Wireless Channel[1, 2]

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Wireless Channel: [1, 2]

- Slides are prepared based on the book **"Fundamentals of Wireless Communication"** by **David Tse** and **Pramod Viswanath**
- Slides are prepared to use in class room purpose, may be used as a reference material
- All the slides are prepared based on the reference material
- Most of the figures/content used in this material are redrawn, some of the figures/pictures are downloaded from the Internet.
- This material is not for commercial purpose.
- This material is prepared for the subject Wireless Communication for M Tech in DECS course as per Visvesvaraya Technological University (VTU) syllabus (Karnataka State, India).

Wireless Channel

- Physical modeling for wireless channels
  1. Free space, fixed transmit and receive antennas
  2. Free space, moving antenna
  3. Reflecting wall, fixed antenna
  4. Reflecting wall, moving antenna
  5. Reflection from a ground plane
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  2. Rayleigh and Rician fading
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Introduction

- The transmission path between transmitter and receiver is varying due to obstruction by buildings, mountains and foliage (trees).
- Wired channels are stationary and predictable, but radio channels are extremely random and difficult to analzye.
- Modeling the radio channels has been one of the most difficult task of mobile radio system design, and is typically done in a statical method based on measurements [2].
- The electromagnetic wave propagation is attributed to reflection, diffraction and scattering.
- The high rise buildings cause severe diffraction loss.
- The received signal strength decreases as the distance between the transmitter and receiver increase.
- Due to multiple reflectors from objects, the electromagnetic wave travel along different paths of varying lengths and causes multipath fading.
- The propagation models that predict the mean signal strength as a function of distance are called large-scale fading.
- The propagation models that characterize the rapid fluctuations of the received signal strength as a function of short distance or short time durations are called sall-scale fading.

- Diffraction of signals: Occurs when the path between transmitter and receiver is obstructed by a sharp edges of an objects.
- Reflection of signals: Occurs when EM wave impinging upon a object which has very large in dimension when compared to the wavelength.
- Scattering of signals: Occurs when the medium through which the wave travels consists of objects which are small in dimension in compared to wavelength and very large number of obstacles per unit volume is large.

Large-scale (slow) fading:

- Large-scale fading is due to path loss of signal as a function of distance and shadowing and is frequency independent.
- Shadowing is caused by the obstacles which are buildings, big trees, and hills in the rural side.

Small-scale (Fast) fading:

- Small-scale fading is occurs due due to the constructive and destructive interference of the multiple signal paths between the transmitter and receiver and is frequency dependent.
- This is depending on the relative phase difference between the received signals.

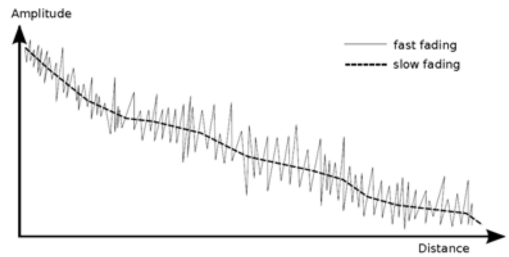


Figure 1: Slow and fast fading

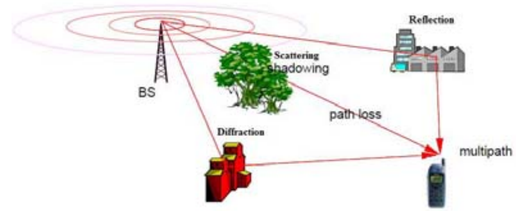


Figure 2: Multipath propagation

Free space, fixed transmit and receive antennas

- Consider a fixed antenna radiating its electromagnetic field into the free space.
- The electric and magnetic field at any given location are perpendicular to each other and are to the direction of propagating antenna.
- Consider an electromagnetic sinusoid wave with frequency f i.e, $cos2\pi ft$ is transmitted.
- The electric far field received at a point **u** with a distance r is

$$E(f,t,(r,\theta,\psi)) = \frac{\alpha_s(\theta,\psi,f)cos2\pi f(t-r/c)}{r}$$

- where, (r,θ,ψ) represents the point **u** at which the electric field measured.
- (θ,ψ) represents the vertical and horizontal angles from the antenna to u respectively.
- Constant c is the speed of light
- $\alpha_s(\theta,\psi,f)$ is the radiation pattern of the sending antenna at frequency f in the direction (θ,ψ); it also contains a scaling factor to account for antenna losses.
- The phase of the field varies with fr/c, corresponding to the delay caused by the radiation traveling at the speed of light.

- The electric field is proportional to the distance r and given as $E \propto \frac{1}{r}$ and the power per square meter in the free space wave decreases as r^{-2}.
- If we considered electric field as concentric spheres of increasing radius r around the antenna, the total power radiated through the sphere remains constant, but the surface area increases as r^2.
- For a fixed receive antenna at the location $u = (r, \theta, \psi)$, the received electric field is

$$E_r(f, t, u) = \frac{\alpha(\theta, \psi, f) cos 2\pi f(t - r/c)}{r}$$

- The magnetic field at a distance **u** is

$$H(f) = \frac{\alpha(\theta, \psi, f) e^{-j2\pi fr/c}}{r}$$

- Then the E and H are related as:

$$E_r(f, t, u) = \Re[H(f)e^{j2\pi ft}]$$

Free space, moving antenna

- Consider a fixed antenna and a receiving antenna is moving away from the transmitting antenna with a speed v.
- The receive antenna is at a moving location described as $\mathbf{u}(t) = (r(t), \theta, \psi)$ with $r(t) = r_0 + vt$.
- The free space electric field at the moving point $\mathbf{u}(t)$ is

$$E(f, t, (r_0 + vt, \theta, \psi)) = \frac{\alpha_s(\theta, \psi, f) cos 2\pi f(t - r_0/c - vt/c)}{r_0 + vt}$$

$$f(t - r_0/c - vt/c) = f(1 - v/c)t - fr_0/c$$

- The sinusoid at frequency f has been converted to a sinusoid of frequency $-fv/c$ there has been a Doppler shift of $-fv/c$ due to the motion of the observation point.
- The electric field at a point $\mathbf{u}(t)$, is

$$E(f, t, (r_0 + vt, \theta, \psi)) = \frac{\alpha_s(\theta, \psi, f) cos 2\pi f[(1 - v/c)t - fr_0/c]}{r_0 + vt}$$

- The amount of doppler shift depends on the frequency f.

