 102

Q 2000-FEB) What should be the value of pure resistance to be connected across the terminals A and B in the network shown in Figure 103 so that maximum power is transferred to the load? What is the maximum power?

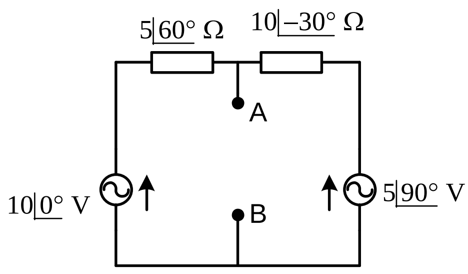

Figure 103

Solution:

The open circuit voltage V_{TH} for the circuit is as shown in Figure 101 is

$$\begin{aligned} x(5\angle 60^o + 10\angle - 30^o) + 5\angle 90^o - 10\angle 0^o &= 0 \\ x(2.5 + j4.33 + 8.66 - j5) + j5 - 10 &= 0 \end{aligned}$$

$$\begin{aligned} x(11.16 - j0.67) &= 10 - j5 \\ &= \frac{10 - j5}{11.16 - j0.67} \\ &= \frac{11.18\angle - 26.56}{11.18\angle - 34.36} \\ &= 1\angle - 23.13 \end{aligned}$$

$$\begin{aligned} V_{AB} &= 10\angle 0^o - x(5\angle 60^o) \\ &= 10 - (1\angle - 23.13)(5\angle 60^o) \\ &= 10 - (5\angle 36.87) \\ &= 10 - (4 + j3) \\ &= 6 - j3 \\ V_{TH} &= 6.7\angle - 26.56) \end{aligned}$$

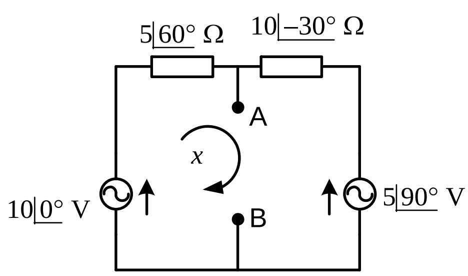

Figure 104

The Thevenin impedance Z_{TH} for the circuit is as shown in Figure 105 is

$$
\begin{aligned}
Z_{TH} &= \frac{5\angle 60^o \times 10\angle -30^o}{5\angle 60^o + 10\angle -30^o} \\
Z_{TH} &= j10 + \frac{j10 \times (-j20)}{j10 - j10} \\
&= \frac{50\angle 30^o}{(2.5 + j4.33) + (8.66 - j53)} \\
&= \frac{50\angle 30^o}{11.18\angle -34.36^o} \\
&= 4.47\angle 26.56^o \\
&= 4 + j2
\end{aligned}
$$

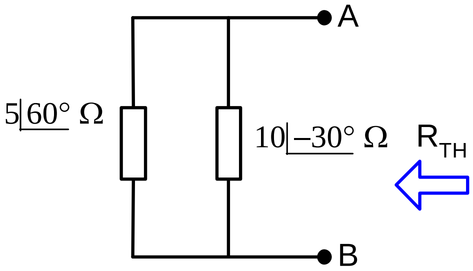

Figure 105

The load impedance is 4-j2 Ω
The Thevenin circuit is as shown in Figure 106. The current through the load is

$$
\begin{aligned}
I_L &= \frac{6.708\angle -26.56}{4 + j2 + 4 - j2} = \frac{6.708\angle -26.56}{8} \\
&= 0.8385\angle -26.65^o \\
P_L &= I_L^2 R_L \\
&= (0.8385)^2 \times 4 \\
&= 2.8123W
\end{aligned}
$$

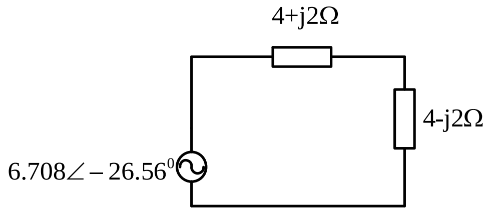

Figure 106

Important: All the diagrams are redrawn and solutions are prepared. *While preparing this study material most of the concepts are taken from some text books or it may be Internet. This material is just for class room teaching to make better understanding of the concepts on Network analysis: Not for any commercial purpose*