Reflecting wall, fixed antenna

- Consider a fixed antenna transmitting the sinusoid $cos 2\pi ft$, a fixed receive antenna, and a single perfectly reflecting large fixed wall as shown if Figure.
- The electromagnetic field received at the antenna is the sum of the free space field coming from the transmit antenna and a reflected wave from the wall.
- It is assumed that the presence of the receive antenna does not affect the reflected wave.
- This means that the reflected wave from the wall has the intensity of a free space wave at a distance equal to the distance to the wall and then back to the receive antenna, i.e., (2d-r) (except for a sign change).
- The electric field is given by

$$E_r(f,t) = \frac{\alpha cos 2\pi f(t - r/c)}{r} - \frac{\alpha cos 2\pi f(t - (2d - r)/c)}{2d - r}$$

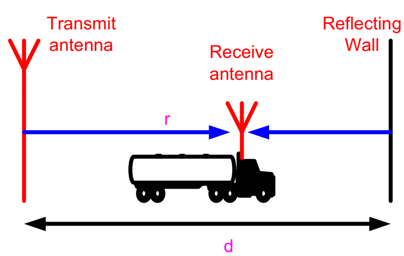


Figure 3: Illustration of a direct and Reflected path

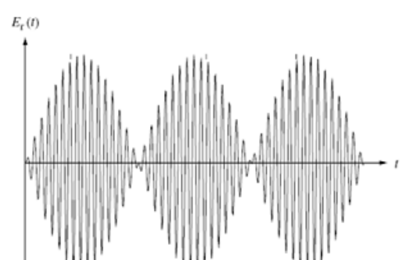
The phase difference between the two waves is

$$\Delta\theta = \left(\frac{2\pi f(2d - r)}{c} + \pi\right) - \left(\frac{2\pi fr}{c}\right) = \frac{4\pi f}{c}(d - r) + \pi$$

- When the phase difference is an integer multiple of 2π, the two waves add constructively, and the received signal is strong.
- When the phase difference is an odd integer multiple of π, the two waves add destructively, and the received signal is weak.
- As a function of r, this translates into a spatial pattern of constructive and destructive interference of the waves.
- The distance from a peak to a valley is called the **coherence distance**:

$$\Delta x_c = \frac{\lambda}{4}$$

- where $\lambda = c/f$ is the wavelength of the signal. At distances much smaller than Δx_c, the received signal at a particular time does not change appreciably.
- The distance in space over which a **fading channel** appears to be unchanged.



- The constructive and destructive interference pattern also depends on the frequency f: for a fixed r, if f changes by

$$\frac{1}{2}\left(\frac{2d-r}{c}-\frac{r}{c}\right)^{-1}$$

- from a peak to a valley.
- The **delay spread** of the channel, that is the difference between the propagation delays along the two signal paths is defined as.

$$T_d = \frac{2d-r}{c}-\frac{r}{c}$$

- The constructive and destructive interference pattern does not change appreciably if the frequency changes by an amount much smaller than $1/Td$ (coherence bandwidth).
- **Coherence bandwidth:** The range of frequencies over which the channel can be considered flat. GSM is of 200 KHz bandwidth, if coherence bandwidth is less than 200 KHz then interference will occur.

Reflecting wall, moving antenna

- Consider a receive antenna which is moving with a velocity v as shown in Figure.
- When it moves, the received signal strength increases or decreases due to the pattern of constructive and destructive interference created by the two waves.
- The phenomenon is known as multipath fading.
- The time taken to travel from a peak to a valley is $c/(4fv)$, and it is called the coherence time of the channel.

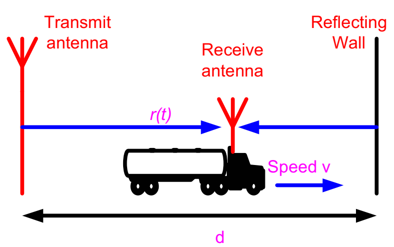


Figure 4: Illustration of a direct and Reflected path

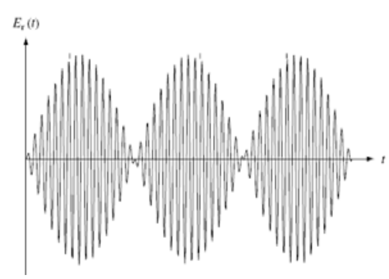


Figure 5: Received signal oscillating at frequency f with slowly varying envelope at frequency $D_s/2$

- Consider a receive antenna located at r_0 at time 0 and taking $r = r_0 + vt$.

$$E_r(f,t) = \frac{\alpha cos 2\pi f[(1 - v/c)t - r_0/c]}{r_0 + vt} - \frac{\alpha cos 2\pi f[(1 + v/c)t + (r_0 - 2d/)c]}{2d - r_0 - vt}$$

- The first term, the direct wave, is a sinusoid at frequency $f(1 - v/c)$, experiencing a Doppler shift $D_1 := -fv/c$. The second is a sinusoid at frequency $f(1 + v/c)$, experiencing a Doppler shift $D_2 := +fv/c$. The Dopper shift is positive, if the mobile is moving toward the direction of arrival of the wave and is negative, if the mobile is moving away from the direction of arrival of the wave. The Doppler spread is defined as

$$D_s = D_2 - D_1$$

- Example, Consider a mobile is moving at 60 km/h and the carrier frequency of the signal is f = 900 MHz, then the Doppler spread is

$$D_1 = fv/c = [(900x10^6)x16.66]/(3x10^8) = 49.8 \simeq 50Hz$$

$$D_s = D_2 - D_1 = 50 + 50 = 100Hz$$

- Consider a mobile is moving at 60 miles/h and the carrier frequency is f = 1850 MHz, then the Doppler spread is

$$D_1 = fv/c = [(1850x10^6)x26.82]/(3x10^8) = 49.8 \simeq 16Hz$$

$$D_s = D_2 - D_1 = 16 + 16 = 32Hz$$

- When the attenuations are roughly same for both paths, then approximate the denominator of the second term by $r = r_0 + vt$.
- Then, combining the two sinusoids,

$$E_r(f,t) = \frac{2\alpha sin 2\pi f[vt/c + (r_0 - d)] sin 2\pi f[t - d/c]}{r_0 + vt}$$

- This is the product of two sinusoids, one at the input frequency f, which is typically of the order of GHz, and the other one at $fv/c = D_s/2$, which might be of the order of 50 Hz.
- Thus, the response to a sinusoid at f is another sinusoid at f with a time-varying envelope, with peaks going to zeros around every 5 ms.
- The envelope is at its widest when the mobile is at a peak of the interference pattern and at its narrowest when the mobile is at a valley.
- Thus, the Doppler spread determines the rate of traversal across the interference pattern and is inversely proportional to the **coherence time** of the channel.
- **Coherence time:** The time duration in which channel impulse response is invariant.

$$T_c = \frac{1}{f_m}$$

- Where f_m is maximum doppler shift. If the reciprocal of the baseband of the signal is greater than the coherence time then the channel will change during the transmission of the baseband message thus causes the distortion at the receiver.

Reflection from a ground plane

- Consider a transmit and a receive antenna, both above a plane surface as shown in Figure.
- When the horizontal distance r between the antennas becomes very large relative to their height of antenna the difference between the direct path length and the reflected path length goes to zero as r^{-1} with increasing r.
- When r is large enough, this difference between the path lengths becomes small relative to the wavelength c/f .
- Since the sign of the electric field is reversed on the reflected path, these two waves start to cancel each other out.
- The electric wave at the receiver is then attenuated as r^{-2}, and the received power decreases as r^{-4}.
- This situation is important in rural areas where base-stations are placed on roads.

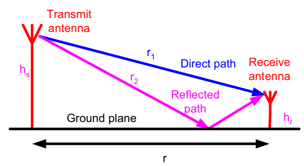


Figure 6: Illustration of a direct path and reflected path off a ground plane

$$E_r(f,t) = \frac{E_0 d_0 cos[\omega_c(t - d^{'}/c)]}{d'} + (-1)\frac{E_0 d_0 cos[\omega_c(t - d^{''}/c)]}{d''}$$

Power decay with distance and shadowing

- The reflection from a ground plane the received power can decrease with distance faster than r^{-2} in the presence of disturbances to free space.
- There are several obstacles between the transmitter and the receiver and, these obstacles might also absorb some power while scattering the remaining power.
- The empirical evidence from experimental field studies suggests that while power decay near the transmitter is r^{-2}, at large distances the power can even decay exponentially with distance.
- With a limit on the transmit power (either at the base-station or at the mobile), the largest distance between the base-station and a mobile at which communication can reliably take place is called the **coverage of the cell**.
- For reliable communication, a minimal received power level has to be met and thus the fast decay of power with distance constrains cell coverage.
- The rapid signal attenuation with distance is also helpful; it reduces the interference between adjacent cells.
- In engineering jargon, the cell is said to be capacity limited instead of coverage limited.
- The size of cells has been steadily decreased to micro cells and pico cells.
- With capacity limited cells, the inter-cell interference may be intolerably high.
- The inter-cell interference, is minimized by using the **different parts of the frequency spectrum** in neighboring cells.
- Rapid signal attenuation with distance allows frequencies to be **reused** at closer distances.

Shadowing

- The density of obstacles between the transmit and receive antennas depends very much on the physical environment.
- For example, outdoor plains have very little by way of obstacles while indoor environments pose many obstacles.
- This randomness in the environment is captured by modeling the density of obstacles and their absorption behavior as random numbers; the overall phenomenon is called shadowing.
- The effect of shadow fading differs from multipath fading in an important way.
- The duration of a shadow fade lasts for multiple seconds or minutes, and hence occurs at a much slower time-scale compared to multipath fading.

Moving antenna, multiple reflectors

- Dealing with multiple reflectors, using the technique of ray tracing, is in principle simply a matter of modeling the received waveform as the sum of the responses from the different paths rather than just two paths.
- The reflected field model is valid only at distances from the wall that are small relative to the dimensions of the wall.
- At very large distances, the total power reflected from the wall is proportional to both d^{-2} and to the area of the cross section of the wall.
- The power reaching the receiver is proportional to $[d - r(t)]^{-2}$.
- Thus, the power attenuation from transmitter to receiver (for the large distance case) is proportional to $[d - r(t)]^{-2}$ rather than to $[2d - r(t)]^{-2}$.
Scattering
- The type of reflection which occur in the atmosphere or in reflections from very rough objects is known as scattering.
- Here there are a very large number of individual paths, and the received waveform is better modeled as an integral over paths with infinitesimally small differences in their lengths.

The wireless channel as a linear time-varying system

- Consider a sinusoidal signal $\phi(t) = cos2\pi ft$ is transmitted over multipath channel and the received signal is the summation of the signal from all the paths is

$$y(t) = \sum_i a_i(f,t)\phi[t - \tau_i(f,t)]$$

- where $a_i(f,t)$ and $\tau_i(f,t)$ are **attenuation** and **propagation delay** at time t on path i.
- The overall attenuation is the product of the attenuation factors due to the antenna pattern of the transmitter and the receiver, the nature of the reflector, and a factor that is a function of the distance from the transmitting antenna to the reflector and from the reflector to the receive antenna.
- If $a_i(f,t)$ and $\tau_i(f,t)$ do not depend on frequency f, then input/output relation to an arbitrary input $x(t)$ is:

$$y(t) = \sum_i a_i(t)x[t - \tau_i(t)]$$

- The attenuation and delays for the direct path is:

$$a_1(t) = \frac{|\alpha|}{r_0 + v(t)} \quad \tau_1(t) = \frac{r_0 + v(t)}{c} - \frac{\angle\phi_1}{2\pi f}$$

- The attenuation and delays for the reflected path is:

$$a_2(t) = \frac{|\alpha|}{2d - r_0 - v(t)} \quad \tau_1(t) = \frac{2d - r_0 - v(t)}{c} - \frac{\angle\phi_2}{2\pi f}$$

- where ϕ_j is phase change at the transmitter, reflector, and receiver.
- In this case there is a phase reversal at the reflector hence $\phi_1 = 0$ and $\phi_2 = \pi$.

- Consider an impulse $x(t)$ transmitted at time $(t-\tau)$ through the channel h its input/output relationship at time t is:

$$y(t) = h(t) \otimes x(t) = \int_{-\infty}^{\infty} h(\tau, t)x(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

- The impulse response for the fading multipath channel is

$$h(\tau, t) = \sum_i a_i(t)\delta[\tau - \tau_i(t)]$$

- When the transmitter, receiver and the environment are all stationary, $a_i(t)$ and $\tau_i(t)$ do not depend on time t, then linear time-invariant channel with an impulse response

$$h(\tau) = \sum_i a_i(t)\delta[t - \tau_i]$$

- For the time-varying impulse response $h(\tau, t)$ the time varying frequency response

$$H(f, t) = \int_{-\infty}^{\infty} h(\tau, t)e^{-j2\pi f\tau}d\tau = \sum_i a_i(t)e^{-j2\pi f\tau_i(t)}$$

Baseband equivalent model

- In typical wireless applications, communication occurs in a passband $[f_c - W/2, f_c + W/2]$ of bandwidth W around a center frequency f_c.
- The processing of signals, such as coding/decoding, modulation/demodulation, and synchronization is done at the baseband.
- At the transmitter, the last stage of the operation is to "up-convert" the signal to the carrier frequency and the first step at the receiver is to "down-convert" the RF signal to the baseband.
- Therefore it is important to consider a baseband equivalent representation of the system.

- Consider a real signal $s(t)$with Fourier transform $S(f)$, band-limited in $[f_c - W/2, f_c + W/2]$ with $W < 2f_c$.
- Define its complex baseband equivalent $s_b(t)$ as the signal having Fourier transform:

$$S_b(f) = \sqrt{2}S(f + f_c) \quad f + f_c > 0$$

The signal $s(t)$ is real, its Fourier transform satisfies $S(f) = S^*(-f)$, and $s_b(t)$ contains exactly the same information as $s(t)$ and is as shown in Figure:7.

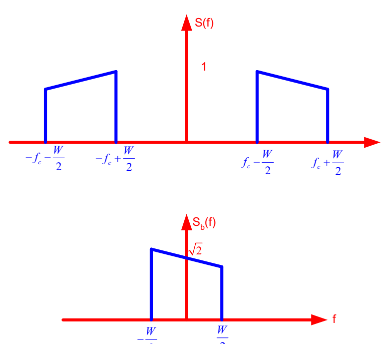


Figure 7: Illustration of passband spectrum and baseband equivalent

- The signal $s_b(t)$ is band-limited in $[-W/2, W/2]$.

$$\sqrt{2}S(f) = s_b(f - f_c) + s_b^*(-f - f_c)$$

- Taking inverse Fourier transform

$$s(t) = \frac{1}{\sqrt{2}}[s_b(t)e^{j2\pi f_c t} + s_b^*(t)e^{-j2\pi f_c t}]$$

$$= \sqrt{2}\Re[s_b(t)e^{j2\pi f_c t}]$$

- The relationship between $s(t)$ and $s_b(t)$ is shown in Figure:8.
- The passband signal $s(t)$ is obtained by modulating $\Re[s_b(t)]$ by $\sqrt{2}cos2\pi f_c t$ and $\Im[s_b(t)]$ by $-\sqrt{2}sin2\pi f_c t$ and summing, to get $\sqrt{2}\Re[s_b(t)e^{2\pi f_c t}]$(up-conversion).
- The baseband signal $\Re[s_b(t)]$ is obtained by modulating $s(t)$ by $\sqrt{2}cos2\pi f_c t$ and $\Im[s_b(t)]$ is obtained by modulating $s(t)$ by $-\sqrt{2}sin2\pi f_c t$ followed by ideal low-pass filtering at the baseband $[W/2, W/2]$ (down-conversion).

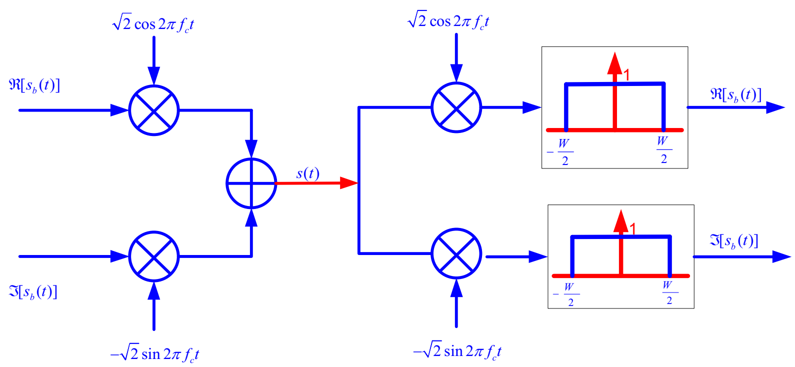


Figure 8: Illustration of up and down conversion

- Consider a multipath fading channel with impulse response given by

$$y(t) = \int_{-\infty}^{\infty} h(\tau, t)x(\tau - t)d\tau$$

- Let $x_b(t)$ and $y_b(t)$ be the complex baseband equivalents of the transmitted signal $x(t)$ and the received signal $y(t)$, respectively.
- Figure 10 shows the system diagram from $x_b(t)$ to $y_b(t)$.
- This implementation of a passband communication system is known as quadrature amplitude modulation (QAM).
- The signal $\Re[x_b(t)]$ is sometimes called the in-phase component I and $\Im[x_b(t)]$ the quadrature component Q (rotated by $\pi/2$).

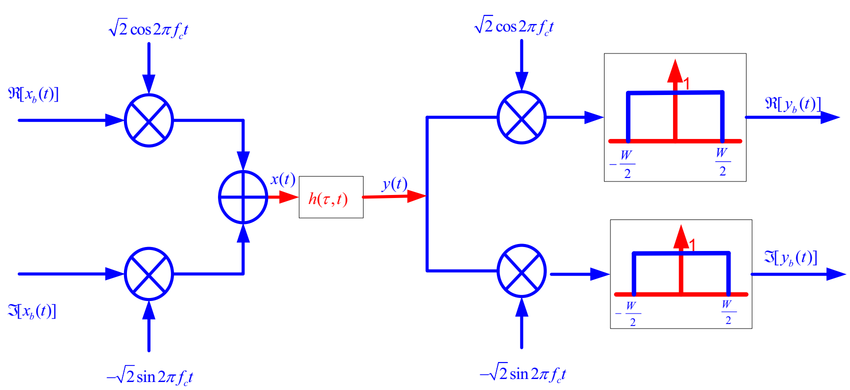


Figure 9: Illustration of up and down conversion

- The estimation of its baseband equivalent channel is as follows.

$$y(t) = \sum_i a_i(t)x[t - \tau_i(t)]$$

$$x(t) = \sqrt{2}\Re[x_b(t)e^{j2\pi f_c t}] \;\; and \qquad y(t) = \sqrt{2}\Re[y_b(t)e^{j2\pi f_c t}]$$

$$\begin{aligned} \Re[y_b(t)e^{j2\pi f_c t}] &= \sum_i a_i(t)\Re[x_b(t - \tau_i(t))e^{j2\pi f_c(t-\tau_i(t))}] \\ &= \Re\left[\left\{\sum_i a_i(t)x_b(t - \tau_i(t))e^{-j2\pi f_c \tau_i(t)}\right\} e^{j2\pi f_c t}\right] \end{aligned}$$

- Similarly,

$$\Im[y_b(t)e^{j2\pi f_c t}] = \Im\left[\left\{\sum_i a_i(t)x_b(t - \tau_i(t))e^{-j2\pi f_c \tau_i(t)}\right\} e^{j2\pi f_c t}\right]$$

- The baseband equivalent channel is

$$y_b(t) = \sum_i a_i^b(t)x_b(t - \tau_i(t))$$

- where

$$a_i^b(t) = a_i(t)e^{-j2\pi f_c \tau_i(t)}$$

The baseband equivalent impulse response is

$$h_b(\tau, t) = \sum_i a_i^b(t)\delta(t - \tau_i(t))$$

- The baseband output is the sum, over each path, of the delayed replicas of the baseband input.
- The magnitude of the ith such term is the magnitude of the response on the given path.
- The phase is changed by $\pi/2$ when the delay on the path changes by $1/(4f_c)$, or equivalently, when the path length changes by a quarter wavelength, i.e., by $c/(4f_c)$.
- If the path length is changing at velocity v, the time required for such a phase change is $c/(4f_c v)$.
- The Doppler shift D at frequency f is fv/c, and noting that $f \approx f_c$ for narrowband communication, the time required for a $\pi/2$ phase change is $1/(4D)$.
- For the single reflecting wall example, this is about 5 ms (assuming f_c = 900MHz and v = 60km/h). The phases of both paths are rotating at this rate but in opposite directions.

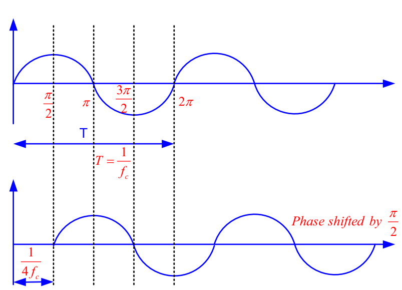


Figure 10: Sinusoidal signal phase shifted by $1/f_c$

A discrete-time baseband model

- Consider an input waveform is band-limited to W and its baseband equivalent is then limited to W/2 and can be represented as

$$x_b(t) = \sum_n x[n] sinc(Wt - n)$$

- where $x(n)$ is given by $x_b(n/W)$ and $sinc(t)$ is defined as

$$sinc(t) = \frac{sin(\pi t)}{\pi t}$$

- The baseband output is given by

$$y_b(t) = \sum_n x[n] \sum_i a_i^b(t) sinc(Wt - W\tau_i(t) - n)$$

- The sampled outputs at multiples of 1/W, $y[m] = y_b(m/W)$, are then given by

$$y[m] = \sum_n x[n] \sum_i a_i^b(m/W) sinc[m - n - \tau_i(m/W)W]$$

- Let $\ell = m - n$.

$$\begin{aligned} y[m] &= \sum_\ell x[m-\ell] \sum_i a_i^b(m/W) sinc[\ell - \tau_i(m/W)W] \\ &= \sum\nolimits_n h_\ell[m] x[m-\ell] \end{aligned}$$

- where $h_\ell[m]$ is denoted as filter tap at time m and is depend on the gains $a_i^b(t)$ of the paths, whose delays $\tau_i(t)$ are ℓ/W as shown in Figure:12.

$$h_\ell = \sum\nolimits_i a_i^b(m/W) sinc[\ell - \tau_i(m/W)W]$$

- When the gains $a_i^b(t)$ and the delays $\tau_i(t)$ of the paths are time-invariant, then the channel is linear time-invariant.

$$h_\ell[m] = \sum\nolimits_i a_i^b sinc[\ell - \tau_i W]$$

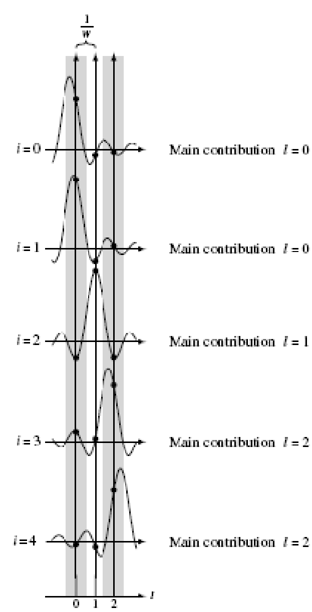


Figure 11: Plot of $h\ell[m]$

The ℓth tap can be interpreted as the sample (ℓ/W)th of the low-pass filtered baseband channel response $h_b(\tau)$ convolved with $sinc(W\tau)$.

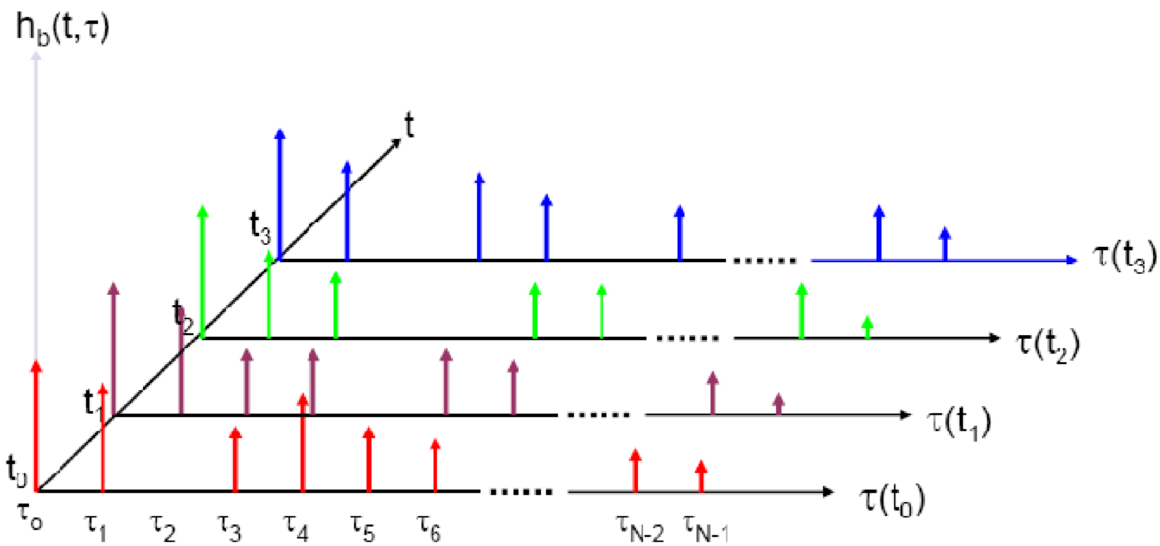


Figure 12: Discrete-Time Impulse Response Model for a Multipath Channel

- The *sampling operation* can be interpreted as *modulation and demodulation* in a communication system.
- At time n, the complex symbol $x[m]$ (in-phase plus quadrature components) modulated by the sinc pulse before the up-conversion.
- At the receiver, the received signal is sampled at times m/W at the output of the low-pass filter.

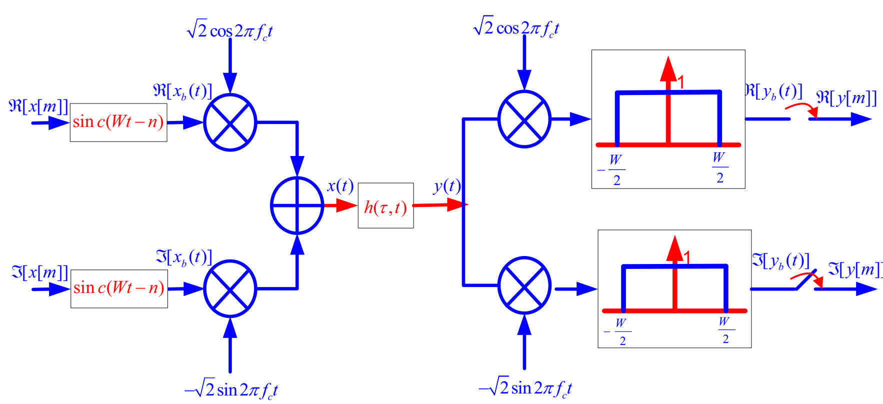


Figure 13: System diagram of transmitter and receiver.

Additive white noise

- Consider an additive noise in our input/output model.
- We make the standard assumption that $w(t)$ is zero-mean additive white Gaussian noise (AWGN) with power spectral density $N_0/2$ (i.e., $E[w(0)w(t) = (N_0/2)\delta(t)$.
-

$$
\begin{aligned}
y(t) &= \sum_i a_i(t)x(t-\tau_i(t)) + w(t) \\
y[m] &= \sum_l h_l[m]x[m-l] + w[m]
\end{aligned}
$$

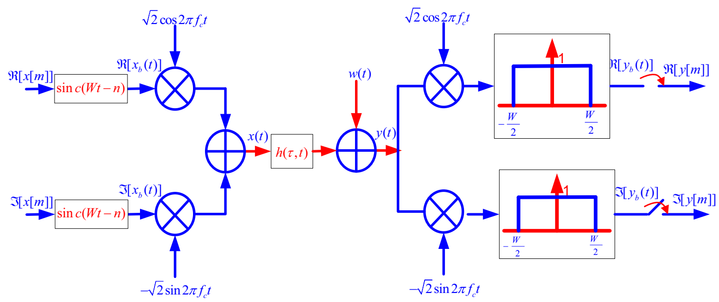


Figure 14: A complete system diagram.

- where w[m] is the low-pass filtered noise at the sampling instant m/W.
- The white noise w(t) is down-converted, filtered at the baseband and ideally sampled.

$$
\begin{aligned}
\Re(w[m]) &= \int_{-\infty}^{\infty} w(t)\psi_{m,1}(t)dt \\
\Im(w[m]) &= \int_{-\infty}^{\infty} w(t)\psi_{m,2}(t)dt
\end{aligned}
$$

- where

$$
\begin{aligned}
\psi_{m,1}(t) &= \sqrt{2W}cos(2\pi f_c t)sinc(Wt-m) \\
\psi_{m,2}(t) &= -\sqrt{2W}sin(2\pi f_c t)sinc(Wt-m)
\end{aligned}
$$

Doppler spread and coherence time

The channel parameter is the time-scale of the variation of the channel. The variation of the channel parameter i.e., taps $h_\ell[m]$ as a function of time m defined as

$$h_\ell[m] = \sum_i a_i(m/W)e^{-j2\pi f_c \tau_\ell(m/W)} sinc[\ell - \tau_\ell(m/W)W]$$

- From this expression the significant changes in a_i occur over periods of seconds.
- Significant changes in the phase of the ith path occur at intervals of $1/(4D_i)$, where $D_i = f_c\tau_i^{'}(t)$ is the Doppler shift for that path.
- Due to the different Doppler shifts from different paths will have significant changes in the magnitude of $h_\ell[m]$.
- The Doppler spread D_s is defined as the largest difference between the Doppler shifts:

$$D_s = max\ f_c|\tau_i^{'}(t) - \tau_j^{'}(t)|$$

- The Doppler spread D_s is a measure of spectral broadening caused by motion.
- If the baseband signal bandwidth is much greater than D_s then the effect of Doppler spread is negligible at the receiver.

Coherence Time

The coherence time T_c of a wireless channel is defined as the interval over which $h_\ell[m]$ changes significantly as a function of m.

$$T_c = \frac{1}{4D_s}$$

- If the symbol period of a baseband signal is *greater* than the coherence time, then the signal will distort, since channel will change during the transmission of the signal.
- Coherence time definition implies that two signals arriving with a time separation greater than T_c are affected differently by the channel.
- If the coherence time is defined as the time over which the time correlation function is above 0.5, then it is approximated as

$$T_c = \frac{9}{16f_m} = \frac{0.179}{f_m}$$

where f_m = maximum Doppler shift and is given by $f_m = f_{d,\ max} = \frac{v}{\lambda} = v\frac{f}{c}$

- The channels are categorized as *fast fading* and *slow fading*.
- The channel is fast fading if the coherence time T_c is much shorter than the delay requirement of the application, and slow fading if T_c is longer.
- A channel is fast or slow fading depends not only on the environment but also on the application.
- For example in voice communication, typically has a short delay requirement of less than 100 ms, while some types of data applications can have a laxer delay requirement

Delay spread and coherence bandwidth

- The multipath delay spread, T_d, is defined as the difference in propagation time between the longest and shortest path, counting only the paths with significant energy.

$$T_d = max|\tau_i^{'}(t) - \tau_j^{'}(t)|$$

- In cellular communication coverage distance is of few kilometers or less, hence path difference is less than 300 to 600 meters and this makes the path delays are of one or two microseconds.
- As cells become smaller due to increased cellular usage, T_d also shrinks.
- The delay spread T_d is much smaller than the coherence time T_c.
- The bandwidths of cellular systems range between several hundred kilohertz and several megahertz, and thus, for the above multipath delay spread, all the path delays lie within the peaks of two or three sinc functions.
- Adding a few extra taps to each channel filter because of the slow decay of the sinc function, the cellular channels can be represented with at most four or five channel filter taps.

- The delay spread of the channel dictates its frequency coherence.
- Wireless channels change both in time and frequency.
- The time coherence shows us how quickly the channel changes in time, and similarly, the frequency coherence shows how quickly it changes in frequency.
- The frequency response at time t is

$$H(f,t) = \sum_i a_i(t)e^{-j2\pi f \tau_i(t)}$$

- The contribution due to a particular path has a phase linear in f.
- For multiple paths, there is a differential phase, $2\pi f(\tau_i(t) - \tau_k(t))$
- This differential phase causes selective fading in frequency.
- This says that $E_r(f,t)$ changes significantly, not only when t changes by $1/(4D_s)$, but also when f changes by $1/(2D_s)$. The coherence bandwidth, Wc, is given by

$$W_c = \frac{1}{2T_d}$$

- The **coherence bandwidth** is reciprocal to the multipath spread.
- When the bandwidth of the input is less than W_c, the channel is referred as *flat fading*.
- In this case, the delay spread T_d is much less than the symbol time 1/W, and a single channel filter tap is sufficient to represent the channel.
- When the bandwidth is much larger than W_c, the channel is said to be **frequency-selective**, and it has to be represented by multiple taps.
- The flat or frequency-selective fading is depends the relationship between the bandwidth W and the coherence bandwidth T_d

Table 1: physical parameters of the channel

Key channel parameters and time-scales	Symbol	Representative values
Carrier frequency	f_c	1 GHz
Communication bandwidth	W	1 MHz
Distance between transmitter and receiver	d	1 km
Velocity of mobile	v	64km/h
Doppler shift for a path	$D = f_c(v/c)$	50 Hz
Doppler spread of paths corresponding to a tap	D_s	100 Hz
Time-scale for change of path amplitude	d/v	1 minute
Time-scale for change of path phase	$1/(4D)$	5 ms
Time-scale for a path to move over a tap	$c/(vW)$	20 s
Coherence time	$T_c = 1/(4D_s)$	2.5 ms
Delay spread	T_d	1 μs
Coherence bandwidth	$W_c = 1/2T_d$	500 kHz

Table 2: Types of wireless channels

Types of channel	Defining characteristic
Fast fading	$T_c \ll$ delay requirement
Slow fading	$T_c \gg$ delay requirement
Flat fading	$W \ll W_c$
Frequency-selective fading	$W \gg W_c$
Underspread	$T_d \ll T_c$

Modeling philosophy

- All analytical work is done with simplified models, like white Gaussian noise is often assumed in communication models but the model is valid only for small frequency bands.
- Doppler spread, multipath spread, etc. are defined for wireless channel with probabilistic models, but these channels are very different from each other and cannot be characterized by probabilistic models.
- Consider a continuous time multipath fading channel defined as

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t)) + w(t)$$

- The discrete-time baseband model in terms of channel filter taps is

$$y[m] = \sum_\ell h_\ell[m]x[m - \ell] + w[m]$$

- where $h_\ell[m]$

$$h_\ell[m] = \sum_i a_i(m/W)e^{-j2\pi f_c \tau_\ell(m/W)} sinc[\ell - \tau_\ell(m/W)W]$$

Rayleigh Fading Distribution

The delays associated with different signal paths in a multipath fading channel change in an unpredictable manner and can only be characterized statistically. When there are a large number of paths, the central limit theorem can be applied to model the time-variant impulse response of the channel as a complex-valued Gaussian random process. When the impulse response is modeled as a zero mean complex-valued Gaussian process, the channel is said to be a Rayleigh fading channel.

- In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component
- The envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution
- The simplest probabilistic model for the channel filter taps is based on the assumption that there are a large number of statistically independent reflected and scattered paths with random amplitudes in the delay window corresponding to a single tap.

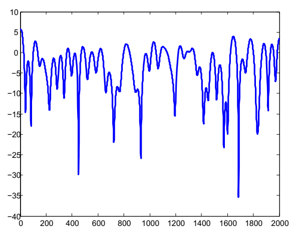


Figure 15: Rayleigh distribution.

- The phase of the ith path is $2\pi f c\tau_i$ modulo 2π. It is assumed that the phase for each path is uniformly distributed between 0 and 2π and that the phases of different paths are independent.

$$f_c\tau_i = d_i/\lambda$$

- where d_i is the distance traveled by the ith path and λ is the carrier wavelength.
- Since the reflectors and scatterers are far away relative to the carrier wavelength, i.e., $d_i \gg \lambda$,
- It is assumed that the phase for each path is uniformly distributed between 0 and 2π and that the phases of different paths are independent.

- The contribution of each path in the tap gain $h_\ell[m]$ is

$$h_\ell[m] = \sum_i a_i(m/W)e^{-j2\pi f_c \tau_i(m/W)} sinc[\ell - \tau_i(m/W)W]$$

- It follows that $\Re(h_\ell[m])$ is the sum of many small independent real random variables, and so by the Central Limit Theorem, it can reasonably be modeled as a zero-mean Gaussian random variable.
- Similarly, because of the uniform phase,$\Re(h_\ell[m]e^{j\phi})$ is Gaussian with the same variance for any fixed ϕ.
- The magnitude $h_\ell[m]$ of the ℓth tap is a Rayleigh random variable with probability density function (pdf) is given by

$$p(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) \qquad x \geq 0$$

- where x is the envelope amplitude of the received signal
- $\sigma =$ rms value of the received voltage signal before envelop detection
- σ^2 = time-average power of the received signal before envelop detection

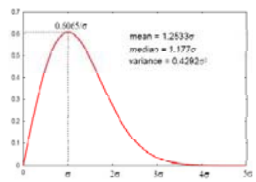


Figure 16: Rayleigh distribution.

Ricean Distribution

- When there is a dominant stationary (non-fading) signal component present, such as a *line-of-sight* propagation path, the small-scale fading envelope distribution is *Ricean*
- The pdf of the Ricean distribution is given by

$$p(x) = \frac{x}{\sigma^2} \exp\left(-\frac{x^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{Ax}{\sigma^2}\right) \qquad A \geq 0,\ x \geq 0$$

- where $A=$ peak amplitude of the dominant (LOS) signal
- $I_0(.)$ = modified Bessel function of the first kind and zero-order
- $k = \frac{A^2}{2\sigma^2}$
- Along with line-of-sight propagation (specular) path and if there are also a large number of independent paths.
- In this case $h_l[m]$ at least for one value of l can be modeled as

$$h_\ell[m] = \sqrt{\frac{k}{k+1}}\sigma_l e^{j\theta} + \sqrt{\frac{1}{k+1}} CN(0, {\sigma_l}^2)$$

- The first term corresponding to specular path and second term corresponding to the aggregation of the large number of reflected and scattered paths.
- The parameter k is the ratio of the energy in the specular path to the energy in the scattered paths.

Tap gain auto-correlation function

- Modeling each $h_\ell[m]$ as a complex random variable provides the statistical description.
- A statistical quantity that models the rate of *channel* variation is known as the tap gain *auto-correlation* function, $R_\ell[n]$. It is defined as

$$R_\ell[n] = \mathcal{E}\{h_\ell[m]h_\ell[m+n]\}$$

- For each tap ℓ, $R_\ell[n]$ gives the auto-correlation function of the sequence of random variables assuming that this is not a function of time m.
- $h_\ell[m]$ is independent of $h_\ell[m^{'}]$ for all $\ell = \ell^{'}$ and all $m, m^{'}$. This final assumption is intuitively plausible since paths in different ranges of delay contribute to $h_\ell[m]$ for different values of ℓ.
- The coefficient $R_\ell[0]$ is proportional to the energy received in the ℓth tap.
- The multipath spread T_d can be defined as the product of 1/W times the range of ℓ which contains most of the total energy $\sum_{\ell=0}^{\infty} R_\ell[0]$
- The coherence time T_c defined as the smallest value of $n > 0$ for which $R_\ell[n]$ is significantly different from $R_\ell[0]$.

Clarkes model

- This is a popular statistical model for flat fading.
- The transmitter is fixed, the mobile receiver is moving at speed v, and the transmitted signal is scattered by stationary objects around the mobile.
- There are K paths, the ith path arriving at an angle $\theta_i : 2\pi i/K, \quad i = 0 \ldots K-1$, with respect to the direction of motion.
- The scattered path arriving at the mobile at the angle θ has a delay of $\tau_\theta(t)$ and a time invariant gain $a_\theta(t)$, and the input/output relationship is given by

$$y(t) = \sum_{i=0}^{K-1} a_{\theta_i} x(t - \tau_{\theta_i}(t))$$

- The received power distribution $p(\theta)$ and the antenna gain pattern $\alpha(\theta)$ are functions of the angle θ
- It is assumed as uniform power distribution and isotropic antenna gain pattern, i.e., the amplitudes $a_\theta = a/\sqrt{K}$ for all angles θ.
- The details of the description is as shown in Figure 17 known as the **one ring model**.
- The amplitude of each path scaled by $\sqrt{K}$ so that the total received energy along all paths is a^2.

- When the communication bandwidth W is much smaller than the reciprocal of the delay spread, then the complex baseband channel can be represented by a single tap at each time:

$$y[m] = h_0[m]x[m] + w[m]$$

- The phase of the signal arriving at time 0 from an angle θ is $2\pi f_c \tau_\theta(0) \bmod 2\pi$, where f_c is the carrier frequency.
- Making the assumption that this phase is uniformly distributed in $[0, 2\pi]$ and independently distributed across all angles θ, the tap gain process $h_l[m]$ is a sum of many small independent contributions, one from each angle.
- By the Central Limit Theorem, it is modeled the process as Gaussian with an autocorrelation function $R_0[n]$ as shown in Figue 18given by:

$$R_0[n] = 2a^2\pi J_0(n\pi D_s/W)$$

where $J_0(.)$ is the zeroth-order Bessel function of the first kind

$$J_0(x) = \int_0^\pi e^{jxcos\theta} d\theta$$

and $D_s = 2f_c v/c$ is the Doppler spread. The power spectral density $S(f)$, defined on [-1/2, +1/2], is given by

$$S(f) = \frac{4a^2 W}{D_s\sqrt{1-(2fW/D_s)^2}}$$

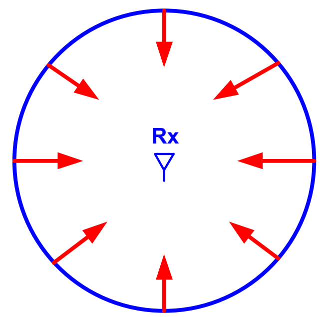


Figure 17: The one-ring model.

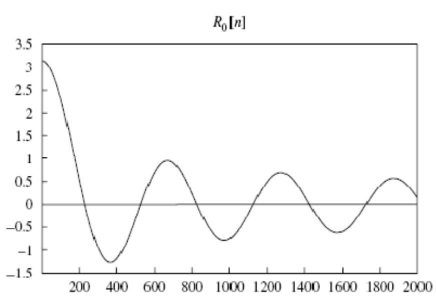


Figure 18: Auto-correlation function

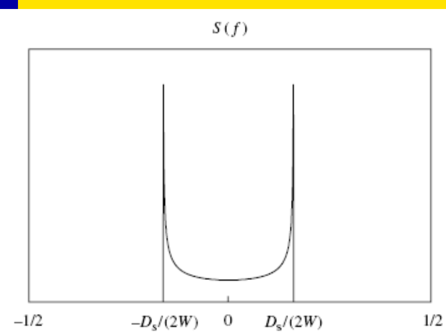


Figure 19: Doppler spectrum

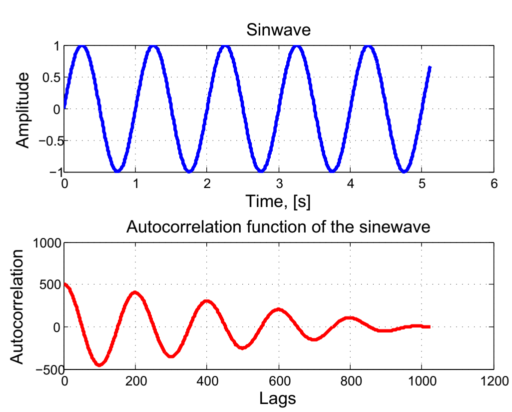


Figure 20: Auto-correlation function

For the following data calculate doppler shift
fc 1850 MHz, 60mile/hour (mph)

Solution

$\lambda = c/f_c = 3 \times 10^8/1850 \times 10^6 = 0.162m$

$v = 60mph = 26.82m/s$

The mobile is moving toward the transmitter, fd = 26.82 / 0.162 = 1850.0 Hz

The mobile is moving away the transmitter, fd = - 1850.0 Hz

$Tc = 9/16fm = 2.22ms$

If a digital transmission is used, max. symbol rate $Rc = 1/T_c = 454\ bps$.

Distortion could result from multipath time delay spread

Using the practical rule, $Tc = 0.423/fm = 6.77ms$, max. symbol rate $Rc = 1/Tc = 150bps$

An aircraft is heading towards a control tower with 500 kmph, at an elevation of 20°. Communication between aircraft and control tower occurs at 900 MHz. Find out the expected Doppler shift.

Solution

v = 500 kmph

The horizontal component of the velocity is

$v^{'} = vcos\theta = 500 \times cos20. = 130m/s$

Hence, it can be written that $\lambda = \frac{900\times10^6}{3\times10^8} = 13m$

$fd = \frac{130}{1/3} = 390Hz$

If the plane banks suddenly and heads for other direction, the Doppler shift change will be 390Hz to -390Hz.

References

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